Coupled Electromagnetics-Multiphase Porous Media Model for Microwave Combination Heating

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Goals

Convection-Radiant Heating
- Used for cooking
- Slow

Microwave Heating
- Fast and convenient
- Non-uniform heating
- Mostly used for reheating

Combination Heating
- Fast and convenient
- Can be used for cooking
- Can provide custom cooking ability

Processing Variables
- Heating Modes
- Power levels
- Cycling

Food Factors
- Physical properties
  - Thermal
  - Dielectric
- Size

Desired Product
- Uniform heating
- Crust formation
- Appropriate texture
Modeling Methods

**Primary Variables:** microwave energy deposition, temperature, moisture, pressure

Microwave heating (Electromagnetics)

- **Model:** Exponential decay (empirical) - 1D, 2D
- **Variables:** energy deposition - not valid in general

Maxwell's equations - 3D

Model: Transport

- **Model:** Heat Transfer only
- **Model:** Heat and mass transfer (no pressure driven flow)
- **Variables:** temperature and moisture

Increasing complexity (physics + geometry)

Present work

**Fully Coupled Electromagnetics:** Heat and Mass Transfer with pressure driven flow (multiphase porous media model) - in 3D
Process Description

- Microwave, hot air, radiant heating
- Multiphase transport in sample treated as a porous medium\textsuperscript{1,2,3}

\begin{itemize}
  \item Water
    \begin{itemize}
      \item bulk flow/convection
      \item capillary flow
      \item phase change
    \end{itemize}
  
  \item Gas (Vapor + Air)
    \begin{itemize}
      \item bulk flow
      \item binary diffusion
      \item phase change
    \end{itemize}
\end{itemize}

\textsuperscript{1}Whitaker S., Advanced Heat Transfer, \textbf{13}, 119-203 (1997)
Governing equations: Electromagnetics

- Maxwell’s equations:
  \[ \nabla \times \mathbf{E} = -j\omega \mu \mathbf{H} \quad \mathbf{E} = \text{Electric field intensity} \]
  \[ \nabla \times \mathbf{H} = j\omega \varepsilon \varepsilon_0 \mathbf{E} \quad \mathbf{H} = \text{Magnetic field intensity} \]
  \[ \nabla \cdot (\varepsilon \mathbf{E}) = 0 \quad \omega = \text{microwave angular frequency} \]
  \[ \nabla \cdot \mathbf{H} = 0 \quad \text{dielectric loss} \]

- Relative permittivity:
  \[ \varepsilon = \varepsilon' - j\varepsilon'' \]
  \[ \varepsilon' = \text{dielectric constant} \]

- Boundary condition (oven walls):
  \[ E_{\text{tangential, oven wall}} = 0 \]

- Power absorbed (by the sample):
  \[ Q_{\text{mic}} = \frac{1}{2} \omega \varepsilon_0 \varepsilon'' |\mathbf{E}|^2 \]
Governing Equations: Multiphase Porous Media Model

- Momentum balance
  > Darcy’s Law:

\[ v_i = -\frac{k_i k_{r,i}}{\mu_i} \nabla P \]

for water and gas (vapor/air)
G.E.- Mass balance

1. Liquid phase (water):

\[
\frac{\partial c_w}{\partial t} + \nabla \cdot \left( n_w \right) = -\dot{I}
\]

\[
\begin{align*}
n_w &= -\rho_w \frac{kk_r}{\mu} \nabla \left( P - p_{cap} \right) = -\rho_w \frac{kk_r}{\mu} \nabla P + \rho_w \frac{kk_r}{\mu} \frac{\partial p_{cap}}{\partial S_w} \nabla S_w \\
D_w &= -\frac{kk_r}{\phi \mu} \frac{\partial p_{cap}}{\partial S_w} \\
c_w &= \rho_w \phi S_w
\end{align*}
\]

\[
\frac{\partial c_w}{\partial t} + \nabla \cdot \left( \rho_w v_w \right) = \nabla \cdot \left( D_w \nabla c_w \right) - \dot{I}
\]

2. Gas phase (vapor and air):

\[
\frac{\partial c_g}{\partial t} + \nabla \cdot \left( \rho_g v_g \right) = \dot{I}
\]

bulk flow

bulk flow phase change

bulk flow phase change
G.E.- Mass balance (contd..)

- **Vapor:**

\[
\frac{\partial c_v}{\partial t} + \nabla \cdot \left( \rho_g \omega_v v_g \right) = \nabla \cdot \left( \varphi S_g \frac{C^2 M_a M_v D_{eff,g}}{\rho_g} \nabla x_v \right) + \dot{I}
\]

\[
\omega_v + \omega_a = 1
\]

- **Phase change#** (evaporation/condensation):

\[
\dot{I} = K \frac{M_v}{RT} (p_{v,eq} - p_v)
\]

---

For the mixture:

\[
\frac{\partial}{\partial t} \left[ \sum_{i=s,w,v,a} \left( c_i c_{p,i} T \right) \right] + \nabla \cdot \left[ \sum_{i=w,v,a} \left( c_{p,i} n_i T \right) \right] = \nabla \left( k_{\text{eff}} \nabla T \right) - \lambda I + Q_{\text{mic}}
\]

Fluxes:

\[
\begin{align*}
n_v &= \rho_g \omega_v v_g \\
n_a &= \rho_g \omega_a v_g \\
n_w &= \left[ \rho_w v_w - D_{w,\text{cap}} \nabla c_w \right] \\
\end{align*}
\]

Average thermal conductivity:

\[
k_{\text{eff}} = (1 - \varphi) k_s + \varphi \left\{ S_w k_w + S_g \left( \omega_v k_v + \omega_a k_a \right) \right\}
\]
Boundary Conditions

\[ P = P_{amb} \]
\[ n_v = h_m c_v \]
\[ n_w = \rho_w u_w, \quad \text{when } S_w = 1 \]

\[ -k \frac{\partial T}{\partial n} = h(T - T_{oven}) - n_v c_{pv} T - n_w c_{pw} T \]

\[ \text{when } S_w = 1 \]

Oven at temp, \( T_{oven} \)

- Insulated
- Hot air & radiant heating
- Liquid pumping
- Vapor convection

Sample
Computational Scheme

- Microwave combination oven geometry
- Solve Maxwell’s equations of Electromagnetics
  - Source term for microwave heating
  - Solve multiphase porous media model in the sample
- Update dielectric properties
Implementation

- Equations coupled using scripting in COMSOL

Problems

- 3D first time- EM (~2.4M DOFs) and Porous media (~200k DOFs): 42 hrs for 10 min heating: 16 Gb RAM
- Different mesh and solver (stationary, transient, iterative/direct) needed for the two physics
- To make equations implementable in COMSOL additions terms were added-convergence problems
Computed Results

Radiant & hot air

Microwave (10s/50 on), radiant & hot air

Temperature

Moisture
Temperatures & moisture distributions after 10 min of heating

Radiant & hot air

Microwave (10s/50 on), radiant & hot air

Temperature

Moisture
Microwave combination heating increases the speed of heating while maintaining the uniformity.
Conclusions

 First study - complex coupling of Maxwell’s equations with a multiphase porous media model
 Optimum combination of heating parameters can be developed that can speed up the process and maintain heating uniformity at the same time
 Results can be used to develop design recommendations for combination heating for different thermal processes
Ongoing Work

- Experimental validation using Magnetic Resonance Imaging (MRI): UC Davis
- Coupled multiphase porous media-solid mechanics model to study processes with large volume change (e.g. microwave puffing)
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Questions?

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