Model Order Reduction for COMSOL
A Compact Model of a Wireless Power Transfer System

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Wireless Power Transfer (WPT) systems facilitate the use of battery powered devices ranging from hand-held devices to electrically powered vehicles. Electrical plugs are made redundant as electrical energy is transmitted without the need of a wired electrical connection. Such systems feature two inductors and magnetic components. Energy is transferred to the mobile component by means of magnetic resonance coupling.

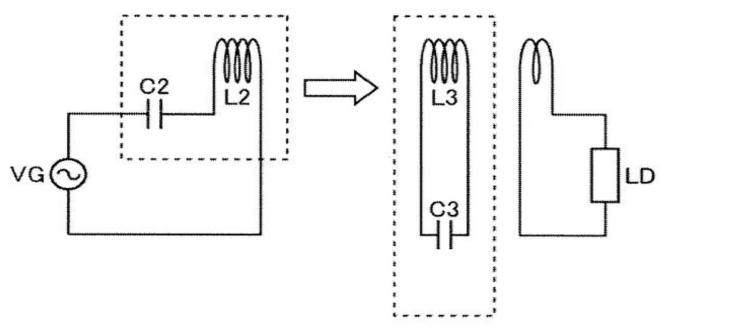

Figure 1. Magnetic resonance coupling

Nowadays, a new **technological trend** towards WPT is driven by modern applications, such as medical implants, mobile devices, electrical vehicles, drones, and home appliances, which all profit of cable-free energy supply.

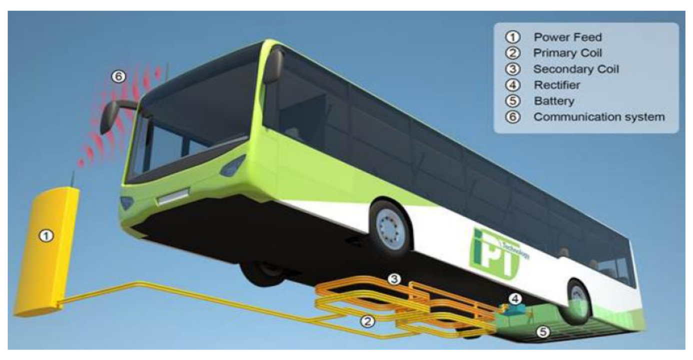

Figure 2. Conductix WPT electric bus. Source: [2]

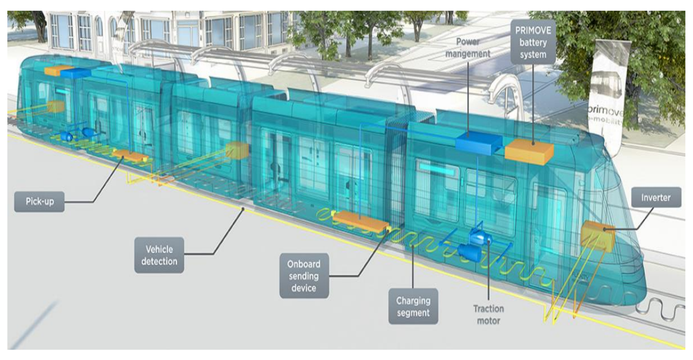

Figure 3. Bombardier WPT tram schematic. Source: [2]

A 3D model was built in **COMSOL Multiphysics®** following the specifications in [3]. The design goal was to maximize the mutual inductance, while avoiding its sensitivity to winding misalignment. Spatial discretization of Maxwell equations by finite element method (FEM) resulted in a model with 110.000 degrees of freedom (DOFs).

