Fluid Flow Behavior in Steady and Transient Force Injection Systems

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Introduction and Objective

• Injection is an effective technique used for delivering drugs by parenteral administration.

• It is one of the most commonly used health care procedures in the world.

• Modern automated injection systems facilitate the application of precision operator-controlled injections.

• This presentation described modeling work performed at RPI to better understand the dynamics of the injection process with the objective of designing improved automated injection systems.
General Description of the Model

• The model was based on fundamental principles of fluid mechanics and was designed to simulate the behavior of fluid within an injection system (i.e. syringe).

• The steady state version of the model simulates the behavior of the fluid when used with a constant pressure to achieve a desired injection time.

• The transient model can be used to determine injection time based on a spring strength history.
Schematic Representation of a Typical Syringe System
Assumptions and Tools

- Becton Dickinson Syringe
  - 1mL Long Barrel
  - 27 Gage Thin Wall Needle (I.D. 0.26 mm)

- Regeneron Dupixent Drug Product

- Hagen-Poiseuille Law (for model calibration purposes)
- COMSOL Multiphysics Software (for finite element analysis)
Cases Investigated

- **Steady State** (Constant Pressure)
- **Transient State** with Changing Pressure and Volume over Time
  - Auto-Injector
  - Spring - Driven Plunger applies Pressure to Fluid

- **In all Cases:**
  - Creeping Flow Conditions in the Barrel (Re<<1)
  - Laminar Flow Conditions in Hub and Needle (1<Re<2100)
Input Data

• **BD 1mL Long Syringe**
  - Syringe Diameter: 6.35mm
  - Syringe Length: 35mm
  - Hub Diameter: 1mm
  - Hub Length: 5.2mm
  - Needle Diameter: 0.26mm
  - Needle Length: 13mm

• **Flow Rate (Steady Case)**
  - 1.43E-7 m³/s

• **Spring Force (Transient Case)**
  - F High Load = 31.65 N
  - F Low Load = 10.86 N

• **Drug Product**
  - \( \mu = 0.0142 \text{ Pa}\cdot\text{s} \)
  - \( \rho = 1073 \text{ kg/m}^3 \)
  - Volume: 1mL
Analytical Solution
(for model calibration)

• Hagen-Poiseuille Law

\[ \Delta P = \frac{128\mu LQ}{\pi d^4} \]

• Laminar Pipe Flow: Poiseuille Velocity Field

\[ \nu = \frac{dP/dx}{4\mu} (a^2 - r^2) \]
Steady State Pressure Drops

Barrel

$\Delta P_1 = 2 \text{ Pa}$

$\Delta P_2 = 430 \text{ Pa}$

$\Delta P_3 = 235361 \text{ Pa}$
Analytical Needle Velocity Field

\[ n := 5.39 - 318744577 \cdot r^2 \]

\[ \text{plot}(n, r = 0 \ldots 0.00013) \]

\[ v = \frac{G}{4\mu} (a^2 - r^2) \]
Governing Equations

Navier-Stokes, Continuity, Cylindrical Coordinates, Axisymmetric

\[
\begin{align*}
\rho \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} \right) &= -\frac{\partial p}{\partial r} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_r}{\partial r} \right) + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} \right] + \rho g_r \\
\rho \left( \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} \right) &= -\frac{\partial p}{\partial z} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_z}{\partial r} \right) + \frac{\partial^2 u_z}{\partial z^2} \right] + \rho g_z \\
\frac{1}{r} \frac{\partial}{\partial r} (r u_r) + \frac{\partial u_z}{\partial z} &= 0.
\end{align*}
\]
Finite Element Model
Steady State Case
Needle Flow - Steady State Case
Transient Case
Starting Pressure Drops

$\Delta P_1 = 8.5 \text{ Pa}$

$\Delta P_2 = 1823 \text{ Pa}$

$\Delta P_3 = 998068 \text{ Pa}$
Transient Case
Ending Pressure Drops

\[ \Delta P_1 = 2.91 \text{ Pa} \]
\[ \Delta P_2 = 626 \text{ Pa} \]
\[ \Delta P_3 = 342465 \text{ Pa} \]
Starting and Ending Needle Velocity Fields (Analytical)

\[ n = 22.84 - 1351663267 r^2 \]

\[ \rho \phi r(n, r = 0...0.00013) \]

\[ e = 7.85 - 464644745 r^2 \]

\[ \rho \phi r(e, r = 0...0.00013) \]
Finite Element Modelling (Transient Case)

Parameters

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Parametric Sweep of tt with time
Finite Element Model
Transient Case

Geometry varies with $t$, which changes over time. Allows Geometry to change over time.
Transient Model Animations
Syringe Flow Field - Transient Case
Time = 0.2 s
Syringe Flow Field - Transient Case
Time = 1 s
Syringe Flow Field - Transient Case
Time = 2 s
Syringe Flow Field - Transient Case
Time = 3 s
Hub and Needle Flow Field - Transient Case
Time = 0.2 s
Hub and Needle Flow Field - Transient Case
Time = 3 s
Summary of Results

Transient Model

- Initial Volumetric Flow Rate
  6.3E-7 m³/s
- Final Volumetric Flow Rate
  2.2E-7 m³/s
- Injection Time for Average Flow Rate Injection Time
  3.1 seconds

Steady State Model

- Steady State Flow Rate
  1.43E-7 m³/s
- Steady State Injection Time
  7 seconds
Summary

• Steady State Model
  • Given Injection Time
  • Estimated fluid velocities
  • Estimated Pressure Drop

• Transient Model
  • Given Applied Force
  • Estimated fluid velocities
  • Estimated Injection Time