Finite Element Based Improved Characterization of Viscoelastic Materials

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Introduction: Viscoelastic materials exhibit several non-trivial effects such as highly non-linear dynamic behavior, advocating the use of finite element software. The objective of this project is to fully characterize a viscoelastic material. This is done by firstly performing a Dynamical Mechanical Analysis (DMA), see Figure 1. Simultaneously we investigated the deformation with advanced image processing algorithms – hence the two dots seen in Figure 1. These experimental results are subsequently used to determine and optimize parameters of a Weak Form COMSOL model.

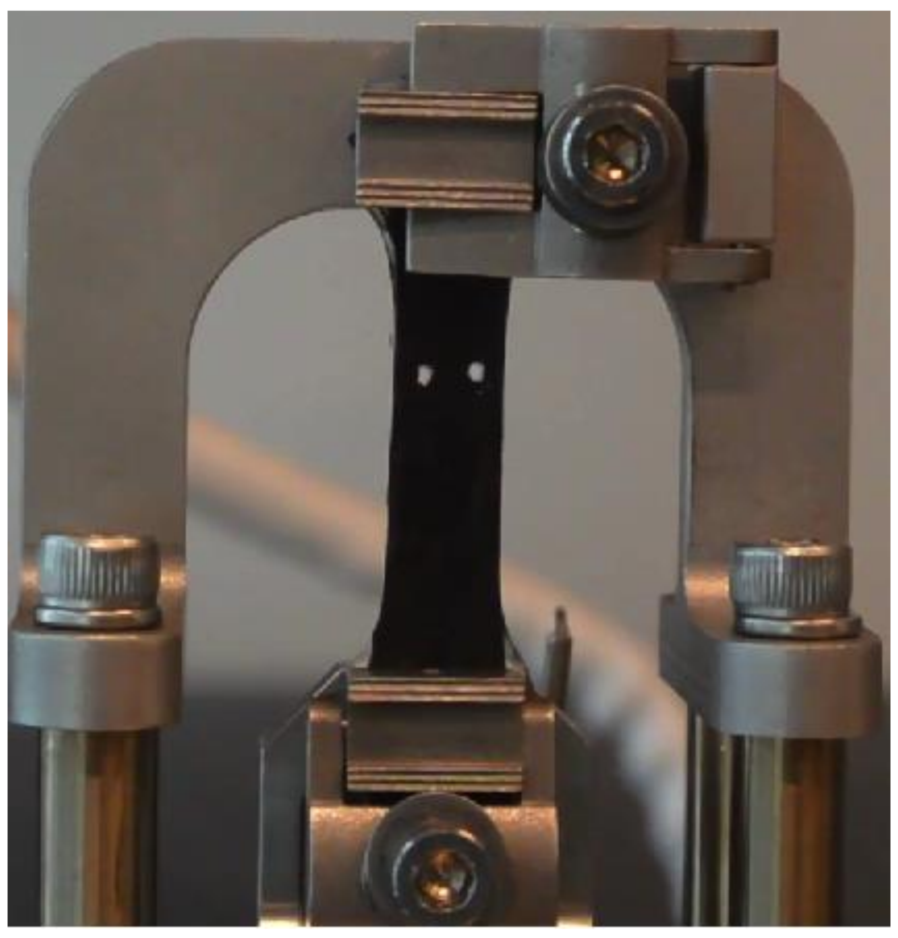


Figure 1. The stretched material sample

Computational methods: Hamilton's principle is used to derive the governing equation for the deformation of the viscoelastic material:

$$\delta I = \int_{t_1}^{t_2} \int_B \left(\rho_R \frac{\partial u_i}{\partial t} \frac{\partial \delta u_i}{\partial t} - \delta W \right) dB dt.$$

Using COMSOL's weak form PDE physics interface, three mathematical models are developed. In the three models, the strain-energy density function W depends on different hyperelastic constitutive relations. They are:

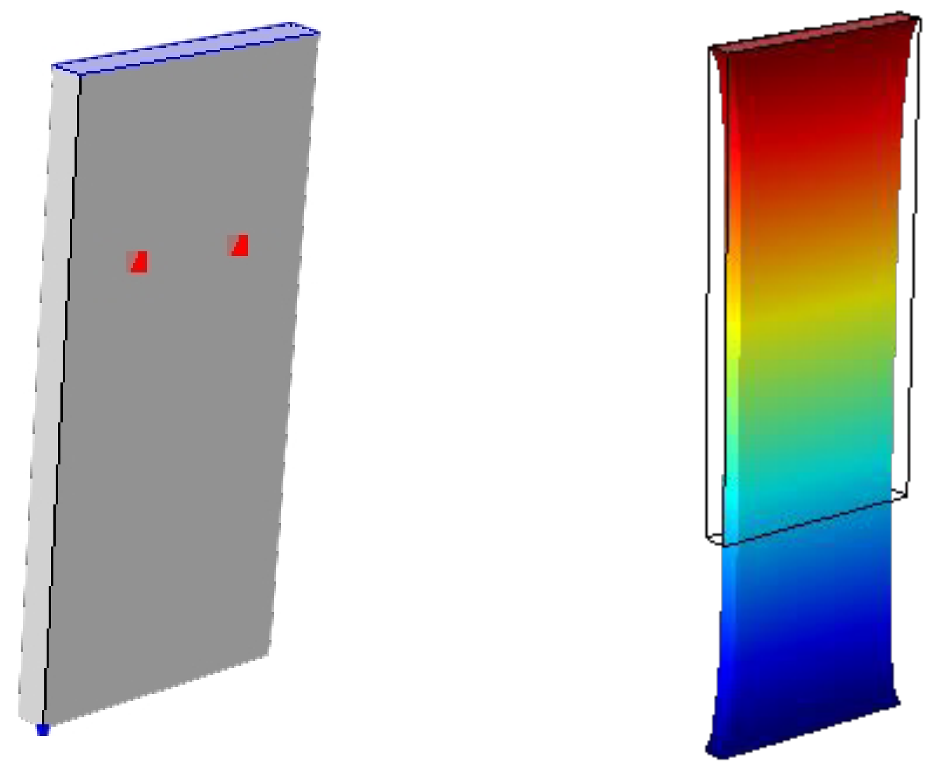
- Mooney-Rivlin [1],
- Yeoh [2]
- and Arruda-Boyce [3].

