Modelling of Viscoelastic Phenomena in Concrete Structures

A.A. Pomarico, G. Roselli (STMicroelectronics-Lecce)
D. Caltabiano (STMicroelectronics-Agrate Brianza)
Outline

• Introduction
  • Modelling sensors in concrete
  • Viscoelastic modelling of concrete
  • Shrinkage strain modelling of concrete
  • Analysis of a sensor in concrete
  • Conclusions
Introduction

Market demand:

**Structural Health Monitoring (SHM):**
Understand the health of the structure and the needs for maintenance intervention

Monitoring the pressure in various strategic points of the structure and its evolution over time

*OUR OBJECTIVE:*

Develop a *method* that allows us to model *concrete mechanical properties and their variation over time*, and to model the behaviour of a *mechanical sensor embedded in it* when external forces are applied to the concrete structure.

• Introduction

• Modelling sensors in concrete

• Viscoelastic modelling of concrete

• Shrinkage strain modelling of concrete

• Analysis of a sensor in concrete

• Conclusions
COMSOL for modelling sensors in concrete environments

For a reliable design of an electromechanical sensor in a concrete structure:

- Appropriate modelling of main concrete properties that can impact on an embedded sensor response
- Modelling of the sensor features
- Modelling of the total system and of the combined effect of an external force and of the concrete-induced effects on the sensor
Introduction

Modelling sensors in concrete

Viscoelastic modelling of concrete

Shrinkage strain modelling of concrete

Analysis of a sensor in concrete

Conclusions
Concrete - Creep

- **Creep** is the property of materials by which they continue deforming over considerable length of time under sustained stress.
- **Concrete** under stress undergoes creep (gradual increase of strain with time)
- In concrete, creep deformations are generally larger than elastic deformation and thus creep represents an important factor affecting the deformation behavior.

Concrete under constant axial compressive stress

MIT: Mechanics and design of Concrete structures. 2004
Concrete viscoelasticity is classically **described by means of the Creep Function**, representing the stress dependent strain per unit stress.

Extract from **ModelCode2010**:

Unless special provisions are given the relations are valid for ordinary structural concrete (15 MPa ≤ $f_{cm}$ ≤ 130 MPa) subjected to a compressive stress $|\sigma| \leq 0.4 f_{cm}(t_0)$ at an age at loading $t_0$ and exposed to mean relative humidities in the range of 40 to 100 % at mean temperatures from 5 °C to 30 °C. The age at loading should be at least 1 day.

$f_{cm}$ is the mean compressive strength in [MPa] at an age of 28 days.

The stress dependent strain at time $t$ (in days) may be expressed as:

$$\varepsilon_{c\sigma}(t,t_0) = \sigma_c(t_0) \left[ \frac{1}{E_{ci}(t_0)} + \frac{\varphi(t,t_0)}{E_{ci}} \right] = \sigma_c(t_0) J(t,t_0)$$

The Creep Function $J(t,t_0)$ of concrete can be calculated according to the concrete equation theory, for given **concrete class**, **sample size** and **humidity conditions**, and after specifying the **concrete age at the loading instant**.
Modelling viscoelasticity in concrete (2/2)

**STRATEGY:** creep in concrete can be more easily analysed modelling the material by means of a so-called **Kelvin chain**

![Kelvin chain diagram](image)

**Kelvin chain parameters** for Concrete Modelling can be obtained by means of appropriate fitting of the Creep curves.

**“Creep function”** for a loading instant $\tau$:

$$ J(t - t_0) = \frac{1}{G_0} + \sum_{i=1}^{n} \frac{1}{G_i} \left[ 1 - e^{-\frac{t-t_0}{\tau_i}} \right] $$

$$ \tau_i = \frac{\eta_i}{G_i} $$

retardation time per branch (for each branch, estimates the time required for the creep process to approach completion)

We built a **new mathematical model for Viscoelasticity**, exploiting Equation-based Modelling capabilities of COMSOL.
Kelvin chain model of viscoelasticity in COMSOL (1/2)

We built in COMSOL a **new mathematical model for Viscoelasticity**, exploiting COMSOL **Equation-based Modelling** capabilities:

Implementation of Kelvin chain model by introducing differential equations in a “Linear Elastic Material” framework:
Kelvin chain model of viscoelasticity in COMSOL (2/2)

Viscoelastic material is described as a domain obeying a set of equations, and all the material parameters are manually introduced as “Variables”:

Values used for Kelvin chain:

(Values had been obtained by fitting of the exact creep function calculated for a given creep sample, using Kelvin-chain parametric equation)
The model has been applied to a cylinder (with given concrete material parameters), L=20cm, R=5cm, with an applied load of 1MPa.
The viscoelastic material can be only a portion of the modelled structure. Other domains, having no viscoelastic behaviour, can be built in the same COMSOL file.

