one applying the Beam interface and the second one the Solid Mechanics (SM) interface.

Further, in case the design needs adjustments, the results for both models will be compared to decide whether the Beam model may be used to make design adjustments before re-modelling the complete solid by the SM interface. The equations of the two models are the following:

Solid Mechanics

The conservation equation is:

$$0 = \nabla \cdot \sigma + Fv \qquad (1)$$

where σ is stress tensor and Fv are the volumetric forces.

Then, for linear elastic materials the relationship between the stress tensor and the small strain tensor is given by:

$$\sigma = C : \varepsilon = C(E, V) \qquad (2)$$

which corresponds to the Hooke's Law, where C is the elasticity or stiffness tensor, ε is the small strain tensor, E is Young's modulus and V is the Poisson's ratio.

Beam model

Timoshenko formulation:

$$\frac{\partial N}{\partial x} = f_x \qquad \frac{\partial T_y}{\partial x} = f_y \qquad \frac{\partial M_z}{\partial x} + T_y = m_z$$
$$\frac{\partial T_z}{\partial x} = f_z$$
$$\frac{\partial M_x}{\partial x} = m_x$$
$$\frac{\partial M_y}{\partial x} - T_z = m_y \qquad (3)$$

where N is the normal force, T is the shear, M are the bending moments and m are the twisting moments.

3. Methods

The geometry of the tridimensional frame is imported in Comsol Multiphysics® by means of the CAD Import Module capabilities. Then the Solid Structure Module is used to define two different 3D FEM models, one applying the Beam interface and the second one the Solid Mechanics (SM) interface.

Using different colors for each force, Figure 2 shows the loads applied in the different areas of the tricycle, while Table 1 gives the combination of loads for acceleration, steady pedalling and horizontal impact cases.

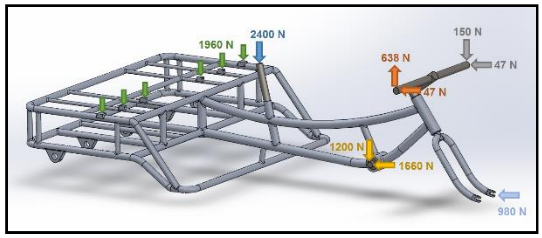

Figure 2. Loading values.

Table 1. Loading cases.

Loading cases	■	■	■	■	■	■
1. Acceleration		√	√	√		√
2. Steady Pedalling					√	√
3. Horizontal Impact	√					√

The acceleration case will involve pedaling forces at the bottom bracket, pulling and pushing forces at the handlebar and the passenger weight. The steady pedaling case takes into account only the passenger and driver weight as the tricycle rolls along the surface. Finally, the impact case simulates a sudden impact of the tricycle against a wall. In this case, we assume that the rider leaves contact with the seat then only the passenger weight is considered as well as the impact force. The British Standard – Mountain-bicycles – Safety requirements and test methods (2005) describes the different test used to examine safety of mountainbike frames. In particular, the Section 4.8.2 indicates how the frame is constrained from movement during impact testing.

In the computational model, the frame is constrained from movement in the rear axle, the fork is allowed to slide only along the horizontal X and Y axes and the vertical Z displacement is set to zero on the front axle boundaries. MUMPS (Beam) and SPOOLES (SM) are the solvers. For the SM Model, the frame consists of around 6×10^5 tetrahedral elements (3.3×10^6 DOFs in the computations). Figure 3 depicts the mesh of some components close to the bottom bracket.

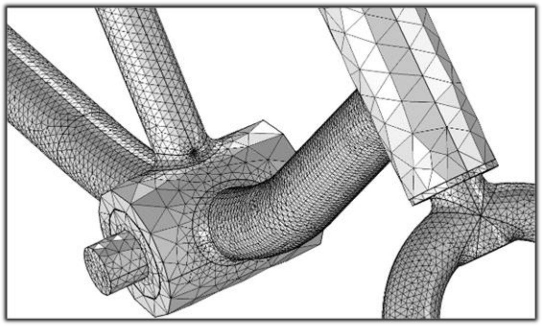

Figure 3. A partial view of the mesh (SM Model).

4. Computational results and discussion

The computational results show that the Beam model returns higher values both for stresses and displacements compared to the SM model, while the zones for the maximum stresses seem to be the same.

Initially both the Euler-Bernoulli and Timoshenko formulations were used, obtaining with the second one results closer to the SM model's computations. For instance, in loading case 1 the maximum von Mises stress for the Euler-Bernoulli formulation is 1.13E9 N/m^2, while with the Timoshenko formulation we obtained 9.57E8 N/m^2. The second value compares better to the value of 7.08E8 N/m^2 calculated by the SM model. Displacement results get even closer: 0.082 m (Euler-Bernoulli), 0.031 m (Timoshenko) and 0.018 m (SM).

