Depth-Averaged Modeling of Groundwater Flow and Transport

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Outline

- Pedagogical Aspects (“Comsol in Education”)
- Importance of 2-D Depth-Averaged Flow
- Some Examples
Pedagogical Aspects

- We have largely completed a transition to numerical tools to teach:
  - Flow in porous media and
  - Groundwater flow.

- Challenges:
  - Hydrogeologists and Environmental Engineers have their own style of learning numerical methods
  - Little “glitches” in COMSOL appear to students like major stumbling blocks.
A strategy that seems to work: Do a sequence of many problems that improve, simultaneously, grasp of mechanics and mathematical modeling skills.

“Learn on the job” with minimal discussion of numerical methods.

Start from simple problems and end up with actual applications.
Why 2-D Averaged?

- In many practical applications, flow is in thin and (nearly) horizontal layers. Head differences over the depth are small.
- 2-D modeling is easier computationally and in terms of data requirements.
Injection
Extraction
And
MLS Wells
Plan View: Idealized 2-d flow field

- Protective outer loop
- U(VI) reduction zone
- FW103
- FW026
- FW104

Plan View:
Evidence of uranium(VI) reduction

Phase I              Phase II              Phase III

Terminal electron-accepting sequence

Nitrate inhibition

Bicarbonate concentration: U(VI) sorption/desorption model

Sorption site distribution: kinetic U(VI) sorption/desorption

Effective reduction rate: $v_{U(VI)} = 1 \text{mmol U(VI)/mg SRB-day}$
Delivery and Mixing in Wells: Edwards AFB

Idealized vertical cross-section

From Edwards AFB Pilot Study (McCarty et al.)

Time of Operation (days)
\[ \delta_s \frac{\partial S}{\partial t} + \nabla \cdot \left[ -\delta K \left( \rho_f \frac{\partial H}{\partial t} \right) \right] = \delta_Q, \quad H = \frac{p}{(\rho_f g)} + D \]
\[
\delta_S \frac{\partial H}{\partial t} + \nabla \cdot \left[ - \delta_k K / (\rho_g) \nabla (p + p_g D) \right] = \delta Q_S, \quad H = p / (\rho_g) + D
\]
Global Equations

Equation: \( f(u, u_t, u_{tt}, t) = 0 \)

<table>
<thead>
<tr>
<th>Name (u)</th>
<th>Equation ( f(u, u_t, u_{tt}, t) )</th>
<th>Init (u)</th>
<th>Init (ut)</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>phie</td>
<td>OUTe + Q + pi * rw^2 * phiet</td>
<td>16</td>
<td>0</td>
<td></td>
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<tr>
<td>phii</td>
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Base unit system: SI
Delivery and Mixing in Wells: Edwards AFB

Subdomain Settings - Solute Transport (esst)

Equation

\[
\delta_{ts1} \left( \theta_s + \rho_b \frac{\partial c_p}{\partial c} \frac{\partial c}{\partial t} + \nabla \cdot ( - \theta_s D_L \nabla c \right) = -u \cdot \nabla c + R_L + R_p + S_c
\]

Flow and Media

<table>
<thead>
<tr>
<th>Flow Field</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_{ts1})</td>
<td>1/86400</td>
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<tr>
<td>(\theta_s)</td>
<td>(\text{eta*H})</td>
</tr>
<tr>
<td>(u)</td>
<td>(u_{esdl})</td>
</tr>
<tr>
<td>(v)</td>
<td>(v_{esdl})</td>
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<tr>
<td>(Q_s)</td>
<td>0</td>
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</table>

Unit:
- 1: Time-scaling coefficient
- \(\text{eta*H}\): Pore volume fraction
- \(u_{esdl}\): x-velocity
- \(v_{esdl}\): y-velocity
- 0: Liquid source

Subdomains: 1

Group:
- Select by group
- Active in this domain

Buttons:
- OK
- Cancel
- Apply
- Help
Several Other Issues

For example, dealing with transport:

- Recirculation;
- Sorption
- Multiple components and reactions (instantaneous, Monod kinetics, reversible).
Conclusion

For Groundwater Flow and Transport: COMSOL Multiphysics can be a valuable tool in teaching processes and in practical applications.