Computational Modeling of Wave Propagation in a Geophysical Domain

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Overview

- **Objective:**
  Demonstrate the capability of COMSOL Multiphysics to accurately solve wave propagation problems in geophysics

- **Motivation:**
  Reduce reliance on custom software and supercomputers by obtaining solution using commercially available software on high-end desktop computer

- **Approach:**
  - Develop closed-form solutions for
    - Point source in an infinite body
    - Point source on the surface of a semi-infinite body (Lamb’s problem)
  - Develop using solid mechanics module w/ COMSOL
    - Same formulation as acoustics module
    - Three-dimensional
    - Axisymmetric
    - Plane strain
  - Comparison w/ experimental data
    - Hammer blow on surface
Closed Form Solution - Displacement

• Elastic Wave in an Infinite Body

\[ 4 \pi \rho \ u_{ij}(x, t) = \frac{(3 \gamma_i \gamma_j - \delta_{ij})}{r^3} \int_{r/\alpha}^{r/\beta} \frac{f_0(t - \tau) d\tau}{r} + \frac{\gamma_i \gamma_j}{\alpha^2 r} f_0(t - \frac{r}{\alpha}) - \frac{(\gamma_i \gamma_j - \delta_{ij})}{\beta^2 r} f_0(t - \frac{r}{\beta}) \]

• Semi-Infinite Body (Lamb’s Problem - Fixed Poisson ratio)

\[ \begin{align*}
\{w(t)\} &= \frac{\sigma_{33}}{\pi^2 \mu r} \left(\frac{\alpha}{\beta}\right)^2 \left[ \frac{r}{\beta} \int_{-\infty}^{t} df \right]_{t=r/\beta} \quad \{G(\tau - \tau')\} \quad \{R(\tau - \tau')\} \int d\tau'
\end{align*} \]

\[ G(\tau) = \begin{cases}
0, & \tau < 1/\delta \\
\pi \left[ 6 - \left(3\sqrt{3} + 5\right)^{1/2} \right] + \frac{(3\sqrt{3} - 5)^{1/2}}{\left(\sqrt{3} - 1\right)^2}, & 1/\delta < \tau < 1
\end{cases} \\
-\pi/8, & \tau > \gamma
\]

\[ R(\tau) = \begin{cases}
0, & \tau < 1/\delta \\
\pi \left[ 6\kappa(1) - 18\pi(8\kappa^2, k) + (6 - 4\sqrt{3})\pi(20 - 12\sqrt{5})k^2, k \right] + (6 + 4\sqrt{3})\pi(20 + 12\sqrt{5})k^2, k, & 1/\delta < \tau < 1
\end{cases} \\
\pi/8, & \tau > \gamma
\]

Preceding + \frac{\pi \tau}{24} \left(\frac{r^2 - \gamma^2}{\gamma^2}\right)^{1/2}, \quad \tau > \gamma
Point Force Solution – 3D

Quarter Symmetric Model
$N_{DOF} = 574,000$

Long Duration Loading

$\int_{0}^{t} f_0(\tau) d\tau$
Comparison to Analytical Solution – 3D

Long Duration Loading

Path

Subdomain: y-displacement [m]

Analytical solution
FE solution

Arc-length [m]
Point Force Solution – 2D

• Axisymmetric
• Plane Strain

2D Model
\[ N_{DOF} = 195,000 \]
Comparison of 3D and Axisymmetric Vertical Displacement

3D Solution

Axisymmetric solution
Comparison to Analytical Solution - Axisymmetric
Comparison of 3D and Plane Strain
Surface Wave Problem

2D Model
R = 200 m
N_{DOF} = 23,000

Short Duration Loading

\[ f(t) = h \cos^2 \left( \frac{\pi}{T} t \right) \]

\[-\frac{T}{2} \leq t \leq \frac{T}{2}\]

10 ms
Surface Wave Problem – Vertical Velocity

Time = 0.11
Surface: wt/vco*taup

Normalized vertical velocity

Max: 3.65
Min: -5.0

Lamb's Problem

Analysis

FEA

P wave

R = 100 m

Rayleigh
Hammer Data

- Distance = 20 m
- $V = 118$ m/s

Graph showing forces and velocities over time, with a peak at $t=0.17$ s.
Comparison w/ Exp Data

Primary Metrics – arrival time and frequency content

7.5 ms (4%)
Summary

• Closed-form solution developed for
  – Elastic wave in infinite media
  – Elastic wave in semi-infinite media
• Computation models developed using Solid Mechanics Module
  – Three-dimensional
  – Two-dimensional
  • Axisymmetric
  • Plane Strain – not sufficiently accurate for point source
• Comparison with analytical solutions and experimental data
  – Agreement with arrival time, and frequency content
Conclusions

• COMSOL Multiphysics provides a sufficient level of accuracy for the problems of interest
• COMSOL Multiphysics provides a commercially available tool that can solve wave propagation problems on desktop computing resources