Thermal Integrity Analysis of Concrete Bridge Foundations Using COMSOL Multiphysics® Software

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Introduction

The vast majority of bridges in the U.S. are founded on concrete deep foundations, many of which are constructed as cast-in-place structures commonly referred to as drilled shafts. Although drilled shaft foundations are reputed for their strength, reliability, and economy, their construction requires careful attention to quality control and quality assurance. Due to the blind nature of the underground concreting process, defects in drilled shafts can occur. This is particularly true among excavations extending below the water table which are stabilized with drilling fluids (Mullins 2010). Inclusions of soil from the excavation side walls, encapsulation of slurry, or improper flow of concrete through congested reinforcement cages can occur during construction, often without any indication to the contractor. As a result, the as-built shaft may contain areas of degraded concrete quality, exposed rebar, reduced cross section, or combinations thereof. Defects such as these reduce both the structural and geotechnical capacity of the shaft, and allow pathways for corrosion, rendering the shaft unfit for service in many cases.

Even with careful attention to quality control during construction, quality assurance after construction is necessary to validate the integrity of the as-built shaft. Unlike above ground structures, visual inspection of drilled shafts is rarely available. Excavation and core sampling methods can provide some visual confirmation but are expensive, time consuming, and can further compromise the integrity of a shaft. These methods are only employed where strong suspicion warrants. Similarly, proof load testing is expensive and can cause structural damage. (Anderson, 2011)

This two-fold problem - a high probability of defects with a low ability to detect them - lends itself to an array of creative inspection techniques. Non-destructive testing methods developed over the last 40 years have greatly improved the quality assurance process for drilled shafts by allowing owners and contractors to verify structural integrity without the need for excessive coring or load testing.

Developed in the late 1990’s, Thermal Integrity Profiling (TIP) is the most recent method to gain widespread popularity in the post-construction evaluation of shaft integrity. This method excels in its ability to detect anomalies across the entire cross section of a shaft, both inside and outside the reinforcement cage. Multiple studies have proven the effectiveness of TIP for shaft evaluation, and many states and countries have adopted its use. As with any test method, however, the quality of the results depends largely on the level of analysis and the way in which test data is interpreted.

Theory

Concrete hydration is a highly exothermic process, and in large concrete elements, such as drilled shafts, a significant amount of energy is released, causing elevated temperatures in both the shaft and surrounding soil, typically for several days. The amount of temperature increase at any given point depends on the volume of hydrating concrete as well as the cementitious content of that concrete, both of which help to define shaft serviceability. TIP takes advantage of this and detects anomalies based on variations in the thermal profile of a shaft during the curing stages.

Figure 1 illustrates the way in which heat is dissipated from a shaft to its surroundings, and the temperature the resulting distribution. For a perfectly cylindrical shaft, the vertical distribution of temperature is uniform throughout the majority of its length. The exception is near the ends where there is a distinct region of decreasing temperature. This temperature “roll-off” at the top and bottom is due to the added mode of heat loss in the longitudinal direction. The radial temperature distribution is bell-shaped, with peak temperatures occurring at the center of the shaft and decreasing radially towards the surrounding soil. With a typical configuration of access tubes (one tube per 1ft of diameter, evenly spaced around the reinforcing cage, per ASTM D7949), data collected from thermal integrity testing provides a continuous temperature profile vertically and discrete measurements laterally (indicated by red dots in Figure 1). The vertical profile reveals any bulges, necks, or inclusions that may be present, while comparison among tube temperatures indicates lateral cage alignment. (Mullins and Winters, 2012)
Direct observation of measured temperature profiles can provide immediate qualitative information about a shaft, such as general shape, relative cage alignment, and the types of anomalies that may be present. An increase or decrease in all tube temperatures indicates a bulge or neck in the shaft, respectively; whereas an equal but opposite variation of opposing tube temperatures indicates cage eccentricity. Circular shaped temperature roll-offs that extend about one diameter from the top and bottom of the shaft indicate normal end conditions.

While direct observation of profiles is useful in identifying anomalies, a measurable assessment of shaft integrity is obtained by converting temperatures to values of effective shaft radius. Because measured temperatures are affected by both shaft size and cementitious content, it can be conceived that the temperature resulting from an anticipated shaft radius consisting of intact, quality concrete could also result from a larger radius consisting of compromised concrete. In this sense, the term effective radius implies the radius of intact, quality concrete that would produce the measured temperature.

