Electric Field Distributions and Energy Transfer in Waveguide-Based Axial-Type Microwave Plasma Source

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Abstract: In this paper, we examine changes of the electric field distributions in waveguide-based axial-type microwave plasma source (MPS) during tuning procedure. The distributions strongly depend on position of the movable short, so does the wave reflection coefficient of the incident wave. A method of determining tuning characteristics of the MPS consisting in treating the MPS as a two-port network, finding its scattering matrix coefficients and then calculating the reflection coefficient from analytical expressions is proposed. The results of calculations show that the tuning characteristics depend on plasma parameters such as the electron density and on MPS dimensions such as the height of the reduced-height waveguide section. It is possible to find such a position of the movable short for which the reflected wave power is less than 10% of the incident wave power.

Keywords: plasma sources, microwave discharges, modeling, electric field distribution

1. Introduction

Microwave discharges as plasma sources have been used in spectroscopy, technological processes like surface treatment, carbon nanotubes synthesis and sterilization and different kind of gas processing, such as purification of gases, abatement of fluorinated compounds gases, decontamination of chemical warfare agents. Applying microwave discharges for hydrogen production via hydrocarbons reforming have been reported recently [1,2].

In this paper, we analyze electrodynamics properties of a waveguide-based axial-type microwave plasma source (MPS) intended for hydrogen production by methane decomposition, whose sketch is shown in Fig. 1. Microwave power is supplied to the MPS via a standard rectangular waveguide with a reduced-height section in the discharge region. A movable short on the other side of the waveguide is a means of tuning, which allows for impedance matching and efficient transfer of the microwave power to the plasma. The discharge takes place in a quartz tube, which penetrates through the wider waveguide walls. The processed gas is delivered to the discharge region through a metallic tube. The discharge is stabilized by an additional swirled gas flow. The two metallic tubes, through which the gases are introduced, form a coaxial line shorted at the bottom. A shielding metallic tube placed coaxially with the discharge tube in its upper part forms a circular waveguide operating below cut-off. The presented MPS allows for operation at incident microwave power levels up to 6 kW with the gas flow rate 50 – 200 l/min and it needs no special cooling system or sophisticated impedance matching. However, an optimization of the MPS is required to assure stability of the discharge and increase the efficiency of hydrogen production.

One of essential characteristics of any MPS is power transfer from the feeding line to the plasma. It can be expressed as ratio $P_R/P_I$, where $P_I$, $P_R$ are the power of the incident and reflected...
waves, respectively. The ratio is equal to the square of the module of $\Gamma$, the wave reflection coefficient. Tuning characteristics of the MPS is a dependence of $P_R/P_I$ on the position of the movable short normalized to the wavelength. They are easy to obtain experimentally and give an important information about the plasma source. The MPS is efficient when the value of ratio $P_R/P_I$ is small; it is stable when the ratio does not depend on the short position in a wide range.

In general, the tuning characteristics depend on MPS’s geometry and dimensions as well as plasma parameters. They can be obtained theoretically using equivalent circuit theory if characteristic MPS dimensions are much smaller than the wavelength [3,4]. This is not the case for our MPS, for which characteristic dimensions are in the range 0.2...0.5 of the wavelength, so the analysis of the power transfer has to be done in terms of electromagnetic field distributions.

The purpose of this work is to calculate tuning characteristics and 3D distributions of the electric field inside the MPS for different positions of the movable short with the aim to optimize the MPS so that it works efficient and stable for different plasma parameters.

2. Method of determining the tuning characteristics

The tuning characteristics can be obtained using COMSOL by building such a geometry of the MPS in which the movable short is represented by a segment of a rectangular waveguide of regulated length $l$, short-circuited at the end. Changing the length of this segment (which is equivalent to changing the position of the movable short) and determining the wave reflection coefficient for each length allows one to obtain the needed dependence:

$$P_R/P_I = |\Gamma|^2 = f(l). \tag{1}$$

This approach is useful when electromagnetic field distributions inside the MPS during the tuning are of interest. However, it might be not effective if only tuning characteristics are of importance, because it needs recalculation for every position of the short. To obtain the tuning characteristics in a more effective way we propose method as follows.

