Efficiency of a VAWT with Airfoil Pitch Control

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VAWT and HAWT

VAWT: Vertical Axis Wind Turbine

HAWT: Horizontal Axis Wind Turbine
Advantages and Disadvantages

**Advantages:**
- Easier to install and maintain
- No need to point into the wind
- Low risk for human or birds
- Can be installed in urban area

**Disadvantages:**
- Stall start
- Low efficiency
- Dynamic stability: vibration and noise
- Pulsatory torque
VAWT with Pitch and Camber Controls
Governing Equations

Low speed laminar flow: Navier – Stokes equations

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} &= \nabla \cdot [-p \mathbf{I} + \tau] + \mathbf{F} \\
\rho c_p \left( \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T \right) &= -(\mathbf{v} \cdot \mathbf{q}) + \nabla : \mathbf{S} - \frac{T}{\rho} \frac{\partial p}{\partial t} |_{\mathbf{p}} \left( \frac{\partial p}{\partial t} + (\mathbf{u} \cdot \nabla) p + Q \right)
\end{align*}
\]

High speed turbulent flow: \( \kappa - \varepsilon \) turbulent flow model

\[
\begin{align*}
\mu_T &= \rho c_\mu \frac{k^2}{\varepsilon} \\
\rho \frac{\partial k}{\partial t} + \rho \mathbf{u} \cdot \nabla k &= \nabla \cdot \left( \left( \mu + \frac{\mu_T}{\sigma_k} \right) \nabla k \right) + P_k - \rho \varepsilon \\
P_k &= \mu_T \left( \nabla \mathbf{u} : (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} (\nabla \cdot \mathbf{u})^2 \right) - \frac{2}{3} \rho k \nabla \cdot \mathbf{u}
\end{align*}
\]

\[
\begin{align*}
\rho \frac{\partial \varepsilon}{\partial t} + \rho \mathbf{u} \cdot \nabla \varepsilon &= \nabla \cdot \left( \left( \mu + \frac{\mu_T}{\sigma_\varepsilon} \right) \nabla \varepsilon \right) + C_{\varepsilon 1} \frac{\varepsilon}{k} P_k - C_{\varepsilon 2} \rho \frac{\varepsilon^2}{k}
\end{align*}
\]

<table>
<thead>
<tr>
<th>CONSTANT</th>
<th>VALUE</th>
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</thead>
<tbody>
<tr>
<td>(C_\mu)</td>
<td>0.09</td>
</tr>
<tr>
<td>(C_{\varepsilon 1})</td>
<td>1.44</td>
</tr>
<tr>
<td>(C_{\varepsilon 2})</td>
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<td>(\sigma_k)</td>
<td>1.0</td>
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<tr>
<td>(\sigma_\varepsilon)</td>
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Force and Torque

**Force:**

\[
\vec{F} = \int d\vec{F} = \int \vec{p} \ast ds = \int p \ast (-\vec{n}) \ast ds
\]

\[
\vec{n} = \frac{1}{\sqrt{1^2 + (-1/y'(x))^2}} \vec{i} + \frac{-1/y'(x)}{\sqrt{1^2 + (-1/y'(x))^2}} \vec{j}
\]

\[
\vec{F} = \int p \ast \frac{-1}{\sqrt{1^2 + (-1/y'(x))^2}} \ast ds \vec{i} + \int p \ast \frac{1/y'(x)}{\sqrt{1^2 + (-1/y'(x))^2}} \ast ds \vec{j}
\]

**Torque:**

\[
\vec{T} = \oint d\vec{T} = \oint \vec{r} \otimes d\vec{F}
\]
NACA 0012 Airfoil

\[ y = \pm 0.6 \left[ 0.2969 \sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4 \right] \]
COMSOL CFD Model
Velocity Field
Pressure Field
Torque at 30° Support Arm Angle

![Graph showing the relationship between Torque (N) and Attack Angle (°). The graph peaks at an Attack Angle of 100°, with Torque values ranging from 35000 to 51000 N.]
Effects of Wind Speed

![Graph showing the relationship between torque and attack angle for different wind speeds. The graph demonstrates how torque changes with wind speed.](image)

- **5m/s**
- **10m/s**
- **15m/s**
- **20m/s**
Effects of Support Arm Angle
Effects of Support Arm Angle
Effects of Support Arm Angle
Conclusions

• Torque with respect to the VAWT main rotor shaft depends on wind angle of attack and support arm angle

• Wind speed does affect the peak torque. But the torque increase due to higher wind speed is insignificant

• For the NACA 0012 airfoil, the torque always peaks out at 90 degree angle of attack at any given support arm angle
Future Work

- 2D multi-blades model
- 3D model to explore airfoil edge effect
- 3D multi-blades model
- Non-symmetric airfoil other than NACA 0012
Questions?