Studies on Vibration of Beams with Acoustic Black Hole

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Introduction

An Acoustic Black Hole (ABH) is the location where vibration energy is concentrated and is used as a passive technique to control vibration.

Usually ABH is a power-law taper profile due to which the wave velocity gradually reduces to zero.

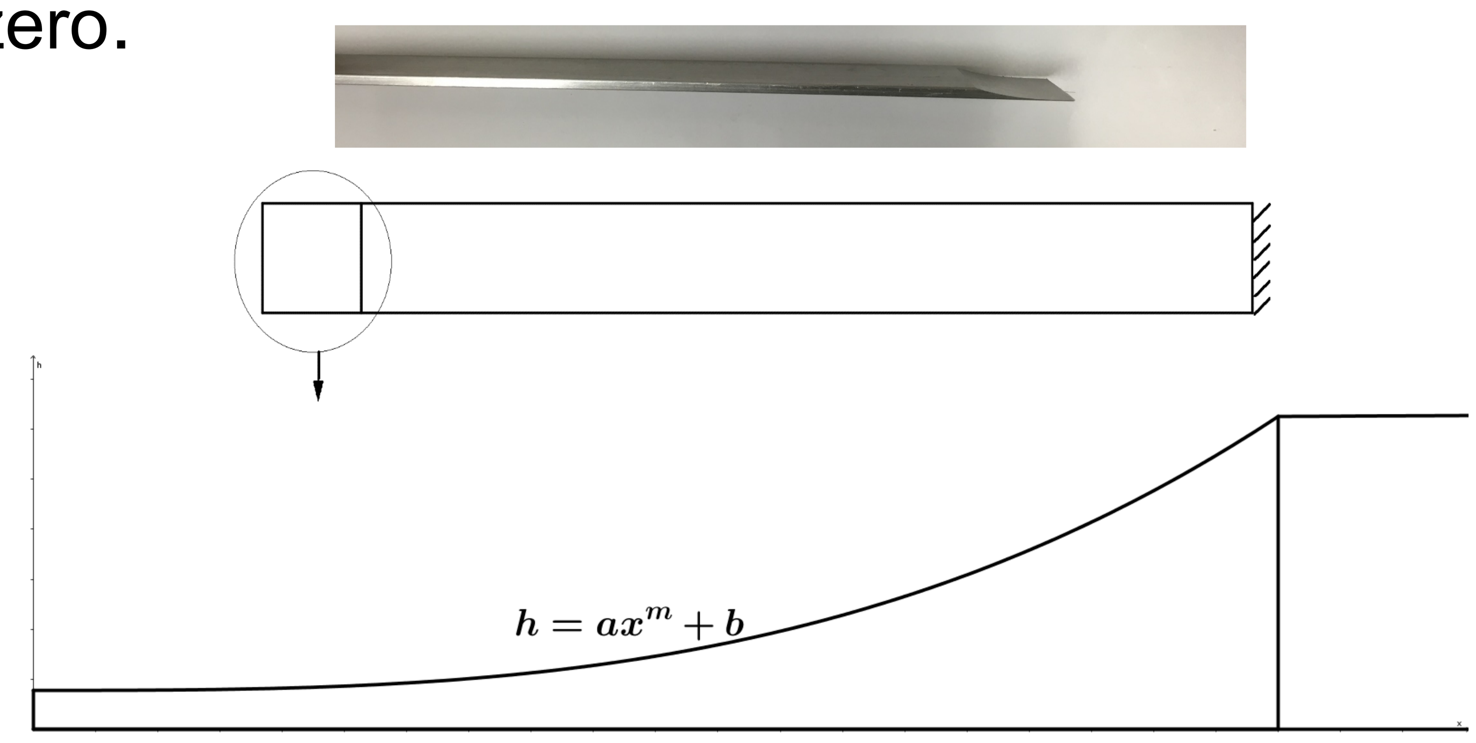

Figure 1. Cantilever Beam with power law profile tapered end

Theoretical Analysis

The equation of the power-law curve is

$$h(x) = ax^m + \mathrm{b}$$

where m is a positive rational number and a is a constant.