Let us treat the MPS as a two-port network terminated with a short-circuited transmission line of the length $l$ (see Fig. 2). The two-port can be represented by its scattering matrix. The reflection coefficient in the input plane of the two-port, $\Gamma_{in}$ can be calculated from [5]:

$$\Gamma_{in} = s_{11} + \frac{s_{12}s_{21}\Gamma_l}{1 - s_{22}\Gamma_l}, \tag{2}$$

where $s_{11}$, $s_{12}$, $s_{21}$, $s_{22}$ are the elements of scattering matrix, $\Gamma_l$ is the reflection coefficient from the short circuit in the output plane of two-port and it can be find from

$$\Gamma_l = \frac{z_l-1}{z_l+1}, \tag{3}$$

where $z_l$ is the normalized impedance of the short circuit at distance $l$. This impedance is given by

$$z_l = j\tan(2\pi l/\lambda_g), \tag{4}$$

where $\lambda_g$ is the wavelength in the waveguide and $j = \sqrt{-1}$.

With the above equations, the tuning characteristics can be determined without doing calculations for every position of the movable short. Instead of this one can calculate only the elements of the scattering matrix $s_{ij}$ and then determine the reflection coefficient for any position of the short from eqs. (2) - (4).

To obtain the elements of the scattering matrix $s_{ij}$ ($i, j = 1, 2$) of a two-port, it should be from the definition - terminated with a matched load. This means, however, that in this method one cannot obtain the electromagnetic field distributions inside the real MPS terminated with the short.
3. Models

3.1. Geometry and boundary conditions

For determining the electromagnetic field distributions in the MPS we use 3D geometry as presented in Fig. 3. The object CO1 represents the main part of the MPS comprising both the waveguide and the axially symmetric parts; the cylinder REV1, placed above the inner conductor of the coaxial line, represents the plasma region; the cuboid BLK1 represents the short-circuited rectangular waveguide of the length \( l \). All the boundary used are perfect electric conductor except for the input plane where microwave power is supplied to the MPS. Port condition with rectangular mode number 10 is imposed on this boundary.

For determining the elements of the scattering matrix we use similar geometry as in fig. 3, but without the cuboid BLK1. To obtain the same conditions as for perfectly matched load we impose the port condition (of number 2) with rectangular mode number 10 on the output boundary.

3.2. Subdomain settings

We assume that entire region of the MPS except for the REV1 cylinder is filled with air. The plasma region (REV1) is filled with homogenous plasma of the complex relative electric permittivity \( \varepsilon_p \) given by the Lorentz formula

\[
\varepsilon_p = 1 - \frac{n}{1 - js}
\]  

(5)

where \( n = n_e/n_c \) and \( s = v/\omega \) are the normalized electron density and electron collision frequency, respectively, \( n_c = \omega^2 \varepsilon_0 m_e/\varepsilon^2 \) is the critical electron density, \( n_e \) is the electron density, \( v \) is the electron-neutral collision frequency, \( \omega \) is the electron-neutral collision frequency for momentum transfer.

3.3. Electromagnetic mode

Application mode type Electromagnetic Waves, Harmonic propagation from RF Module is used for the calculations.

4. Numerical results

4.1. Initial data

All calculations we perform for the field frequency \( f = 2.45 \text{ GHz} \) and a standard rectangular waveguide WR430 as the main part of the feeding line. The lengths of the tapered waveguides and the reduced-height section are \( \lambda_f/4 = 74 \text{ mm} \). According to our experimental results, we take that the cylindrical region occupied by plasma has the length 6 cm and the radius 1 cm and is displaced 0.5 cm up from the gas inlet tube end.

4.2. Electric field distribution and tuning characteristics for MPS with movable short

Distributions of the module of electric field inside the MPS for different positions of the movable short and for plasma parameters \( s = 0.1 \) and \( n = 10 \) are shown in fig. 4. They are normalized such that input power 1 W is assumed for every case. The height of the reduced-height section is \( h = 10 \text{ mm} \). It is seen that the distributions of electric field strongly depend of the short position. Standing wave patterns in the waveguide on both sides of the MPS and in the coaxial line are seen in every case. The electric field strength in the plasma region is high for \( l = 84 \text{ mm} \) and 89 mm, while it is low for \( l = 30 \text{ mm} \) and 95 mm.