**Governing Equations**

Numerical modeling analysis of the thermal effects between concrete and soil requires the finite-difference form of the general heat equation (Eq. 1) based on input shaft dimensions, thermal properties, and boundary conditions.

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + q = \rho c_p \frac{\partial T}{\partial t}$$  \hspace{1cm} (1)

Where,
- \(T\) = temperature
- \(k\) = thermal conductivity
- \(\rho\) = density
- \(c_p\) = specific heat
- \(q\) = rate of heat generation

The thermal properties of concrete vary through the curing stages and are typically expressed as a function of the degree of hydration. The hydration of Portland cement is the result of many different chemical reactions that take place, all of which release heat in the process (i.e. exothermic), though be it at separate times and magnitudes. Since the evolution of heat is a direct indication of completed reactions, it serves as a defining measure for the progression of hydration. Therein, at any given time, the rate of hydration is defined by the instantaneous rate of heat generation, \(q\) (Eq. 2), and the degree of hydration, \(\alpha\), is defined as the fraction of cumulative heat evolved, \(H(t)\), to the ultimate amount of heat available, \(H_u\) (Eq. 3) (Schindler & Folliard, 2002). The variation in time and rate of the multiple types of reactions results in a hydration process that is not constant, but rather occurs in phases. In general, there are five distinguishable stages of hydration: (1) initial hydration, (2) dormant period, (3) acceleration, (4) deceleration, and (5) steady state.

$$q = \text{rate of heat generation} = \frac{d}{dt} H(t)$$ \hspace{1cm} (2)

$$\alpha = \text{degree of hydration} = \frac{H(t)}{H_u}$$ \hspace{1cm} (3)

The most widely accepted method for modeling this hydration behavior involves the concept of equivalent age, \(t_e\), which invokes the Arrhenius theory for rate processes to account for the temperature dependency of reactions (Eq. 4), combined with an exponential formulation which approximates the S-shaped degree of hydration curve (Eq. 5). (Schindler & Folliard, 2002)

$$t_e = \text{Equivalent Age} = \sum_{0}^{E_a} e^{-\frac{E_a}{R(T - T_e)}} \cdot \Delta t$$ \hspace{1cm} (4)

$$\alpha = \alpha_u \cdot \exp \left[ -\left( \frac{T}{T_e} \right)^{\beta} \right]$$ \hspace{1cm} (5)

In Equation 4, \(R\) is the natural gas constant (8.314 J/mol/K) and \(E_a\) is the activation energy, a property which represents the temperature sensitivity of the hydration process. \(T\) is the temperature (°C) of concrete at time \(t\). In Equation 5, \(\alpha_u\), \(\beta\), and \(\tau\) are parameters that describe the shape of the hydration curve, corresponding to the ultimate degree of...
hydration, the rate of the acceleration phase, and the start of the acceleration phase, respectively, as shown in Figures 2 - 4 (Folliard et. al., 2008). These shape parameters, as well as properties $E_a$ and $H_u$, are unique to every concrete batch and are best determined experimentally on an individual basis. They can be found through a combination of isothermal and adiabatic or semi-adiabatic calorimetry testing, wherein $T_r$ is the reference temperature ($^\circ K$) at which testing is conducted.

Bogue (1947) first correlated the total heat of hydration of Portland cement to its major compounds, however several recent studies have since extended this concept to correlate the additional hydration parameters used in the exponential $\alpha$ model and to include a broader range of variables such as supplementary cementitious materials, chemical admixtures, and cement fineness. Some of the most notable and recent work to examine such relationships includes studies from Schindler & Folliard (2005), Ge (2006), and Poole (2007), the latter of which is the most comprehensive. This set of equations is specific to cement compositions as determined by Bogue calculations, which are commonly found on cement manufacturer mill certificates.

