Acoustic Fluid-Structure Interaction Modelling of Gravity Dams in the Frequency Domain

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Adoption of the acoustic formulation:

Previous works:

«... Since the involved motions are small, the relatively simple equations may be used, that apply to sound in liquids. ... »

Author’s assumptions:
• Null global flow
• Rigid dam
• Infinite-length reservoir
• Small displacements of the fluid particles
• Bidimensional problem (2D)
Westergaard added mass method:

Fourier series solution for particle displacement:

\[ \xi = -\frac{agT^2}{\pi^2} \cos\left(\frac{2\pi t}{T}\right) \sum_{n=1,3,5 \ldots}^{n} \frac{1}{n} e^{-qn} \sin\left(\frac{n\pi y}{2h}\right) \]

\[ \eta = \frac{agT^2}{\pi^2} \cos\left(\frac{2\pi t}{T}\right) \sum_{n=1,3,5 \ldots}^{n} \frac{1}{nc_n} e^{-qn} \cos\left(\frac{n\pi y}{2h}\right) \]

\[ c_n = \sqrt{1 - \frac{16wh^2}{n^2 gkT^2}} \]

Reservoir period(s):

\[ T_n = \frac{4h}{c \cdot n} \]

Added Water Mass:

\[ b(y) = \frac{7}{8} \sqrt{hy} \]
The COMSOL® model:

- Pressure acoustics interface – frequency domain Structural mechanics

Boundary conditions:

\[
\lim_{r \to \infty} \sqrt{x} \left( \frac{\partial}{\partial r} + ik \right) \rho = 0,
\]

- Infinite reservoir modeling: (Sommerfeld condition)
  - Plane Wave Radiation
  - Perfectly Matched Layer

- Multiphysics coupling: Acoustics-Structure boundary

- Rigid or deformable dam

- Base excitation

\[
\frac{dp}{dy} = 0,
\]

\[
\ddot{u}(t) = a_0 e^{i\omega t}
\]
Modeling the Infinite

Two approaches:

• **Plane wave radiation (PWR):** Robin boundary condition, second order
  \[
  -n \cdot \left( -\frac{1}{\rho_c} (\nabla p_t - q_d) \right) + i \frac{k}{\rho_c} p + \frac{i}{2k\rho_c} \Delta T p = Q_i^0
  \]
  No monopole or dipole sources

• **Perfectly matched layer (PML):** Complex coordinate transformation
  – Rational scaling:
    \[
    f_r(\xi) = s\Lambda \xi \left( \frac{1}{3p(1-\xi)} + 4 - \frac{i}{3p(1-\xi)} \right)
    \]
  \(\xi =\) Dimensionless coordinate, \([0,1]\)
  \(s =\) Scaling parameter, \(p =\) curvature parameter
  \(\Lambda =\) Typical wavelength parameter
Results: PML vs. PWR - Modal analysis

- Case (a): Rigid dam with perfect bottom reflection ($\alpha=1$)

<table>
<thead>
<tr>
<th>Eigen mode</th>
<th>Analytic ($\frac{nc}{4h}$) [Hz]</th>
<th>PWR [Hz]</th>
<th>PML [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.407</td>
<td>7.817</td>
<td>7.407</td>
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<tr>
<td>2</td>
<td>22.221</td>
<td>22.386</td>
<td>22.222</td>
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<td>3</td>
<td>37.035</td>
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<td>37.036</td>
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<td>4</td>
<td>51.849</td>
<td>51.976</td>
<td>51.851</td>
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<tr>
<td>5</td>
<td>66.663</td>
<td>66.720</td>
<td>66.667</td>
</tr>
</tbody>
</table>

Reservoir depth = 50 m

Plot: First two eigenmodes, height is proportional to water pressure
Results: PML vs. PWR — Frequency response

• Case (a): Rigid dam with perfect bottom reflection ($\alpha=1$)

Plot: Total hydrodynamic pressure at the base of the dam.
Results: Multphysics interaction

• Case (b): Deformable dam with perfect bottom reflection ($\alpha=1$)

*Plot*: Horizontal base reaction at the dam foundation.
Results: Multphysics interaction

• Case (b): Deformable dam with various reflection coefficients (α)

Plot: Horizontal base reaction at the dam foundation.
Results: Multphysics interaction

- Case (b): Deformable dam compared to the added mass method

Plot: Horizontal base reaction at the dam foundation.

Added mass:
- Inaccurate Identification of other peaks
- Approximate Identification of the first mode
- Accurate value of base shear For low frequencies
Results: Multphysics interaction

- Case (b): Variation of dam Young’s modulus

**Plot:** Horizontal base reaction at the dam foundation.
Thanks for your attention!