The tuning characteristics for this MPS is shown in fig. 5. The points marked with letters in brackets correspond to the respective figures of fig. 4. Ratio \( P_R/P_I \) is minimal and equal to about 0.1 for point (d) with the length \( l = 84 \text{ mm} \). It means that for this position of movable short...
about 90% of incident wave power is absorbed in the plasma. It is seen that in this case the electric field intensity in the plasma region is high. Ratio \( \frac{P_R}{P_I} \) is greater than 0.9 for points (a) and (f), what means that only less than 10% of the incident power is absorbed in the plasma for these points. It results from fig.4 and 5 that the electric field intensity in the plasma region is low if ratio \( \frac{P_R}{P_I} \) is close to 1 and increases when the ratio decreases.

Let us note that the electric field intensity inside plasma is much less than outside plasma.

### 4.3. Electric field distributions for MPS terminated with a matched load

The electric field distributions for the MPS terminated with a matched load for two values of

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**Figure 4.** Electric field distribution inside the MPS for different positions of the movable short; plasma parameters are \( s = 0.1, n = 10, h = 10 \) mm.

**Figure 5.** Tuning characteristics of the MPS. The letters in brackets correspond to the respective figures of fig. 4.
normalized electron density $n$ are shown in fig. 6. Standing wave pattern are seen only between the input plane and the main section of the MPS and no standing wave is behind the main part. The scattering matrix elements determined from these configurations are given in table 1.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\text{Re}(s_{11})$</th>
<th>$\text{Im}(s_{11})$</th>
<th>$\text{Re}(s_{21})$</th>
<th>$\text{Im}(s_{21})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.168</td>
<td>-0.617</td>
<td>0.553</td>
<td>-0.306</td>
</tr>
<tr>
<td>50</td>
<td>0.636</td>
<td>-0.558</td>
<td>-0.249</td>
<td>-0.361</td>
</tr>
</tbody>
</table>

Taking into account that the presented plasma source is symmetric, the other elements are $s_{12} = s_{21}$ and $s_{22} = s_{11}$. Having them and eq. (2)-(4), it is easy to obtain the tuning characteristics.

4.4. Influence of the electron density on tuning characteristics

The tuning characteristics for different values on normalized electron density are shown in fig. 7. It is seen that the shape of the characteristics strongly depends on the electron density. The minimum values of $P_R/P_I$ below 0.1 can be obtained for every presented values of $n$ except for $n = 200$. This means that for these smaller values of $n$ it is possible to find such a position of the movable short that coupling between the electromagnetic field and the plasma

is good and almost 90% of incident power is absorbed in the plasma. For electron density $n = 200$, only about 20% of incident power is absorbed in plasma in the best case. The reason is that with increasing electron density also the plasma conductivity increases and the plasma behaves like a good conductor. From the other side it might be an unexpected result since in real discharges the plasma exists because the microwave power is absorbed in the plasma, hence one could expect that more power is absorbed in plasma with greater electron density.

To explain this behavior let us note that our model is not a self-consistent one and does not take into account any discharge processes. In this paper, we have imposed that the plasma is homogenous in the whole region it occupies, while in real discharges plasma is non-homogenous and absorption of microwave power occurs merely in regions with lower electron density. This question is to be examined by us in detail in the future.
4.5. Influence of the height of the reduced-height section on tuning characteristics

Tuning characteristics of the MPS depend not only on plasma parameters but also on dimensions of the MPS. We have examined how the height of the reduced-height sections influences the characteristics.

The results for \( n = 10 \) are presented in fig. 8. It is seen that the height \( h \) is an important parameter of the MPS. By proper choosing its value it is possible to obtain good field - plasma coupling with the ratio \( P_R/P_I \) less than 0.1, as it is for \( h = 6 \) and 10 mm. For \( h = 20 \) mm, the reflected wave power is more than half of the incident wave, yet the minimum is very sharp, which means that the discharge is not stable.

5. Summary and conclusion

We have examined changes of the electric field distributions in waveguide-based axial-type microwave plasma source during tuning the MPS. The distribution strongly depend on position of the movable short so does the wave reflection coefficient of the incident wave.

We have proposed a method of determining tuning characteristics of the MPS consisting in treating the MPS as a two-port network, calculating its scattering matrix coefficients and calculating the reflection coefficient from analytical expressions.

The results show that the tuning characteristics depend on plasma parameters such as the electron density and on MPS dimensions such as the height of the reduced-height waveguide section. It is possible to find such position of the movable short for which the reflected wave power is less than 10% of incident wave power.

6. References


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