The phase velocity C_p and group velocity C_g

$$C_p = \sqrt[4]{\frac{E}{12\rho(1-\nu^2)}}\sqrt{\omega(ax^m + b)}$$

$$C_g = \sqrt[4]{\frac{4E}{3\rho(1-\nu^2)}}\sqrt{\omega(ax^m + b)}$$

E is the Young's modulus,
v is the Poisson's ratio,
ρ is the density,
h is the thickness of the plate
ω is the angular frequency of flexural wave.

When $b = 0$ and $x \to 0$, the phase velocity and group velocity tends to zero, which indicate that the wave will be concentrated at the ABH location.

Simulation of Beam Vibration

Several Aluminum beams which are 10 inch long, 1 inch wide and 1/8 inch thick, with various power-law profile tapered at the free end with the residual thickness b equal to 1/64 inch

The vibration and resulting near field sound radiation results are shown.

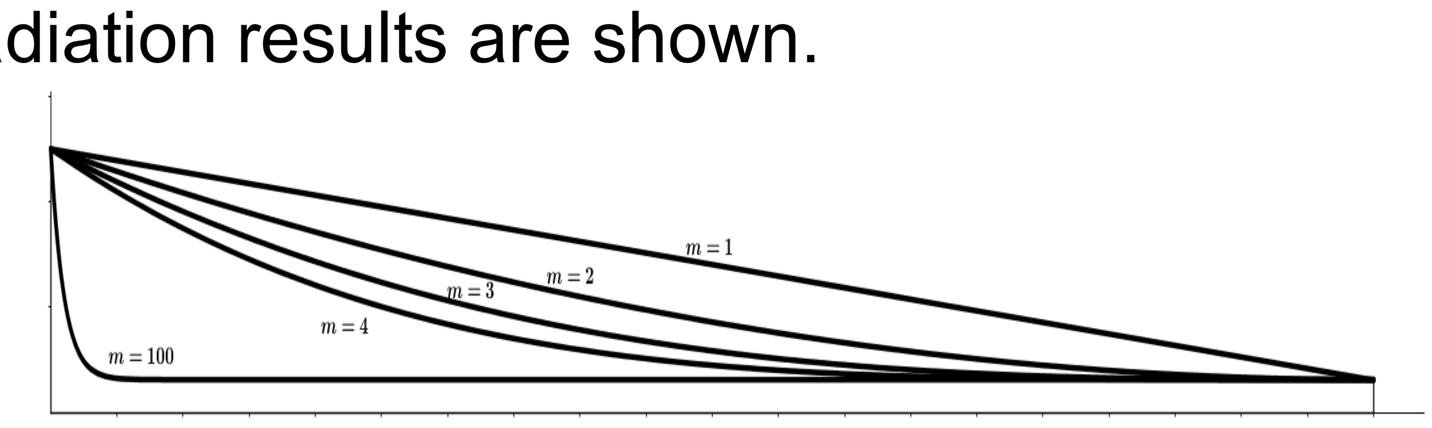

Figure 2. power-law profile with different m values

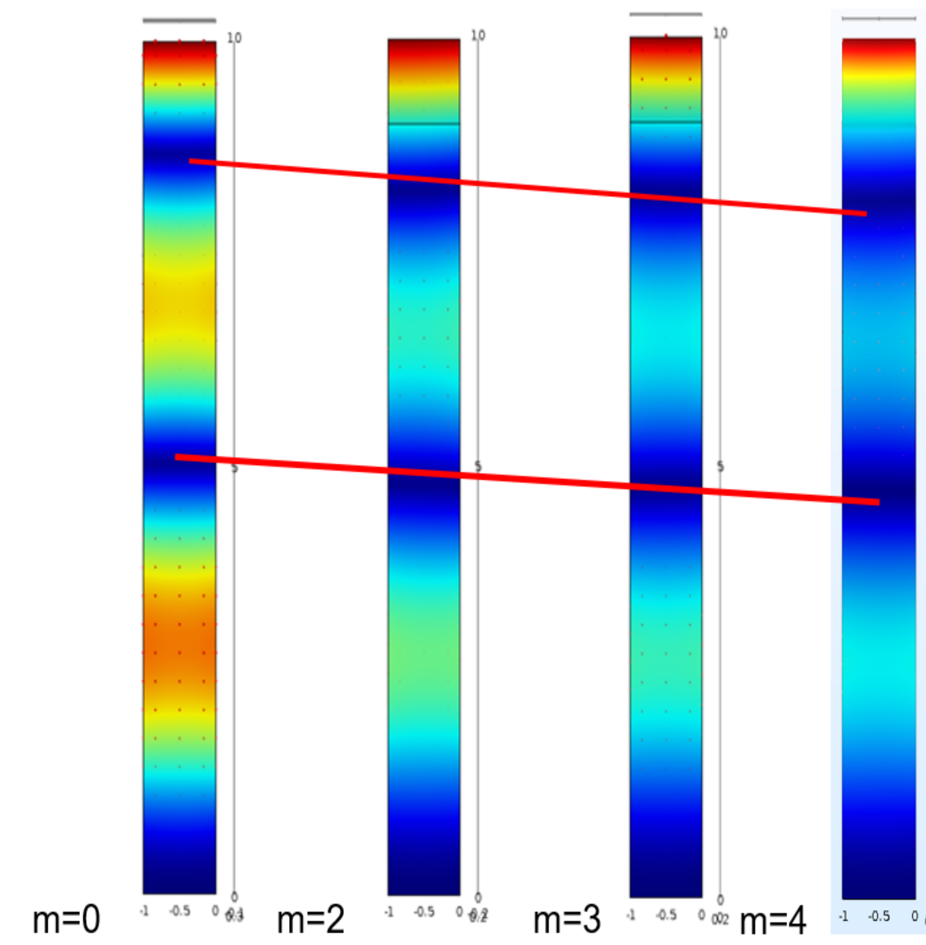

Figure 3. The third vibration mode of normal cantilever beam and beam with ABH with different m values

