Model Order Reduction for COMSOL
A Compact Model of a Wireless Power Transfer System
Jairo A. Pico, Dennis Hohlfeld, Tamara Bechtold
Univ. of Rostock, Inst. of Electronic Appliances and Circuits, Albert-Einstein Str. 2, 18059 Rostock, Germany

Wireless Power Transfer (WPT) systems facilitate the use of battery powered devices ranging from hand-held devices to electrically powered vehicles. Electrical plugs are made redundant as electrical energy is transmitted without the need of a wired electrical connection.

Such systems feature two inductors and magnetic components. Energy is transferred to the mobile component by means of magnetic resonance coupling.

Nowadays, a new technological trend towards WPT is driven by modern applications, such as medical implants, mobile devices, electrical vehicles, drones, and home appliances, which all profit of cable-free energy supply.

A 3D model was built in COMSOL Multiphysics® following the specifications in [3]. The design goal was to maximize the mutual inductance, while avoiding its sensitivity to winding misalignment. Spatial discretization of Maxwell equations by finite element method (FEM) resulted in a model with 110,000 degrees of freedom (DOFs).

By using Ampere’s Law, the magnetic vector potential \( A \) is calculated, and consequently the electric potential \( V \) can be derived.

\[
(j \omega - \omega^2 \varepsilon_0)A + \nabla \times \left( \frac{1}{\mu_0} \nabla \times \hat{A} \right) + (j \omega \varepsilon_0 + \sigma) \nabla V = j_e
\]

The excessive computational effort for the solution of a full-scale model, does not allow its co-simulation with electric circuitry. A compact - but numerically efficient - model is required. Mathemathical model order reduction (MOR) has been proven as a reliable and numerically robust modelling approach [4]. The system matrices were extracted using LiveLink™ for Matlab®, and a compact model with only 20 DOFs was created by MOR using the one-sided and two-sided Second Order Arnoldi (SOAR) algorithms from [5]. This solution time for harmonic analysis was reduced by several orders of magnitude with negligible loss of accuracy.

Sensitivity Analysis of Reduced Order Models
Input-one-sided SOAR models with more than 7 moments (Taylor coefficients of the transfer function, which are matched for the full and reduced model and correspond to the order of reduced model), showed a relative error lower than 2.5%. Two-sided SOAR models proved to be reliable, even with only 4 moments, with a maximum error of \( 5 \times 10^{-3} \) %.

Power Efficiency Analysis
The reduced models were exported in VHDL format and coupled with a circuit schematic using ANSYS Simplorer (see Figure 8). The voltage frequency response (Figure 9. a and b) shows two separate maxima left and right from the operating frequency. This frequency splitting was calculated analytically, yielding similar values to those calculated with ANSYS Simplorer. Finally, the maximum power transfer was computed over the frequencies range of interest (Figure 9. c).

Conclusions
Model order reduction allows to create compact but accurate model and export it as an electrical two-port network, into circuit simulator. Two-sided SOAR reduction method showed an overall better quality compared to one-sided methods. It turned out that only few moments are sufficient to accurately describe a full 3D model.

References:

Excerpt from the Proceedings of the 2016 COMSOL Conference in Munich