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Space-Time Formulation for Finite-Element Modeling of Superconductors

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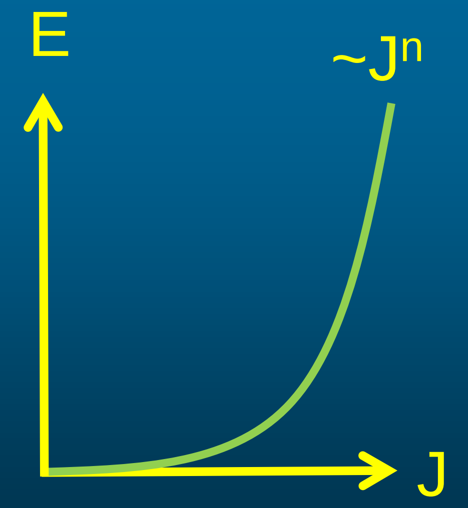
Content

- Modeling high-temperature superconductors (HTS)
  - Motivation
  - Currently used ‘standard’ models
  - Ideas for more efficient modeling
- Periodic space-time (PST) formulation
  - Implementation and results for 3 cases
    - Infinite slab in applied parallel ac field
    - Round wire carrying ac current
    - Multiple conductors carrying arbitrary ac currents
- Conclusion
  - Overall performance of PST
  - Open issues and further work

Motivation

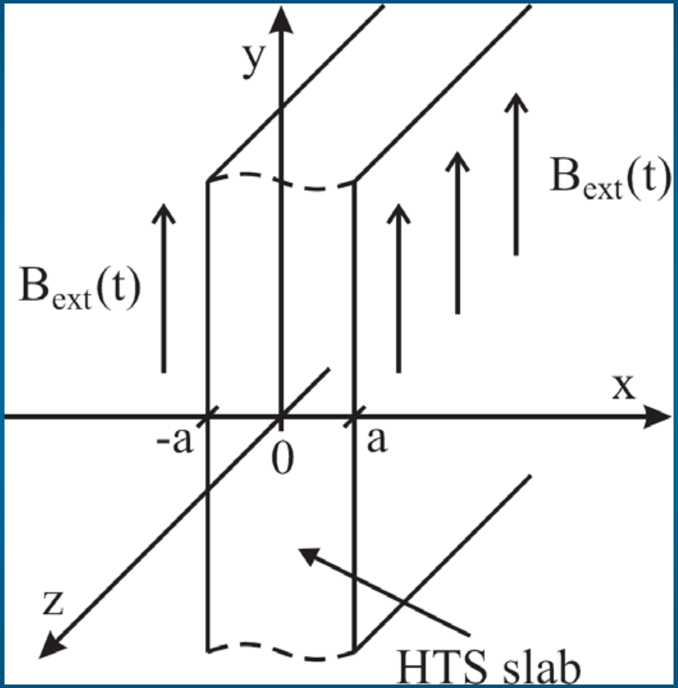
- Ultimate goal: AC losses in HTS wires and devices
  - Necessary to compute current density and field distributions
  - Assumption: constant T, no thermal model
- Electromagnetic part modeled by Maxwell equations
  - Different formulations possible
  - Used magnetic field components as state variables
  - Edge elements to verify zero-divergence equation for B
- Superconductivity: non-linear resistance
  - Power-law: $\rho(J)=E_c/J_c\,|J/J_c|^{n-1}$
  - AC/DC modules cannot be used
  - Numerically challenging (n~25-50)
- Present models work nicely
  - But are quite slow
  - Not ideal for design optimization
    - Parametric studies, numerous simulations