EXAMPLE: model consisting of the same concrete cylinder, with an alumina (not viscoelastic) concentric cylinder inside. A constant 1MPa was applied on top of the concrete:

The time dependence of the strain in alumina is consistent with the time-variable stress on the top of alumina:
• Introduction

• Modelling sensors in concrete

• Viscoelastic modelling of concrete

• **Shrinkage strain modelling of concrete**

• Analysis of a sensor in concrete

• Conclusions
Concrete shrinkage theory in ModelCode

Total shrinkage in concrete structures:

$$\varepsilon_{cs}(t, t_s) = \varepsilon_{cas}(t) + \varepsilon_{cds}(t, t_s)$$

where shrinkage is subdivided into the autogenous shrinkage $$\varepsilon_{cas}(t)$$:

$$\varepsilon_{cas}(t) = \varepsilon_{cas0}(f_{cm}) \cdot \beta_{as}(t)$$

and the drying shrinkage $$\varepsilon_{cds}(t, t_s)$$:

$$\varepsilon_{cds}(t, t_s) = \varepsilon_{cds0}(f_{cm}) \cdot \beta_{RH}(RH) \cdot \beta_{ds}(t - t_s)$$

**MAIN PARAMETERS INFLUENCING THE SHRINKAGE:**

- t is the concrete age (in days)
- $$t_s$$ is the **concrete age at the beginning of drying** (in days)
- (t-$$t_s$$) is the **duration of drying** (in days)
- the coefficient $$\beta_{RH}(RH)$$ takes into account the effect of the **ambient relative humidity RH**
- the function $$\beta_{ds}(t-t_s)$$ describing the time-development, is a function of the **notional size h of the sample**

In COMSOL, shrinkage has been modelled as a thermal contraction, introducing an effective thermal variation $$\Delta T$$. 
Shrinkage strain in concrete

The model has been applied to a cylinder (with concrete material parameters), L=20cm, R=5cm.

No load applied.
ONLY SHRINKAGE STRAIN
maximum strain \(\sim-7e^{-4}\)
• Introduction
• Modelling sensors in concrete
• Viscoelastic modelling of concrete
• Shrinkage strain modelling of concrete
• Analysis of a sensor in concrete
• Conclusions
Analysis of a sensor in a concrete environment

Pressure Sensor structure (a Silicon membrane) in a concrete sample:

- Time: 3.1968E7 s
- Surface: Displacement field, Z component (µm)

10 MPa load applied on top of the concrete sample

Radius: 2mm
Height: 600µm

Thickness: 10µm
Cavity depth: 50µm

Membrane:
Radius: 700µm
Cavity depth: 50µm
Modelling results: creep effects (1/2)

370 days time span for the time-dependent simulation
ONLY CREEP considered (no shrinkage strain)
10MPa CONSTANT EXTERNAL LOAD APPLIED

First observation: Modification in the membrane displacement at its centre over time (assuming the membrane edge as a reference)

Initial value: ~ -0.42 µm
Final value: ~ -0.87 µm

Membrane effective deformation has more than doubled its value

Correlation with a change in the stress state of the membrane?
Modelling results: creep effects (2/2)

Modifications in the membrane radial and angular stress distributions:

RADIAL STRESS:

ANGULAR STRESS:

If piezoresistors are fabricated on the membrane:

*Creep-induced time dependence of the stress will be observed in piezoresistors.*

(relative weight of the two components depending on the actual position of piezoresistors)
Modelling results: adding the shrinkage effect

Only a very small additional modification in the membrane displacement at its centre over time (from –0.42 µm to -0.9 µm)

BUT: Relevant changes are also in this case observed in the stress distributions in the membrane:

RADIAL STRESS:  
ANGULAR STRESS:

If piezoresistors are fabricated on the membrane:
Both creep-induced and shrinkage-induced time dependence of the stress will be observed in piezoresistors.

Time dependent output voltage of the sensor
• Introduction
• Modelling sensors in concrete
• Viscoelastic modelling of concrete
• Shrinkage strain modelling of concrete
• Analysis of a sensor in concrete

• Conclusions
Conclusions

• Concrete creep was modelled by means of a Kelvin-chain model approach exploiting the Equation Based Modelling of COMSOL

• Shrinkage strain of concrete was modelled using an equivalent thermal contraction

• Both viscoelastic creep and shrinkage strain are critical phenomena to take into account for the design of reliable sensors for concrete.
Acknowledgments

STMicroelectronics

Prof. Gabriele Bertagnoli (Politecnico di Torino)

COMSOL

THANK YOU!!!

Anna Pomarico