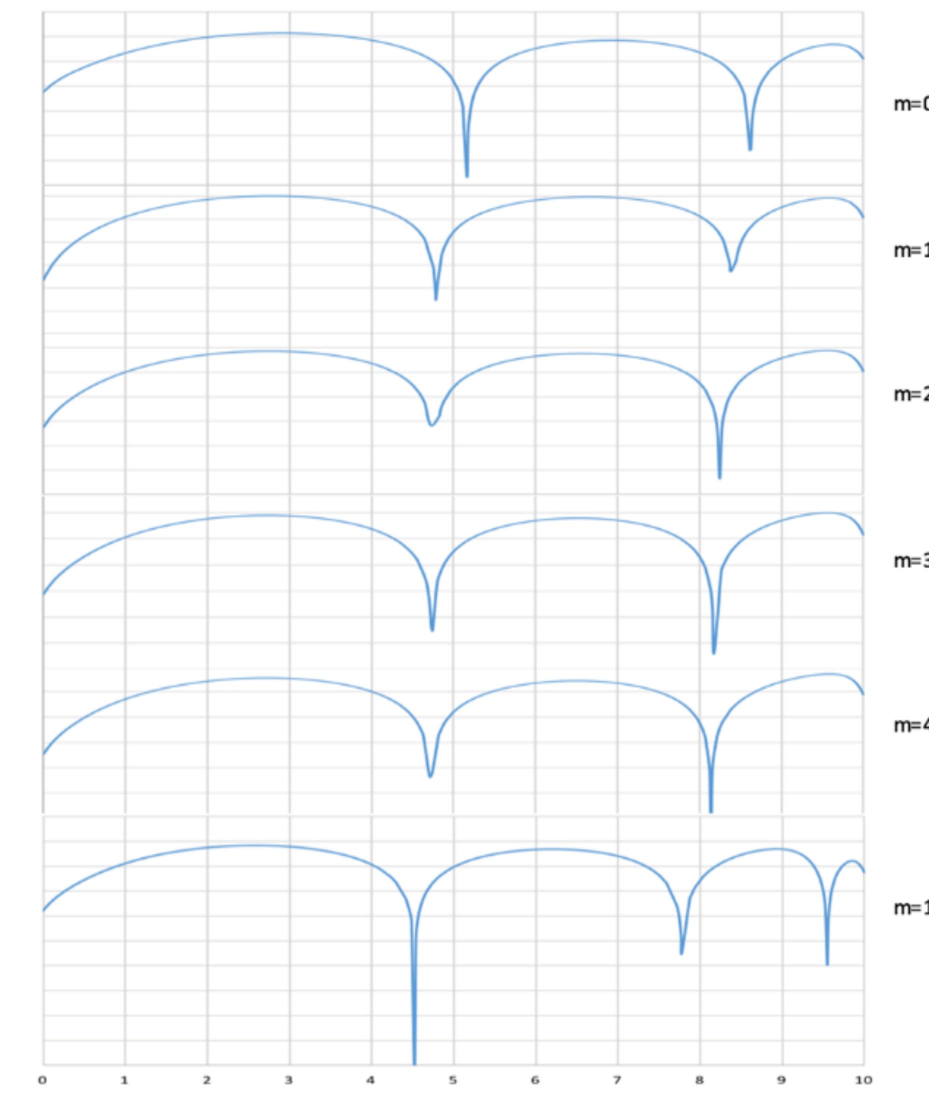

Figure 4. The acoustic near field for various m values

Table 1. The nodes locations and natural frequency for the third mode of various of m values

m value	node 1 location x_1/L	node 2 location x_2/L	Natural Frequency(Hz)
0	0.5	0.863	715.0
2	0.475	0.825	791.3
3	0.478	0.813	802.5
4	0.474	0.811	806.5

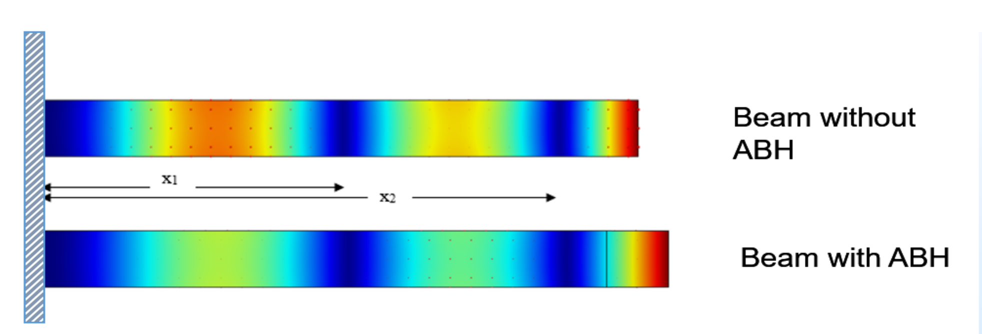

Figure 5. The third mode shape of normal cantilever beam with length correction and with ABH with 900 Hz excitation

Table 3: The peak values for 900 Hz excitation

Peak Value	Beam w/o ABH with length correction 9.4 in	Beam with ABH 10 in
1	0.0019 in	0.0018 in
2	0.0021 in	0.0019 in
Free End	0.0024 in	0.0031 in

Experimental Results

Figure 6. Experiment setup

Table 4: Experimental results of Beam without ABH with correction and the Beam with ABH under 900 Hz excitation

Peak Value (mV)	Beam w/o ABH	Beam with ABH
1	420	405
2	427.5	412.5
Free End	435	465

Table 5. The nodes locations of third mode

Beam w/o ABH	Numerical	Experimental
x_1/L	0.87	0.84
x_2/L	0.51	0.43

Beam with ABH m=2	Numerical	Experimental
x_1/L	0.825	0.8
x_2/L	0.475	0.42

Conclusions: The simulation and experimental results show that the node locations move towards the fixed end with the increase of m value. Also vibration increases at the location of ABH and decreases at the other two anti-node locations, due to energy conservation. The result show that the vibration energy is concentrated at the location of ABH.

References:

1. V.B. Georgiev, J. Cuenca, F. Gautier, L. Simon, V.V. Krylov, "Damping of structural vibrations in beams and elliptical plates using the acoustic black hole effect", Journal of Sound and Vibration 330(11): 2497–2508, (2011).
2. L.Zhao, F. Semperlotti, "Multifunctional Structures for Concurrent Passive Vibration Control and Energy Harvesting Based on Embedded Acoustic Black Holes", NoiseCon, September 2014, Fort Lauderdale, FL,(2014).
3. Chenhui Zhao and M.G. Prasad. "Studies on sound radiation from Beam with Acoustic Black Hole", NoiseCon 2016, Providence, RI, (2016)

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