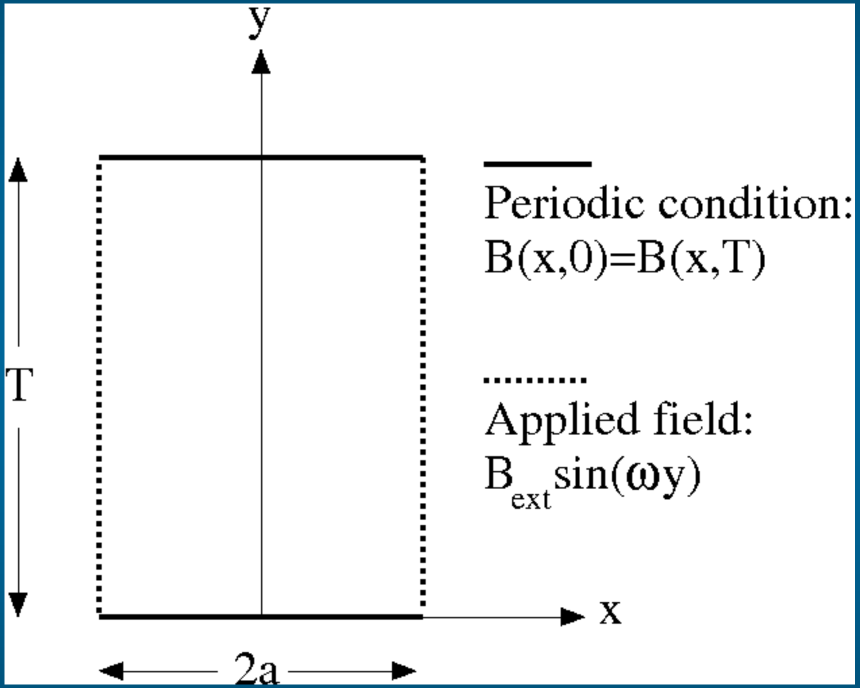
Periodic Space-Time (PST) formulation

- Basic idea: use a space dimension to represent time
  - 1D transient problem becomes 2D static one
  - 2D transient problem becomes 3D static one
- PST should be faster beacuse:
  - 1 time step
  - Problem better handled by (static) solver
  - Solution can start from previously computed solution
    - Useful for parametric studies, e.g. Losses vs current
  - Adaptive mesh
- Considered cases
  1. Infinite superconducting slab in external parallel field
  2. Round superconducting wire carrying AC current
  3. Multiple rectangular superconductors carrying different currents