In all the loading cases, the simulations show that certain regions of the tricycle might suffer stresses above the tensile yield strength of 214 MPa and the fatigue limit of 69 MPa. In the latter case, the fatigue strength value is assumed as the materials' resistance after 500x10^6 fully reversing load cycles, which is approximately 69 MPa for Al 6063.

Table 2. Maximum values for von Mises stresses and displacements.

Load case	Von Mises stress (N/m^2)		Displacement (m)	
	Beam	SM	Beam	SM
1	9.57E8	7.08E8	0.031	0.018
2	9.44E8	1.10E9	0.033	0.021
3	9.61E8	6.47E8	0.030	0.010

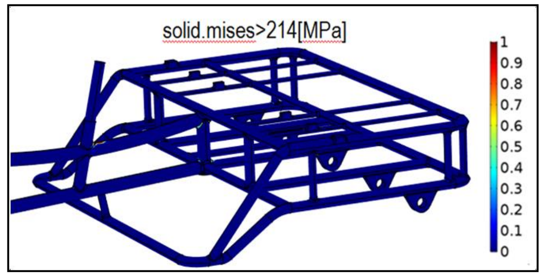

Figure 4. Elastic limit: steady pedalling case, SM model.

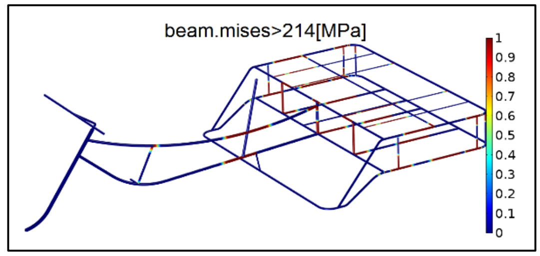

Figure 5. Elastic limit: steady pedalling case, Beam model.

Figures 4 and 5 show the results for the steady pedaling case, where the red areas indicate von Mises stresses greater than the materials elastic limit of 214 MPa.

A critical area is observed at the seat tube-horizontal tube union (the SM model revealing it just behind the union), as it could be expected from the compression that the rider's weight generate in that area. The Beam model extends this condition to the area around the seat tube. For both models, the computational results indicate that another region to verify is the intersection between the horizontal and down tubes with the cage,

In the following plots we depict in red the areas of the frame and fork which are stressed above the fatigue limit resistance of the material, for the steady state pedaling (Fig. 6 and 7) and acceleration (Fig. 8 and 9) loading cases, respectively.

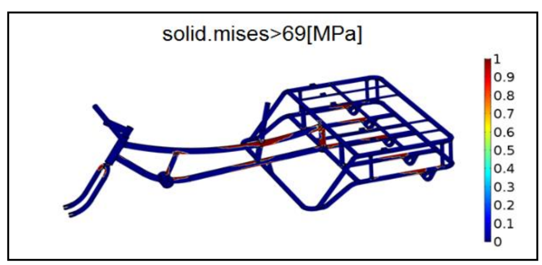

Figure 6. Fatigue strength: steady pedaling case, SM model.

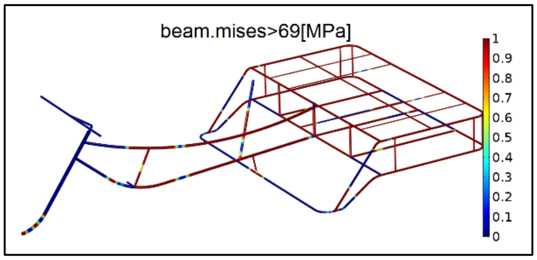

Figure 7. Fatigue strength: steady pedaling case, Beam model.

Although most areas of the frame withstand the static loads, this simple fatigue analysis shows that they need to be strengthened when considering the long-term durability of the frame.

For the steady pedaling loading case, as it would be expected, the area failing under static load (behind the seat tube) extends to the front of the structure and also to the seat tube – down tube union (Figures 6 and 7). The same happens with the area where the horizontal and down tubes meet the cage, showing small weak areas when compared to the elastic limit of the material. This area is larger if compared to the fatigue limit. Additional critical areas highlighted by the computational results are: reinforcement tube unions, cage area in front of the rear axle and the head tube – down tube union, the latter one confirming a typical failure zone of the traditional bicycle design analysis.

For the acceleration loading case (Figures 8 and 9), the same fatigue areas cover a much smaller area of the structure. The head tube – down tube union represents an exception, here the critical region moves to the bottom part of the down tube, being slightly larger than in the steady pedaling case.