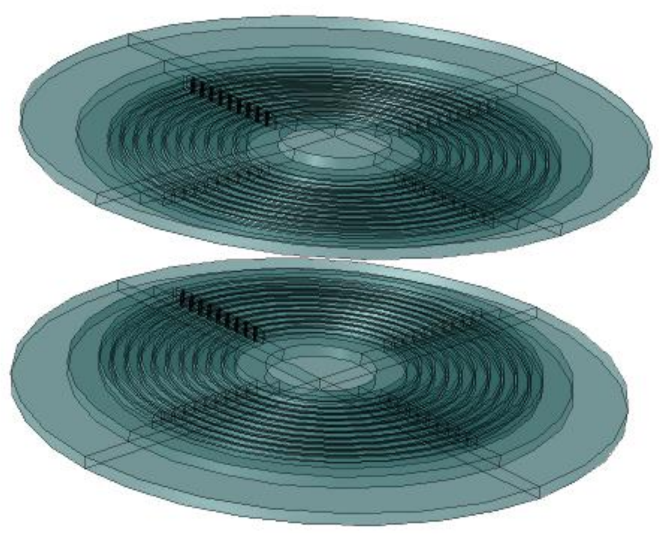

Figure 4. 3D model of a pair of disc coils

By using Ampere's Law, the magnetic vector potential A is calculated, and consequently the electric potential V can be derived.

$$(j\omega\sigma - \omega^2\varepsilon_0)\vec{A} + \nabla \times \left(\frac{1}{\mu_0}\nabla \times \vec{A}\right) + (j\omega\varepsilon_0 + \sigma)\nabla V = \vec{J}_e$$

The excessive computational effort for the solution of a full-scale model, does not allow its co-simulation with electric circuitry. A **compact - but numerically efficient - model** is required. Mathematical model order reduction (MOR) has been proven as a reliable and numerically robust modelling approach [4]. The system matrices were extracted using **LiveLink^{TM} for Matlab ®**, and a compact model with only 20 DOFs was created by MOR using the one-sided and two-sided Second Order Arnoldi (SOAR) algorithms from [5]. This solution time for harmonic analysis was reduced by several orders of magnitude with negligible loss of accuracy.