Infinite slab in applied parallel field

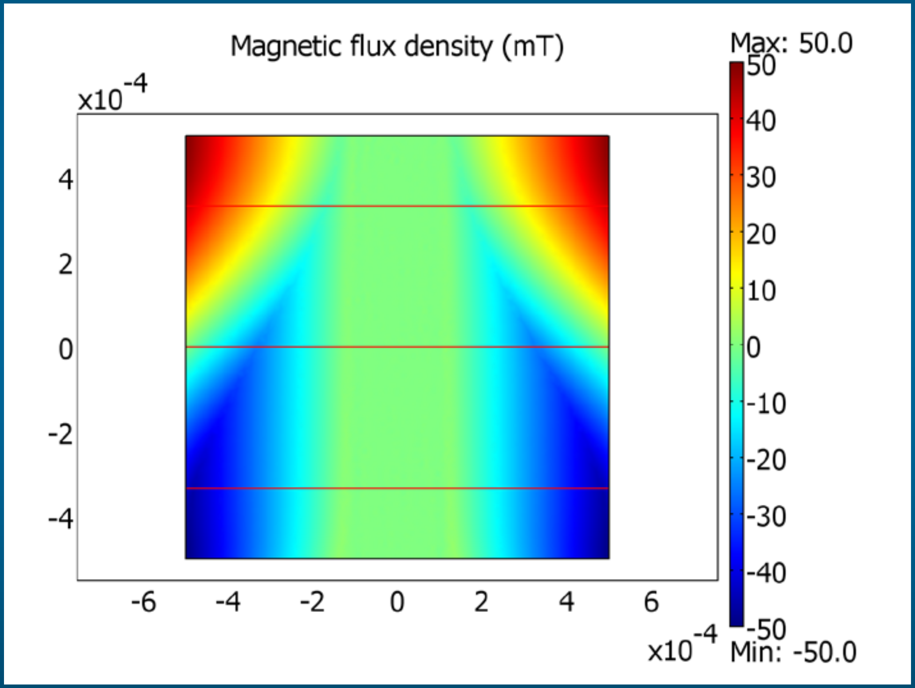


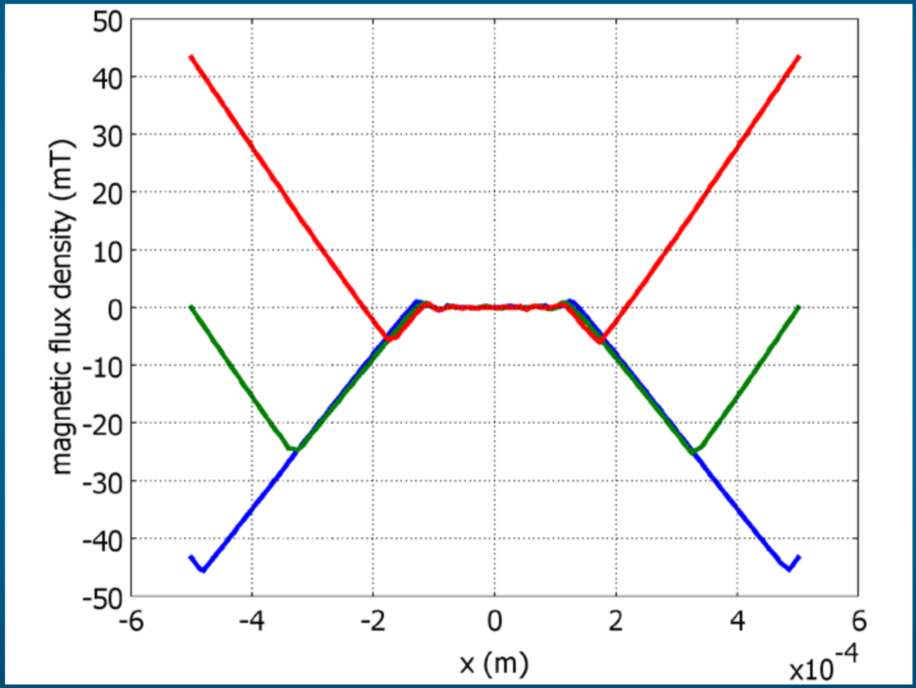
Real problem



PST model

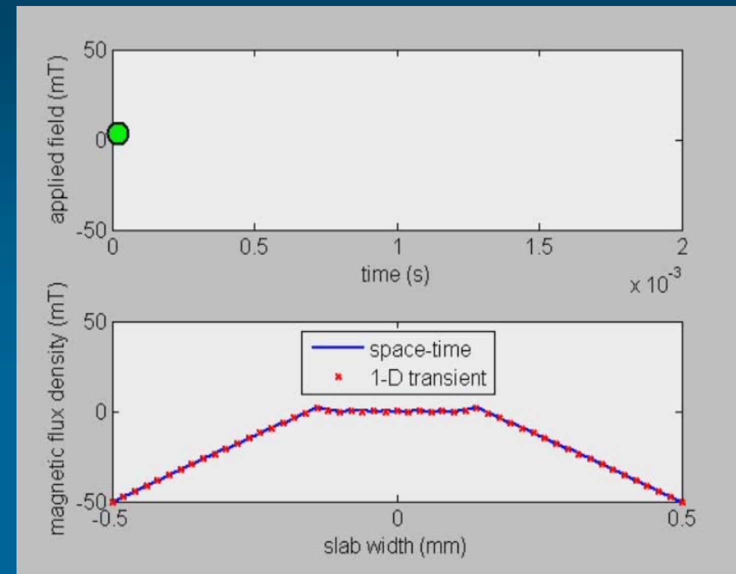
Infinite slab in applied parallel field

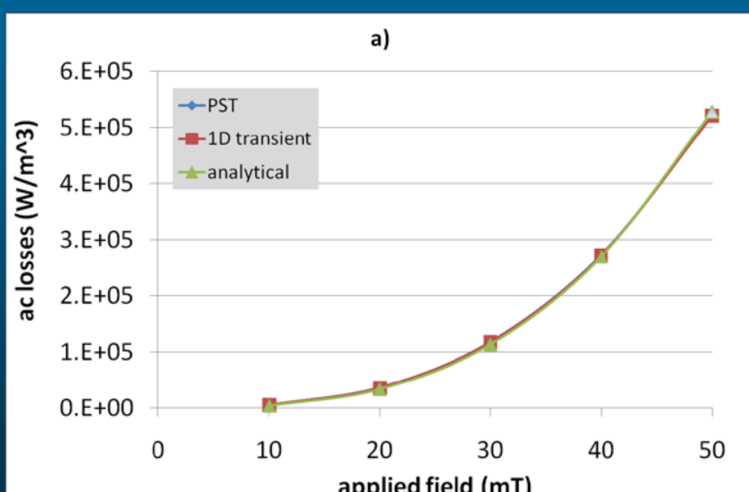


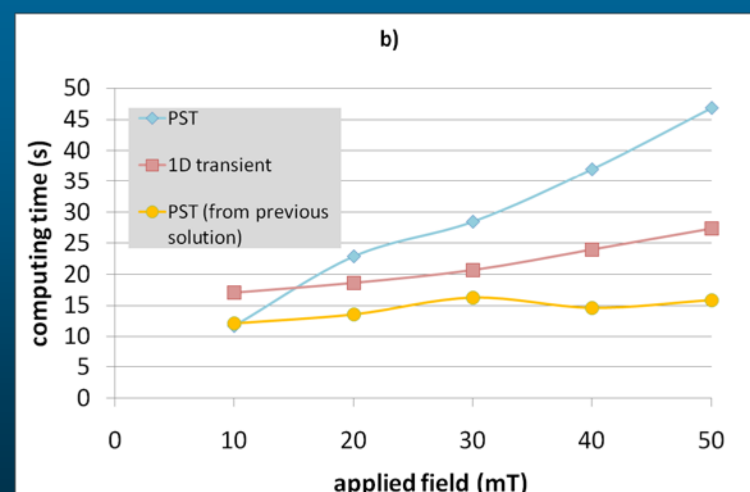


Comparison with transient ('standard') model

- Excellent agreement
  - Magnetic field and current density profiles
