A FEM Study of displacement sensor based on L-L Magnetostrictive/Piezoelectric block magnetoelectric composite material

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Introduction

- **Magnetoelectric effect (ME)** is the phenomenon of inducing magnetic (electric) polarization by applying an external electric (magnetic) field.
Polarization

Mech. Strain

ε

ε

σ

P

E

M

H

N

S

Magnetization

Magneti-

zation

P

+-

++

+-

++

+-

++
Product Property:

\[ ME_{H\text{effect}} = \frac{\text{magnetic}}{\text{mechanical}} \times \frac{\text{mechanical}}{\text{electric}} \]

\[ ME_{E\text{effect}} = \frac{\text{electric}}{\text{mechanical}} \times \frac{\text{mechanical}}{\text{magnetic}} \]
- **Magnetostrictive effect** is a property of ferromagnetic materials that causes them to change their shape or dimensions during the process of magnetization.

- **Piezoelectric effect** is the ability of certain materials to generate an electric charge in response to applied mechanical stress.
Magnetostrictive layer

Piezoelectric layer

Magnetostrictive rod

Piezoelectric plate

Fixed boundary with rigid frame
Modelling

- Magnetostrictive nonlinear constitutive equation

\[
\varepsilon_i = \frac{3}{2} \lambda_s \left( \left( \frac{m_i}{M_i} \right)^2 - \frac{1}{3} \right)
\]

\[
\varepsilon_\perp = \lambda_s \left( \frac{M}{M_s} \right)^2, \quad \varepsilon_\parallel = -\frac{\lambda_s}{2} \left( \frac{M}{M_s} \right)^2
\]

\[
H_e = H + \alpha M + H_\sigma
\]

\[
\sigma = E \left[ \varepsilon - \lambda(\sigma, H) \right]
\]

\[
B = \mu_0 H + \mu_0 M(s, H)
\]

\[
\varepsilon_x = -\frac{\lambda_s}{2} \left( \frac{M_x}{M_s} \right)^2, \quad \varepsilon_y = -\frac{\lambda_s}{2} \left( \frac{M_y}{M_s} \right)^2, \quad \varepsilon_z = \lambda_s \left( \frac{M_z}{M_s} \right)^2
\]
Modelling

- Piezoelectric linear constitutive equation

\[ \sigma_e = c_e \varepsilon_e - eE \]

\[ D = e^T \varepsilon_e + \kappa E \]
Implementation with COMSOL
Implementation with COMSOL
Implementation with COMSOL

- Geometry
Implementation with COMSOL

- **Realization of PZT material model**
  - The linear constitutive equations for piezoelectric material
    \[ s - S_0 = c_E \varepsilon (\varepsilon - \varepsilon_0) - e^T E \]
    \[ D = D_r + e (\varepsilon - \varepsilon_0) + \kappa E \]
  - For solid mechanics, the elastic relations
    \[ \varepsilon = \frac{1}{2} [(\nabla u)^T + \nabla u] \]
    \[ \sigma = s \]
    \[ -\nabla \sigma = F_v \]
  - For electrostatics, the electrical relations:
    \[ \nabla E = \rho_v \]
    \[ E = -\nabla V \]
Implementation with COMSOL

- Realization of magnetostrictive material model
  - The elastic relations

\[
\sigma = c_E \varepsilon_0 (\varepsilon - \varepsilon_0)
\]

\[
\varepsilon_0 = \text{diag} \left( \frac{-\lambda}{2}, \frac{-\lambda}{2}, \frac{-\lambda}{2} \right)
\]
<table>
<thead>
<tr>
<th>Properties</th>
<th>PMN–28PT</th>
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<tbody>
<tr>
<td>$\rho$, kg m$^{-3}$</td>
<td>8060</td>
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<tr>
<td>$c_{11}^E$, GPa</td>
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<tr>
<td>$c_{12}^E$, GPa</td>
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<td>$c_{13}^E$, GPa</td>
<td>102.6</td>
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<td>$c_{33}^E$, GPa</td>
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<td>$c_{44}^E$, GPa</td>
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<td>$c_{66}^E$, GPa</td>
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<td>$\varepsilon_{11}^S / \varepsilon_0$</td>
<td>925</td>
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<td>$\varepsilon_{33}^S / \varepsilon_0$</td>
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<td>$e_{13}$, C m$^{-2}$</td>
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<td>$e_{15}$, C m$^{-2}$</td>
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<tr>
<td>$e_{33}$, C m$^{-2}$</td>
<td>20.5</td>
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</tbody>
</table>
Implementation with COMSOL

- Boundary conditions & Mesh
Results
[Magnetic field, Z component along MS rod's Z axis (A/m)]

- Magnetic field, Z component / A/m
- Z axis position of MS rod / mm

Graph showing the variation of magnetic field along the Z axis of the MS rod as the Z axis position changes.
Total displacement along MS rod's Z axis (mm)
Thanks.