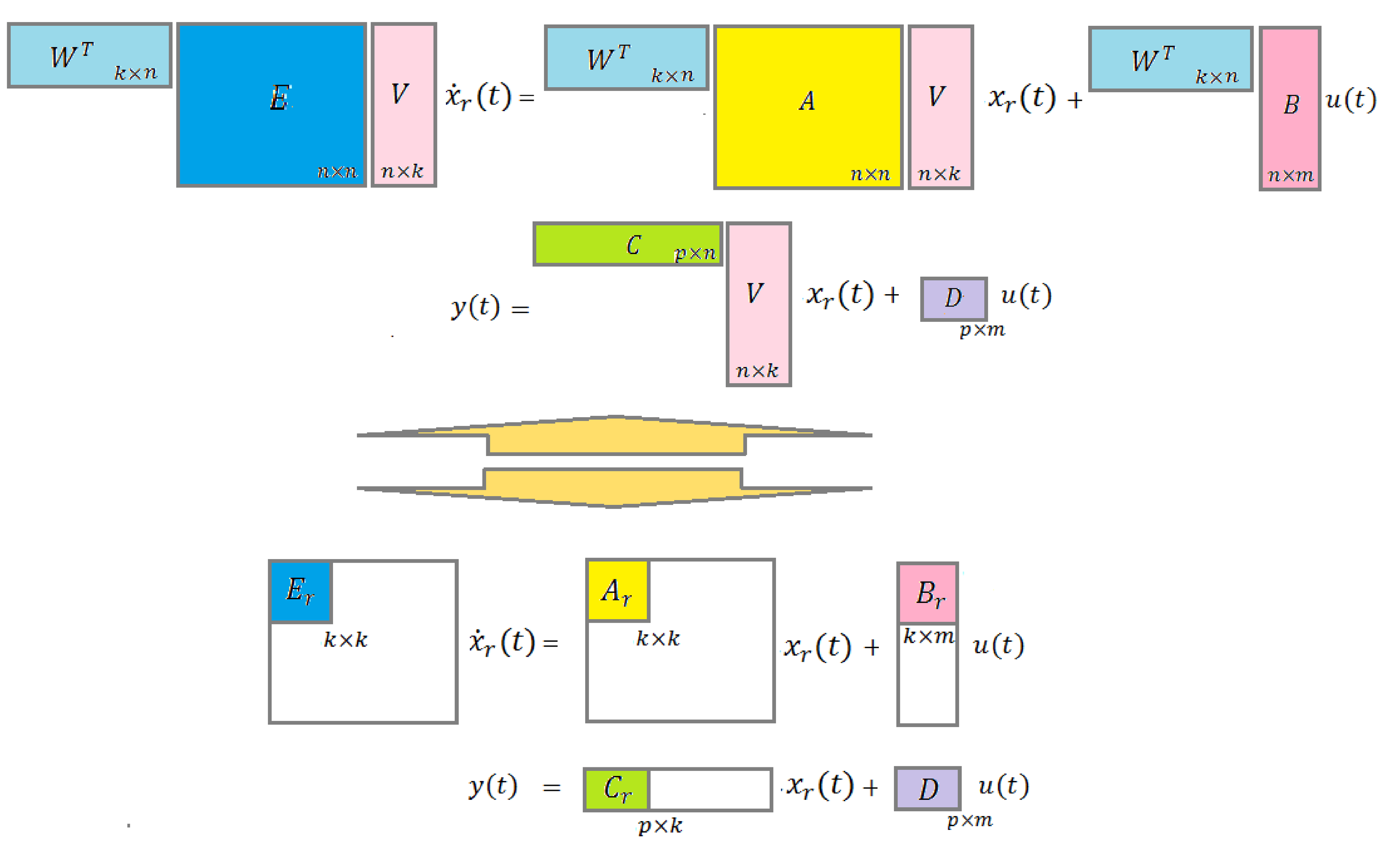

Figure 5. Visualization of MOR process for a first order ordinary differential equation system

Sensitivity Analysis of Reduced Order Models

Input-one-sided SOAR models with more than 7 moments (Taylor coefficients of the transfer function, which are matched for the full and reduced model and correspond to the order of reduced model), showed a relative error lower than 2.5%. Two-sided SOAR models proved to be reliable, even with only 4 moments, with a maximum error of 5×10^{-3} %.