  - AC losses
- Faster if starting from previous solution
  - More substantial advantage expected for more complex cases







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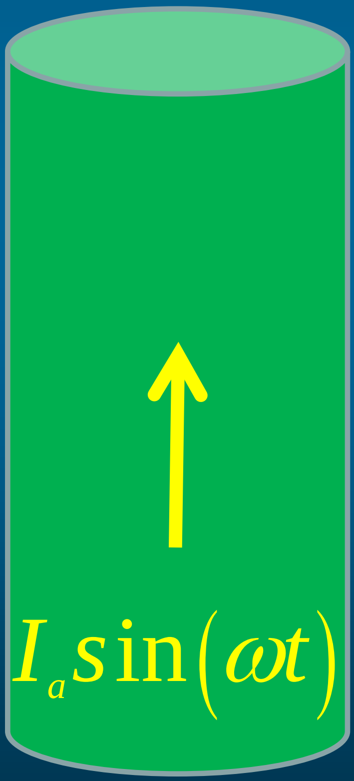
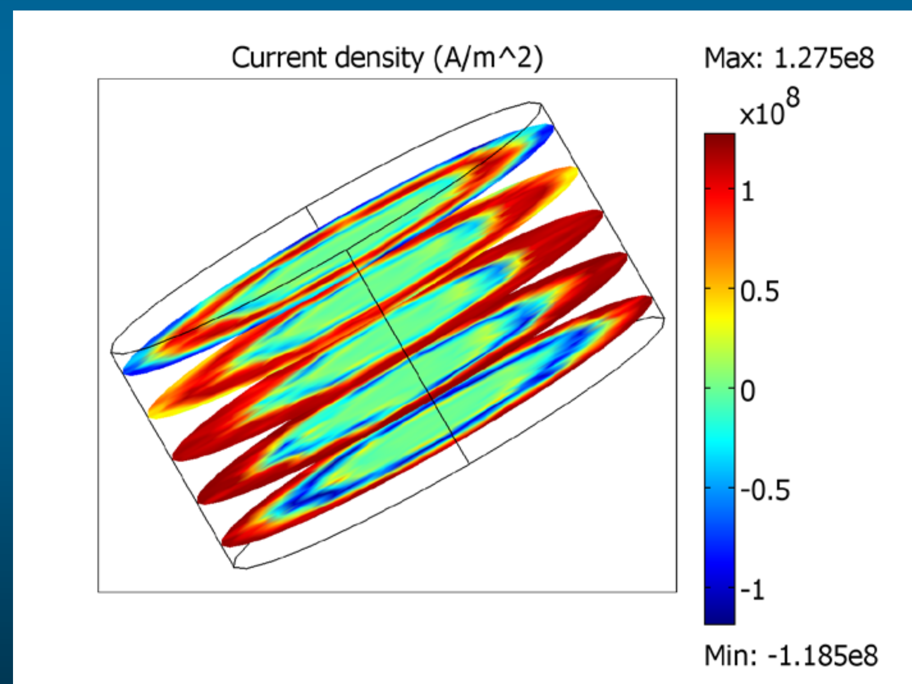
Round conductor carrying AC current

- Current imposed by boundary conditions for the magnetic field
- Cylinder axis represents the time