In order to optimize the respective parameters, we fit the calculated force on the lower surface to the DMA results. The force is calculated from stress as

$$S_{ij} = \rho_E \frac{\partial e}{\partial F_{ij}},$$

$$F = \int_A S_{ij} N_j dS \,.$$

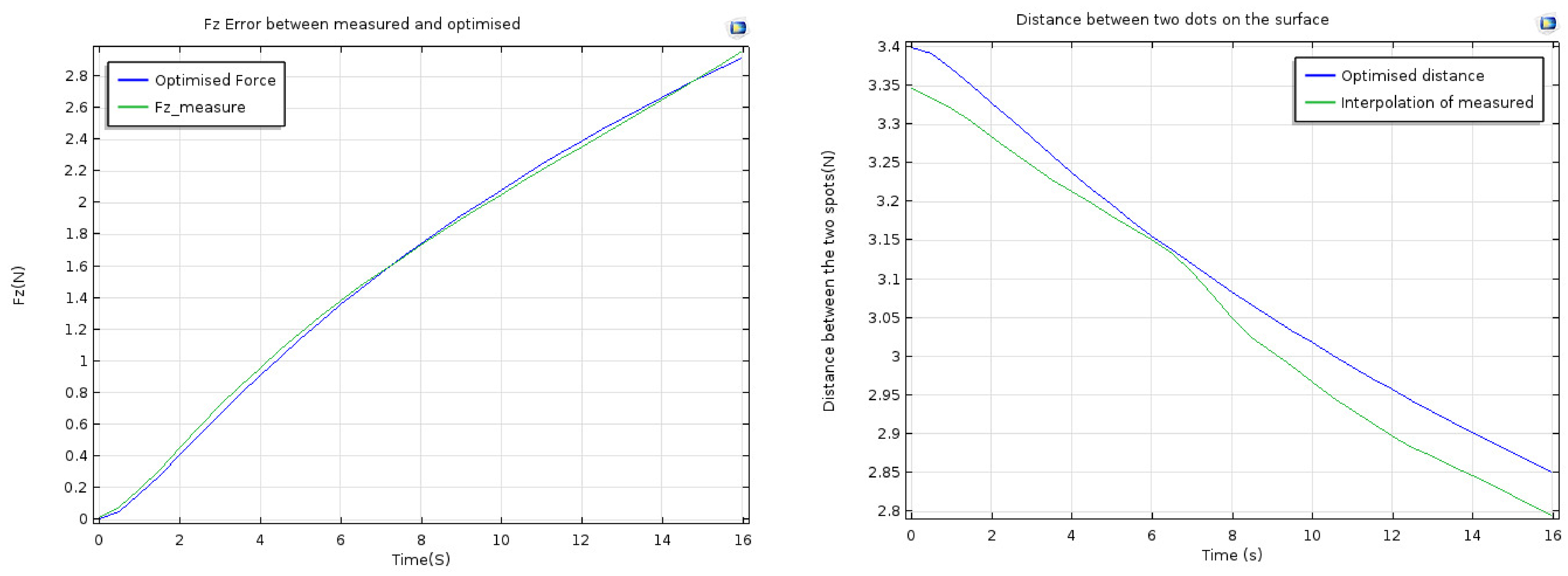
The distance between the probes is fitted to the image processing results as indicated in Figure 2.



(a) (b)

Figure 2. **(a)** Geometry model including two dot probes, **(b)** The resultant deformation

Results: The acquired results are demonstrated and compared to each other. The simulated and measured force, shown in Figure 3(a), using the Arruda-Boyce model are in good agreement. The same, although to a slightly less extent, is observed for the dot distance in Figure 3(b). Still, the deviation in dot distance does not exceed 2% over the complete measurement. The other two models showed similar trends, but slightly larger errors around 7% (not shown here).



(a) (b)

Figure 3. **(a)** Force comparison **(b)** Dot distance comparison

Conclusions: The results of the numerical study demonstrate the correct trend of all three non-linear models, with the Arruda-Boyce model showing the best fitting performance.

References:

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2. O. H. Yeoh, Some forms of the strain energy function for rubber, Rubber Chemistry and Technology, Volume **66**, Issue 5, November 1993, pp. 754-771 (1993)
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