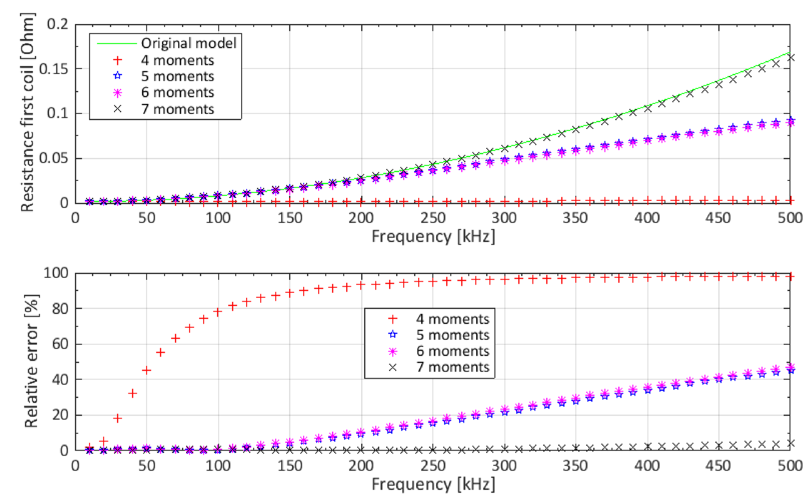

Figure 6. Input one-sided SOAR models

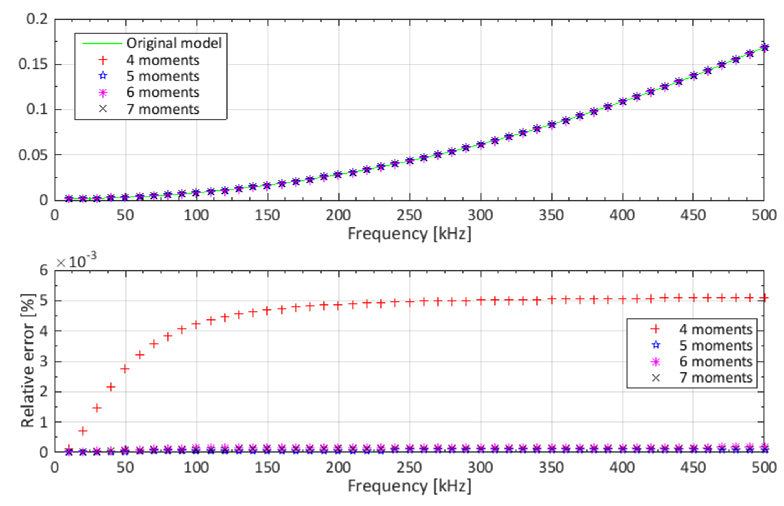

Figure 7. Two-sided SOAR models

Power Efficiency Analysis

The reduced models were exported in VHDL format and coupled with a circuit schematic using ANSYS Simplorer (see Figure 8). The voltage frequency response (Figure 9. a and b) shows two separate maxima left and right from the operating frequency. This frequency splitting was calculated analytically, yielding similar values to those calculated with ANSYS Simplorer. Finally, the maximum power transfer was computed over the frequencies range of interest (Figure 9. c).

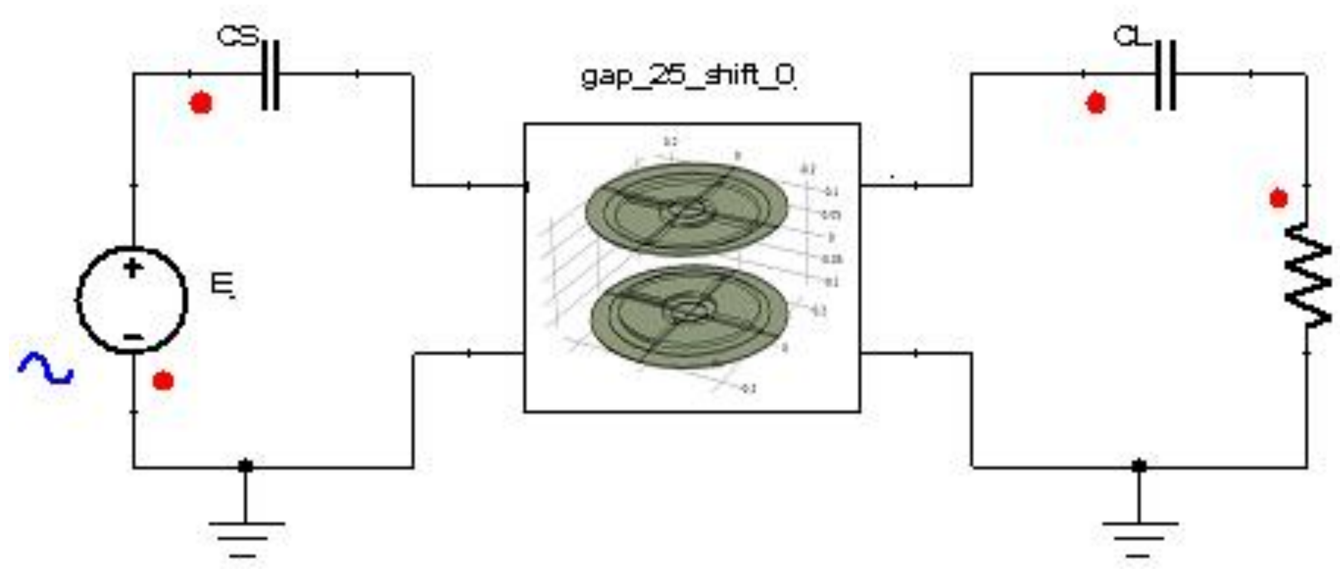

Figure 8. Reduced model connected as a 2-ports network to an external electrical circuit