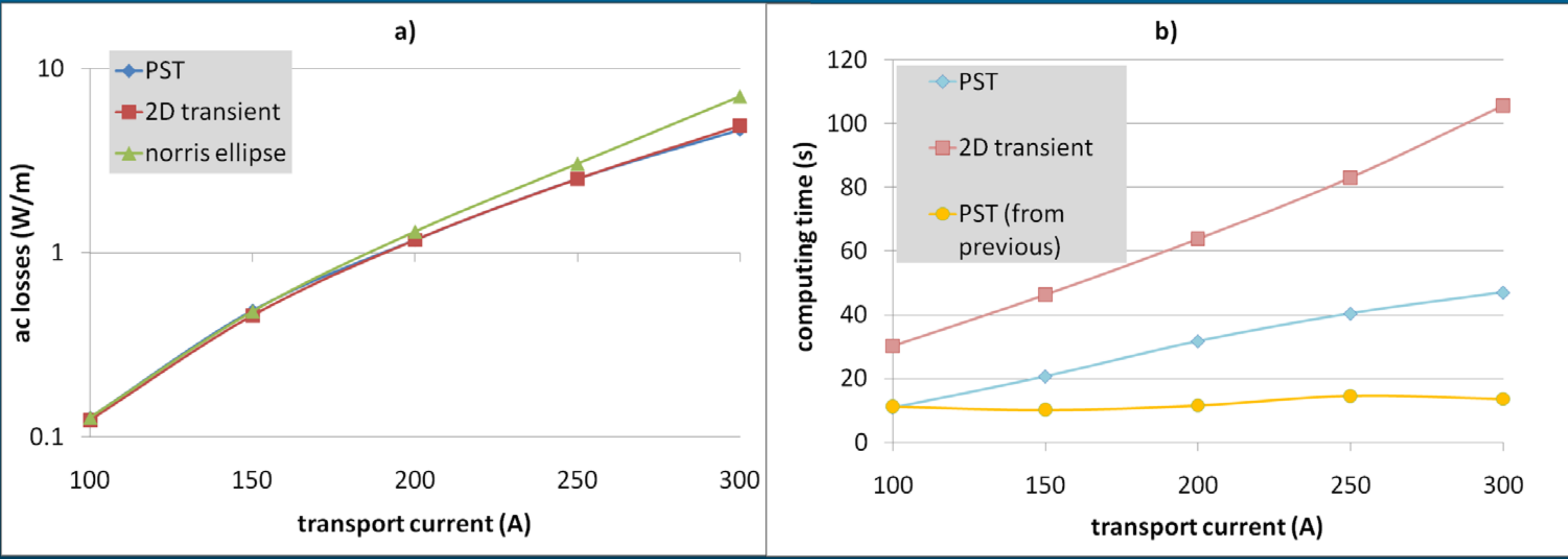
$$H_\phi = \frac{I_a}{2\pi}\sin(\omega t)$$





Round conductor carrying AC current

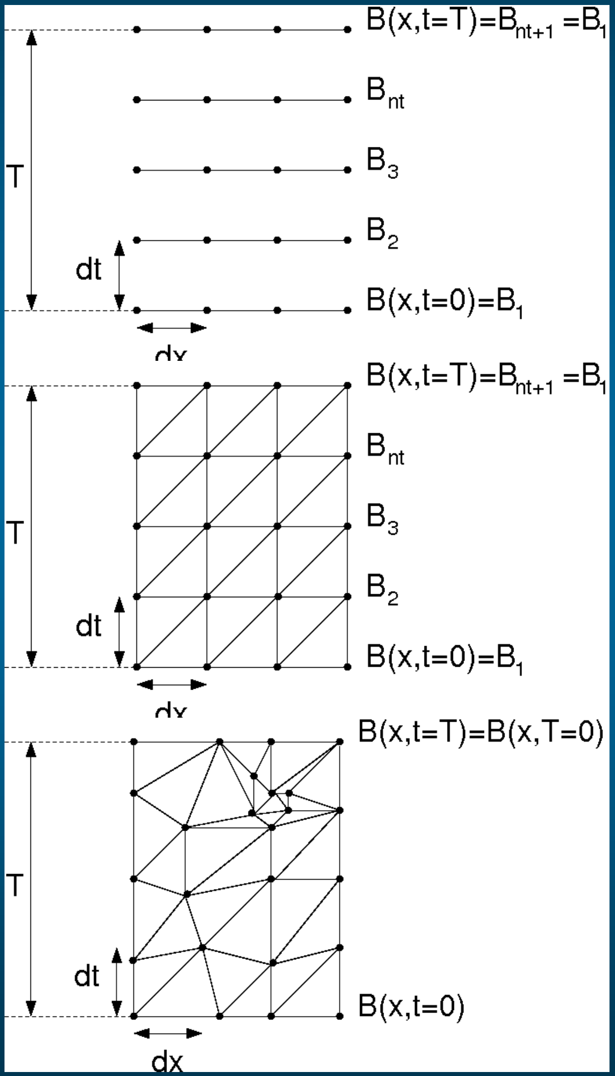
- Very good agreement with the standard transient model
- Dramatic gain in computation speed
  - Especially starting from previous solution



