Modeling of Respiratory Lung Motion as a Contact Problem of Elasticity Theory

R. Werner, J. Ehrhardt, H. Handels

Department of Medical Informatics, University Medical Center Hamburg-Eppendorf, Hamburg, Germany
Motivation / Clinical Background

• Breathing-induced motion of lung tumors and organs is a significant source of error in radiotherapy of lung tumors

• Enlarging safety margins increases radiation dose delivered to healthy tissues
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  Methods to compensate for respiratory motion during radiation delivery
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Requires detailed knowledge about breathing dynamics
⇒ Motion analysis
⇒ Motion modeling
Motivation / Clinical Background

Key aspect in modeling and motion analysis: Estimation of motion fields in 4D (=3D+t) Data

\[ I_i \subset R^3 \quad \text{image data at different breathing phases} \]
\[ u_i : I_i \rightarrow I_{i+1} \quad \text{motion field estimators} \]
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- Application 1: to quantify respiratory motion (Werner et al., Meth Inform Med 2007)
- Application 2: dose accumulation (Keall et al., Med Phys 33, 2007)
Motivation / Clinical Background

Key aspect in modeling and motion analysis: Estimation of motion fields in 4D (=3D+t) Data

How to integrate (a-priori) knowledge about breathing anatomy and physiology into the modeling process?

- Application 1: to quantify respiratory motion (Werner et al., Meth Inform Med 2007)
- Application 2: dose accumulation (Keall et al., Med Phys 33, 2007)
Methods: Modeling Approach

Aspect to model: Process of lung ventilation

Contraction of breathing muscles (diaphragm, intercostals) ↓
Expansion of the thoracic cavity ↓
Intrapleural pressure increases ↓
Pressure states a surface force, applied to the lung surface ↓
Lung expands, following the expansion of the thoracic cavity

Root of the lung
Visceral pleura = inner wall of the thoracic cavity plus diaphragm
Pleural cavity
Tumor

Parital pleura = lung surface

negative pressure to force lung expansion
Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem
(Zhang et al., Medical Physics 31, 2004; Sarrut et al., IEEE Transactions on Medical Imaging 26, 2007)

Starting Point:
- Lung geometry at end-expiration (EE geometry)
- Lung geometry at end-inspiration (EI geometry)

Modeling Process:
- Apply pressure to expand EE geometry
- Increase the pressure until EE and EI geometries nearly match
- Searched for: Displacement field for the deformed state
Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem

Lung boundary at end expiration (red: deformed boundary)
Lung boundary at end inspiration (= limiting boundary)
Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem

Initial situation

Modeling process
Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem

- Searched for: \( u : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \)

- Equilibrium: \( \nabla \cdot \sigma = 0 \) with \( \sigma = \frac{1}{\det F} F S F^T \)
  
  \( \sigma \) : Cauchy stress tensor  
  \( S \) : 2. Piola-Kirchhoff stress tensor  
  \( F = \nabla u + I \) : Deformation gradient

- Constitutive equation (generalized Hook’s law):
  \[
  S = C(E, \nu) \varepsilon \quad \text{with} \quad \varepsilon = \frac{1}{2} \left( \nabla u + \nabla u^T + \nabla u^T \nabla u \right)
  \]

  \( S \) : 2. Piola-Kirchhoff stress tensor  
  \( \varepsilon \) : Green-Lagrange strain tensor  
  \( C \) : elasticity tensor  
  \( E \) : Young’s modulus  
  \( \nu \) : Poisson's ratio
Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem

- **Dirichlet BC’s:**
  \[ u = 0 \] for the limiting geometry and at the root of the lung

- **von Neumann BC’s:**
  \[ \sigma n = p_{\text{int}} n + p_{\text{contact}} n \]
  - \( p_{\text{int}} \): intrapleural pressure
  - \( n \): surface normal
  - \( p_{\text{contact}} \): contact pressure

- **Signorini conditions:**
  \[ g \geq 0 \land p_{\text{contact}} \leq 0 \land p_{\text{contact}} \cdot g = 0 \]
  - \( g \): gap between EI and deformed EE geometries

Contact conditions
Methods: Implementation

• Data base: 4D CT data
  (Ehrhardt, Werner et al., Medical Physics 34, 2007)
  – 12 Lung tumor patients
    » 6 data sets with small tumors (Ø < 3 cm)
    » 3 with “middle sized” tumors (3 cm < Ø < 5 cm)
    » 3 with big tumors (Ø > 5 cm)
  – Spatial resolution: 1.0 x 1.0 x 1.5 mm
  – 14 breathing phases each patient