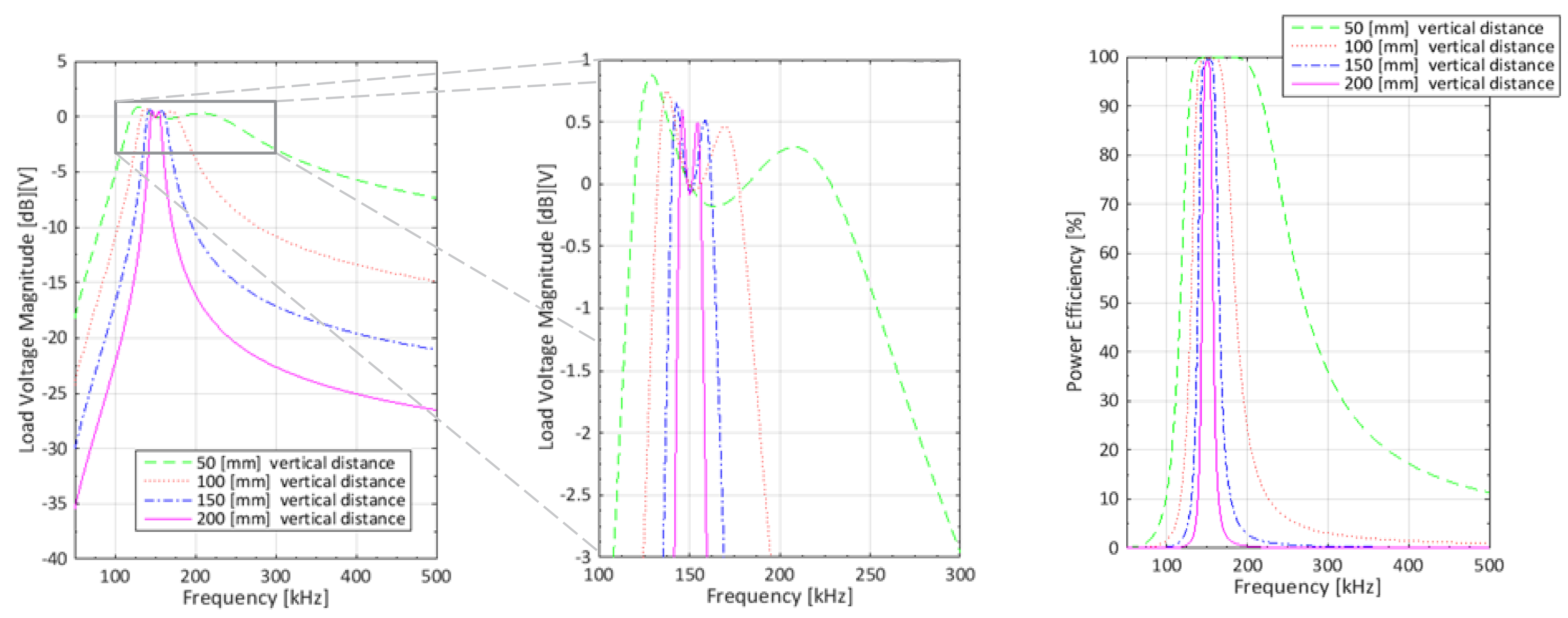

Figure 9. a) Load voltage, **b)** Frequency splitting, **c)** Power efficiency

Conclusions

Model order reduction allows to create compact but accurate model and export it as an electrical two-port network, into circuit simulator. Two-sided SOAR reduction method showed an overall better quality compared to one-sided methods. It turned out that only few moments are sufficient to accurately describe a full 3D model.

References:

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[5] B. Salimbahrami, B. Lohmann, and T. Bechtold, "Two-Sided Arnoldi in Order Reduction of Large Scale MIMO Systems," Scientific Report, University of Bremen, 2002.