Extension to multiple conductors

- Cases presented so far have common characteristic:
  - Boundary conditions are known
  - Used to impose the field (slab) or the current (round conductor)
- What happens with multiple conductors (of arbitrary shape)?
  - We need to simulate air domain
  - We can still use the field to impose the current
  - We don’t have control on individual currents
    - We can simulate only conductors in parallel
    - Not useful for real applications (coils, bifilar winding)
- Impose current by integral constraints
  - Different possible ways to do that

The three methods



1. Use finite differences for dB/dt term, impose current constraint at $z=z_n$ planes: series of coupled 2-D problems

2. Approximate dB/dt with a weak formulation, mesh by extrusion, impose current at $z=z_n$ planes

3. Approximate dB/dt with a weak formulation, mesh whole domain, how to impose current constraints?

Method #1: finite differences

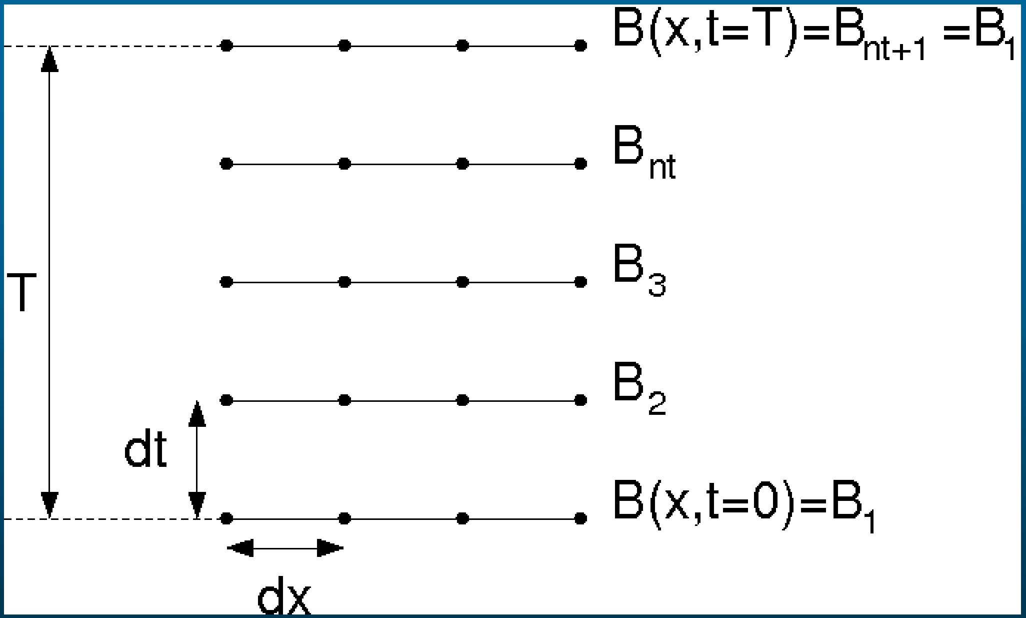
- General diffusion equation $\nabla\times\frac{\rho}{\mu_0}\nabla\times B=-\frac{\partial B}{\partial t}$
- Finite-difference approximation $\frac{\partial B}{\partial t}\approx\frac{B(t+\Delta t)-B(t-\Delta t)}{2\Delta t}$
- A number of coupled "layers"