• Preprocessing:
  – Lung segmentation at EE and EI
  – Generation of lung surface models (Marching Cubes, Laplace smoothing)
  – Import of surface models to COMSOL via STL
Methods: Implementation

• **Meshing:**
  - Tetrahedral meshes, by COMSOL mesher

• **Solving process:**
  - Increase pressure gradually until:
    \[
    \frac{\text{Volume of deformed EE geometry}}{\text{Volume of limiting geometry}} > 0.995
    \]
  - Each pressure value: Solve contact problem by the Augmented Lagrange algorithm
    » Iterative process, solving the problem in a segregated way
    » Part of the Structural Mechanics Module
Results

Modeling process: Expansion of the initial lung geometry

Color coded:
distance of surface points with respect to their initial positions (in mm)

Details:
Approx. 60 000 tetrahedrons
Solving time: approx. 0.5 h
Results

A) Influence of the elastic constants:

![Graph showing the influence of elastic constants on volume quotient vs. intrapleural pressure.]

<table>
<thead>
<tr>
<th>Literature values:</th>
<th>E [kPa]</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>West et al., J Appl Physiol 32, 1972</td>
<td>0.25</td>
<td>0.3</td>
</tr>
<tr>
<td>De Wilde et al., J Appl Physiol 52, 1981</td>
<td>0.73</td>
<td>0.3</td>
</tr>
<tr>
<td>Zhang et al., Med Phys 31, 2004</td>
<td>4</td>
<td>0.35</td>
</tr>
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<td>Sundaram et al., Med Image Anal 9, 2005</td>
<td>0.1</td>
<td>0.2</td>
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<td>0.823</td>
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Results

A) Influence of the elastic constants:

1) Pressure to fulfill a volume ratio of >0.995 depends on E and ν values

2) BUT: Differences in displacement vectors are small (for the given application!): < 0.5 mm in magnitude

⇒ Choice of E and ν values of minor impact!

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Results

B) Modeling accuracy

Evaluation concept:

• Evaluation based on landmarks identified by an medical expert

• 35 to 45 landmarks each lung and each breathing phase (here: only EE and EI)

• Quality measure of model based predicted motion field:

\[ R_k = \delta p_k^{\text{predicted}} - \delta p_k^{\text{actual}} \]

[„Registration residual“]
Results

B) Modeling accuracy

Results averaged over all patients:

- Mean landmark motion observed: 6.6 ± 5.2 mm
- Intraobserver variability: 0.9 ± 0.8 mm
- No systematic prediction error: $R_{CC} \approx R_{AP} \approx R_{ML} \approx 0$
- Registration residual magnitude: 3.3 ± 2.1 mm
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Literature values (non-linear registration), e.g. Vik et al. [Philips Medical Systems], SPIE Medical Imaging 2008:

- 2.5 ± 2.2 mm (surface-tracking)
- 2.9 ± 3.1 mm (B-Spline based reg.)
- 3.3 ± 3.1 mm (non-parametric reg.)
Results

B) Modeling accuracy

Influence of lung tumors on modeling accuracy:

<table>
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<th>Distance from tumor</th>
<th>Registration residual magnitude $| R |$ in mm</th>
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<td>far from the tumor</td>
<td>$2.8 \pm 1.6$</td>
</tr>
<tr>
<td>near the tumor</td>
<td>$3.1 \pm 1.6$</td>
</tr>
<tr>
<td>close to the tumor</td>
<td>$4.2 \pm 2.4$</td>
</tr>
<tr>
<td>mean</td>
<td>$4.6 \pm 2.7$</td>
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⇒ Modeling accuracy decreases with increasing tumor size!
Summary & Discussion

• Biophysical approach to modeling respiratory lung motion
  – Lung ventilation as a contact problem of linear elasticity theory

• Implementation using FEM / COMSOL SME Module

• Evaluation of model accuracy by means of 4D CT data
  – Registration residual of approx. 3 mm

⇒ Modeling approach promising, but …

• Assumption of lung tissue to be homogenous oversimplifying!
  – Explicit modeling of inner lung structures
  – Import of “complex” structures to COMSOL?
Left lung at EE (i.e. the initial, the undeformed geometry)

Deformed left lung (i.e. lung at the final situation, the predicted lung shape at EI)

Gap between initial and final geometry (red: > 20 mm)

Thank you for your attention.