Modeling of Ultrasonic Near-Filed Acoustic Levitation: Resolving Viscous and Acoustic Effects

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Ultrasonic Near-Field Acoustic Levitation Overview

Near-Field Levitation
Levitation Height 5-500 μm

<table>
<thead>
<tr>
<th>Viscous Mechanism</th>
<th>Acoustic Mechanism</th>
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<tbody>
<tr>
<td>$H &lt; 2 \text{ BTL}$</td>
<td>$H &gt; 2 \text{ BTL}$</td>
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*BTL – viscous boundary layer thickness, (for air @ 20 kHz approx. 20 μm)

- Allows levitation of moderately large objects
- Is used for contactless transportation systems (e.g. semiconductor industry)

Details of analytical and experimental work can be found in I. Melikhov, S. Chivilikhin, A. Amosov, R. Jeanson, Visco-acoustic model for near-field ultrasonic levitation, *submitted to Phys. Rev. E*
Physics of Near-Field Levitation

• The air flow between sound source and object is described by
  – Transient Navier-Stokes equations for compressible fluid
  – Energy equation
  – Equation of gas state
  – Viscosity-temperature dependence (Sutherland’s law)

• Leading-order model assumptions
  – $H_0/R \ll 1$
  – $H_0/\lambda_{\text{acoustic}} \ll 1$
  – $\epsilon = \frac{a}{H_0} \ll 1$

→ Pressure, density, viscosity are z-independent
Equations to Solve (Non-dimensional)
First-Order (Frequency Domain)

• Continuity

\[ i \frac{p_1}{\gamma} + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{u}_1) = \frac{i}{2} \]

\[ \partial_r p_1 \bigg|_{r=1} = i K p_1 \]

• Longitudinal velocity

\[ i \gamma K^2 u_1 = - \frac{\partial p_1}{\partial r} + \Sigma \frac{\partial^2 u_1}{\partial z^2} \]

\[ u_1 \bigg|_{z=0} = u_1 \bigg|_{z=1} = 0 \]

\[ \bar{u}_1 = \int_0^1 u_1 \, dz \]

• Transversal velocity

\[ v_1 = - \int_0^z \left( \frac{i}{\gamma} p_1 + \frac{1}{r} \frac{\partial}{\partial r} (r u_1) \right) \, dz' + \frac{i}{2} \]
Equations to Solve (Non-dimensional)
Second-Order (Time-Averaged Values)

- **Continuity**
  \[
  \frac{1}{r} \frac{\partial}{\partial r} \left( r \left\{ \bar{u}_2 + \left[ \left( \frac{p_1}{\gamma} - 1/2 \right) \bar{u}_1 \right]_0 \right\} \right) = 0
  \]

\[
p_2 \bigg|_{r=1} = 0
\]

- **Longitudinal velocity**
  \[
  \gamma K^2 \left( i \left( \frac{p_1 u_{-1}^* - p_{-1}^* u_1}{{\gamma}} \right) + \left[ u_1 \frac{\partial u_1}{\partial r} + v_1 \frac{\partial u_1}{\partial z} \right] \right) 
  = - \frac{\partial p_2}{\partial r} + \sum \frac{\partial^2}{\partial z^2} (u_2 + M[p_1 u_1]_0)
  \]

\[
  u_2 \bigg|_{r=0} = - \frac{1}{2} \frac{\partial}{\partial z} [u_1 + u_1^*]_0, \quad u_2 \bigg|_{z=0} = 0
  \]

\[
  \bar{u}_2 = \int_{0}^{1} u_2 \, dz - (\bar{u}_1 + \bar{u}_1^*)/2
  \]

\[
[a_1 b_1]_0 = a_1 b_1^* + a_1^* b_1 \quad \text{where } * \text{ denotes complex conjugation; } M - \text{ constant came from Sutherland’s law}
\]
COMSOL Implementation: 2D Axisymmetric Case

Equation Coupling

**Domain PDE**

**Velocity equation**

**Linear Projection coupling**

- Averaged velocity, m/s

**Linear Extrusion coupling**

- Pressure, Pa

**Boundary PDE**

- Continuity equation
Modeling Results
Levitation Force vs. Levitation Distance

![Graph showing Levitation Force vs. Levitation Distance](image)

- **RMS error 14%**

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- **Modeling Results**
  - $a=3\mu m$, $R=20mm$
  - $a=3\mu m$, $R=35mm$
  - $a=5\mu m$, $R=20mm$
  - $a=5\mu m$, $R=35mm$

- **Experimental Data**
  - $a=3\mu m$, $R=20mm$
  - $a=3\mu m$, $R=35mm$
  - $a=5\mu m$, $R=20mm$
  - $a=5\mu m$, $R=35mm$
Modeling Results
Pressure Profile

- Pressure profile varies depending on the regime
  - Close-to-uniform pressure in the viscous regime
  - Non-uniform pressure in the acoustic regime

\[ a=3 \mu m, R=20\text{mm}, \text{freq}=20\text{kHz} \]
Conclusions

- We developed and experimentally validated a new efficient model of ultrasonic levitation covering wide range of air flow regimes

- The model consists of five linear stationary PDE – much easier to solve than initial transient non-linear problem

- COMSOL helps to formulate and solve non-standard equations in simple and elegant way

\[\text{Math} + \text{Physics} + \text{COMSOL} = \text{Efficient, fast and accurate models}\]
Non-Dimensional Variables

• Coordinates and time
  \[ r = \hat{r}/R, \quad z = \hat{z}/H_0, \quad t = \omega \hat{t} \]

• Velocities and pressure
  \[ v_z = \hat{v}_z/(\omega H_0), \quad v_r = \hat{v}_r/\omega R, \quad p = \hat{p}/p_0 \]

• Density, viscosity and temperature
  \[ \rho = \hat{\rho}/\rho_0, \quad \mu = \hat{\mu}/\mu_0, \quad T = \hat{T}/T_0 \]

• Gap thickness
  \[ h^{(0)} = H(r)/H_0 \]

• Non-dimensional quantities
  – Acoustic wave number \( K = \left( \frac{\omega^2 R^2 \rho_0}{\gamma p_0} \right)^{1/2} \)
  – Squeeze number \( \Sigma = \frac{\mu \omega R^2}{p_0 H_0^2} \)