$$\nabla\times\frac{\rho}{\mu_0}\nabla\times B_1=\frac{B_2-B_0}{2\Delta t}$$

$$\nabla\times\frac{\rho}{\mu_0}\nabla\times B_2=\frac{B_3-B_1}{2\Delta t}$$

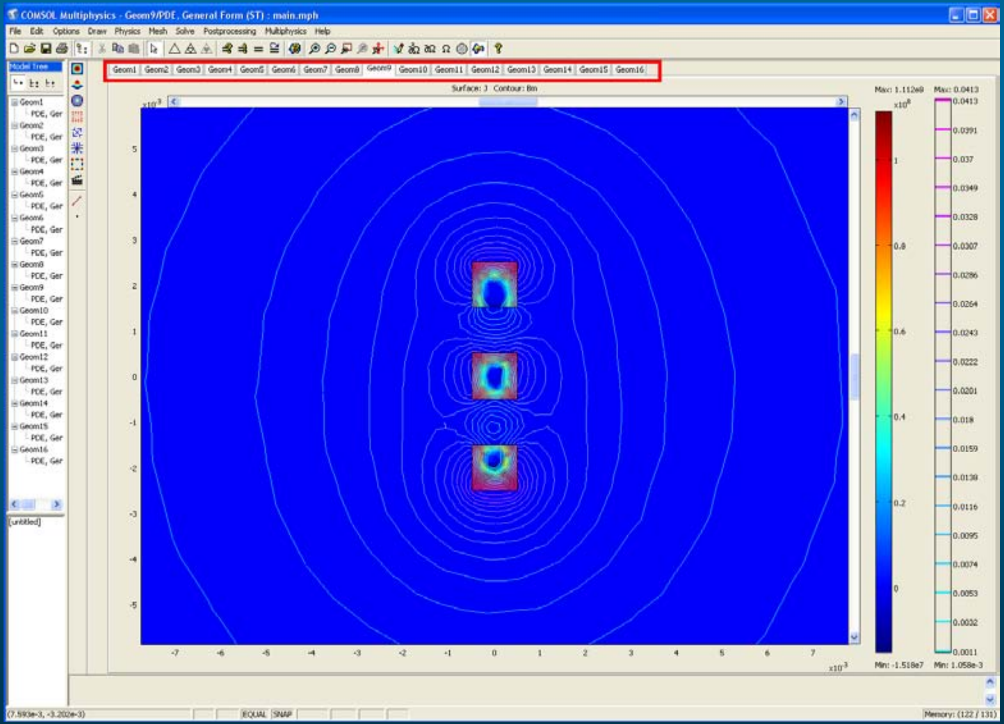
...

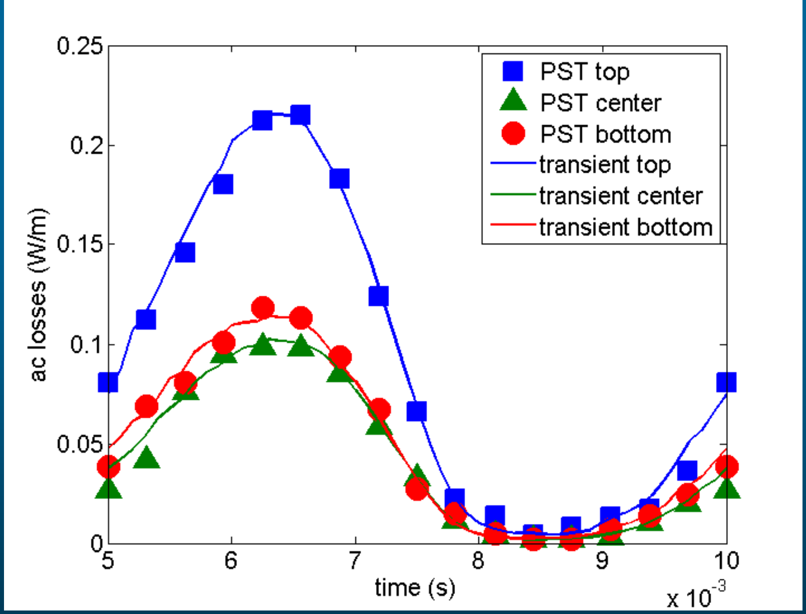
$$\nabla\times\frac{\rho}{\mu_0}\nabla\times B_{n_t}=\frac{B_{n_t+1}-B_{n_t-1}}{2\Delta t}$$



Method #1: finite differences

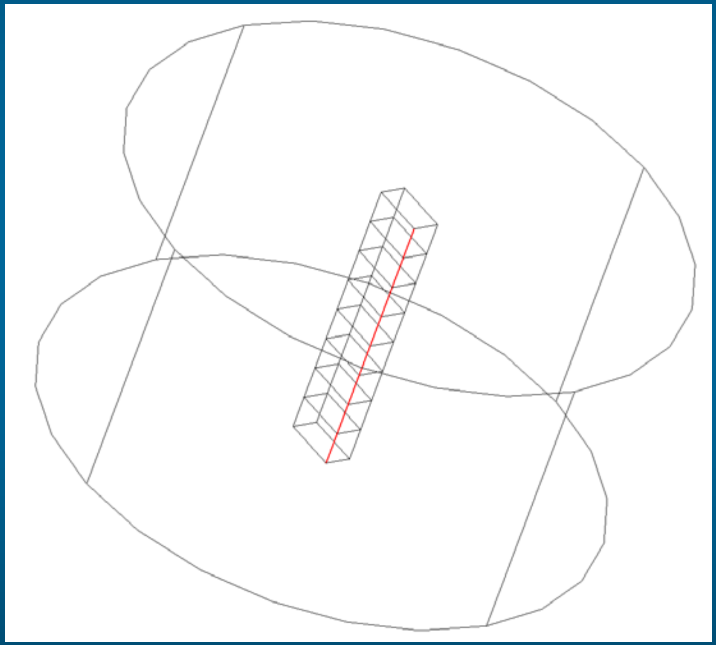
- The method works
  - Proves correctness of the approach for multiple conductors
  - Far from being optimal
    - Very slow compared to corresponding transient model
    - Doesn't really use features of PST (3-D mesh, adaption, etc.)