Due to the low fatigue life expected from the computational results, other material could be used for the tricycle fabrication. According to Dwyer et al. (2012), 6061-T6 aluminum is a more common option in the bicycle industry, because the value of the fatigue life is 96 MPa compared to 69 MPa of the aluminum 6063.

The horizontal impact case is not analyzed in fatigue strength as it represents an occasional condition, not a continuous condition that could debilitate the material in time.

For the horizontal impact loading case, the numerical results are compared with the elastic limit of the material, showing that the frame will withstand the loads (Figures 10 to 12). However, the Beam model points out some critical areas in the cage.

In the fork, which is only modeled in the SM model, we obtain similar results to the rest of the frame. Figure 12 shows that the fork will withstand the impact force with minimum

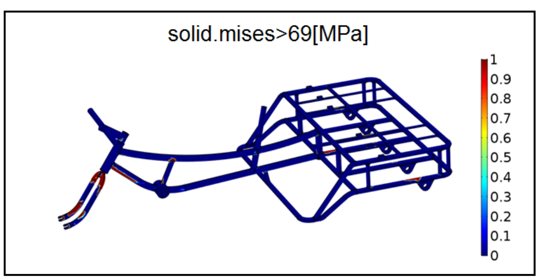

Figure 8. Fatigue strength: acceleration case, SM model.

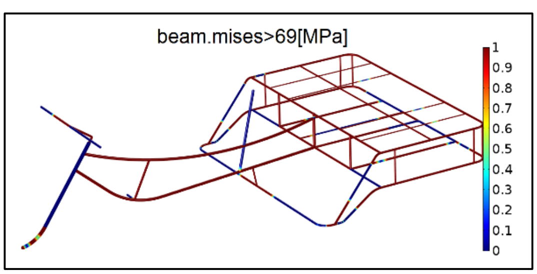

Figure 9. Fatigue strength: acceleration case, Beam model.

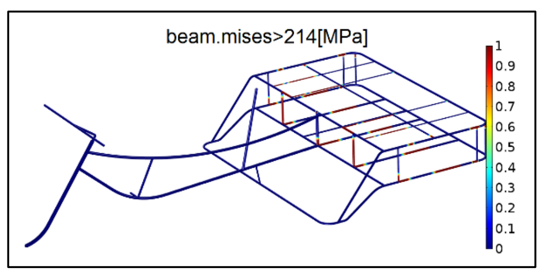

Figure 10. Elastic limit: horizontal impact case, Beam model.

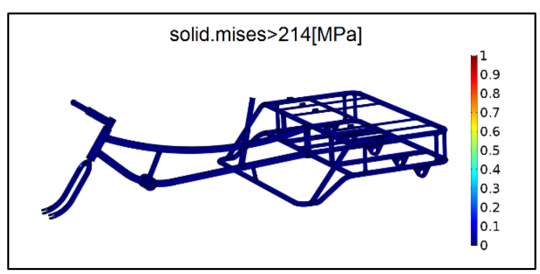

Figure 11. Elastic limit: horizontal impact case, SM model.

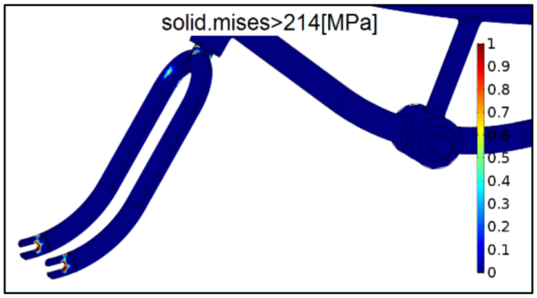

Figure 12. Elastic limit: horizontal impact case, SM model, fork region.

deformation areas deformation (areas with lighter colors), as required by design and test standards. However, under fatigue analysis, the computational results show that this component should be redesigned.

5. Conclusions

- A finite element analysis of an aluminum tricycle frame has been carried out by using Comsol Multiphysics® 5.2.
- For different combinations of loads, the stress and deformation distributions have been evaluated using the Beam and the Solid Mechanics interfaces.
- The analysis of the structural characteristics of the tricycle frame shows that certain regions of the frame will not withstand the loads, needing to fine-tune the frame geometry.
- A simple fatigue analysis reveals that the long-term durability of the design is compromised, and then additional fatigue and impact simulations should be developed in order to improve the design of the tricycle.
- Due to the low fatigue life expected from results, 6061-T6 aluminum could represent a better choice for the tricycle fabrication.
- The FEM simulations have provided useful insights in defining the structural performance of the tricycle, gathering knowledge for future studies.

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