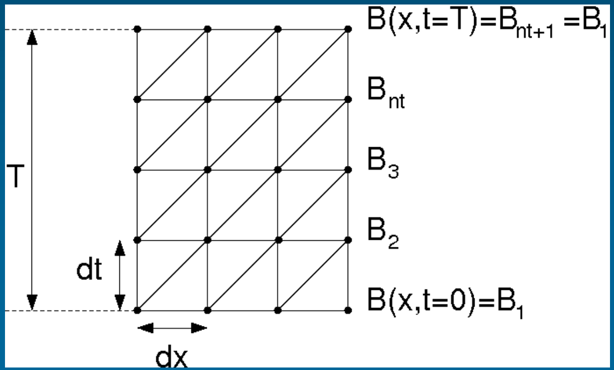


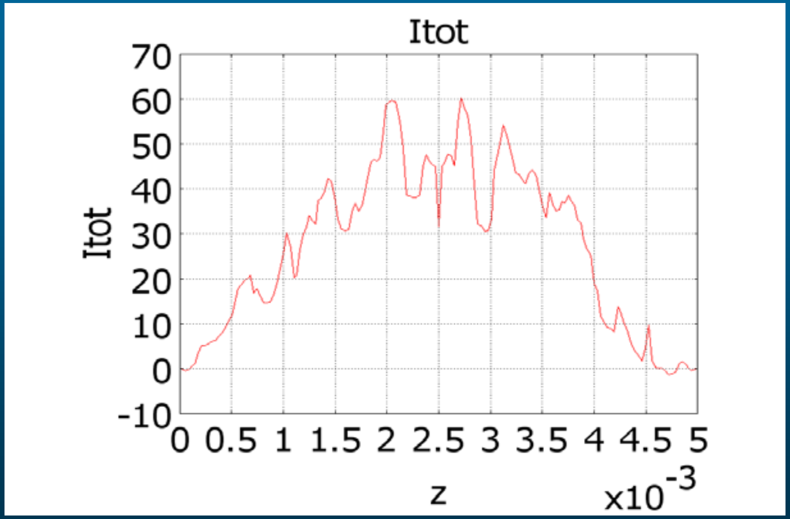


Methods #2 and #3: some difficulties...

- Method #2: the current constraint set on the planes is not satisfied



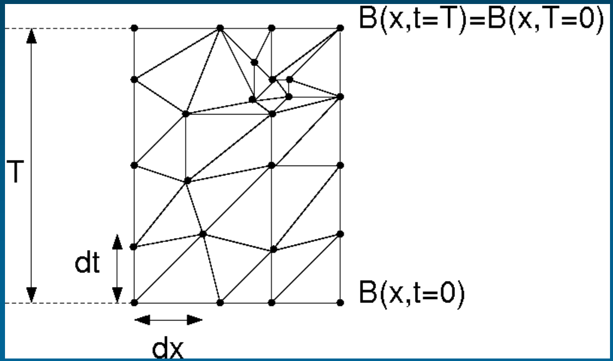




Method #3:

- For each conductor, integrate J(x,y,z) along the conductors’ cross-section

$$I(z)=\int_x\int_y J(x,y,z)dz$$



- Impose I(z) equal to the current we want, e.g. $I_0\sin(\omega t)=I_0\sin(\omega z)$
- How to impose this constraint?
  - Use of projection/extrusion coupling variables
  - Weak forms
- Unsuccessful so far

Conclusion

- Implemented Periodic Space-Time formulation for computing AC losses in high-temperature superconductors
- Developed examples show correctness of the approach
- Simple cases (slab, round conductors) are faster to solve than with standard time-dependent models
- Case of multiple conductors of arbitrary shape is the most interesting for practical application
  - Different approaches possible
  - Finite-difference method works, but not interesting in practice
  - Most flexible approach (‘method 3’) not simple to implement