With a working model for hydration behavior of concrete, thermal properties that are hydration dependent can be determined as they vary with time. Since the rate of heat generation, $q$, is an inherent part of the model definition, it can be found by substituting Equations 3, 4, and 5 into Equation 2 and differentiating. The resulting expression is given in Equation 7.

$$q = H_u W c \left( \frac{1}{\alpha \tau} \right)^\beta \left( \frac{\beta}{\tau} \right)^\alpha \frac{E}{R} \left( \frac{1}{T_r} - \frac{1}{T_c} \right)$$  \hspace{1cm} (7)

**COMSOL® Model**

**Geometry**

Using the COMSOL Multiphysics® software, the governing equations can be applied to both 2-D and 3-D geometries using the Heat Transfer in Solids module along with two separate Coefficient Form PDE modules. Figure 5 shows the geometry for a simple 2-D analysis where a concrete shaft of diameter $D$ is surrounded by a concentric mass of soil with diameter of at least three times the shaft. Based on modeling, this is the minimum recommended ratio of soil-to-shaft diameter, where the edge of the soil domain can be considered unaffected by any heat transfer from the shaft, and the edge boundary condition can be set to constant soil temperature.
2-D model geometry for a concrete shaft surrounded by soil.

**Materials**

The thermal conduction properties of hydrating concrete are both time and temperature dependent. Thermal conductivity, $k$, and specific heat, $C_p$, can be estimated using Equations 8 and 9 in conjunction with empirical values found in literature (Schindler & Folliard, 2002). The density, $\rho$, is calculated from the weight of materials in the given concrete mix.

\[ k = k_{uc}(1.33 - 0.33\alpha) \]  
\[ C_p = \frac{1}{\rho}(W_cC_{ref} + W_c(1 - \alpha)C_c + W_\text{a}C_a + W_wC_w) \]

Where,

$k_{uc} = \text{Thermal conductivity of mature concrete}$

$C_{ref} = 8.4T + 339, \text{ where } T \text{ is temperature in } ^\circ\text{F}$

$C_{c,a,w} = \text{Specific heat of cement, aggregate, water}$

$W_{c,a,w} = \text{Weight of cement, aggregate, water}$

For foundations surrounded by soil and/or rock, the overall thermal diffusivity is generally on the same order of magnitude as that of concrete. However for more advanced analysis, the heat conduction properties of subsurface materials can be correlated to parameters such as soil type, saturation state, and density, much of which is estimated from in-situ soil tests such as Standard Penetration Testing (SPT).

Notable correlations for the thermal properties of soils can be found in publications from Arya (2001), Incropera & Dewitt (2007), and Pauly (2010), the latter of which provides a comprehensive culmination of soil thermal properties with SPT values.

**Physics**

In the case of concrete surrounded by soil, heat is transferred via conduction, thus the *Heat Transfer in Solids* module is used to apply the general heat equation (Eq. 1) to all domains.

Initial values for each domain are set such that the concrete temperature is equal to the ambient air temperature at the time of placement, and initial soil temperature is assumed to be equal to the annual average air temperature of the given geographical location. The latter is particularly true for depths below 10ft, where seasonal temperature variations have a negligible effect.

The concrete shaft domain is designated as a heat source defined by Equation 7 and the outer edge of the soil boundary is constrained to a constant temperature equal to that of the ambient soil.

In addition to the *Heat Transfer in Solids* module, two separate *Coefficient Form PDEs* must be included to complete the model for concrete hydration behavior. The first is used to calculate the equivalent age, $t_e$, of the concrete, in which the source term is defined by Equation 4. It should also be noted that the initial values for this PDE should be set to $t=1s$ and $dt/dt=1$.

The second *Coefficient Form PDE* defines the degree of hydration, $\alpha$, which is defined by Equation 5. The initial values for $\alpha$, as well as its time derivative, are zero.

**Solver Configuration**

A time dependent study was used to analyze the temperature variations throughout the concrete curing period. For proper application of the concrete hydration model, the time dependent solver must be separated into three segregated steps in the following order:

Segregated Step 1 – Equivalent Age ($t_e$)

Segregated Step 2 – Degree of Hydration ($\alpha$)

Segregated Step 3 – Temperature ($T$)
With this solver configuration, thermal modeling can be performed for any size concrete foundation element at any given time after placement.

**Discussion**

Figure 5 shows the heat distribution in a typical drilled shaft surrounded by soil at the peak of hydration.

![Figure 5](image)

**Figure 5.** Temperature distribution in a drilled shaft at peak of hydration.

With a working model, a parametric study can be conducted to determine the temperature-radius relationship for a given measurement location. This type of relationship is illustrated in Figures 6 and 7 for a shaft with a radial measurement location of 30in.

![Figure 6](image)

**Figure 6.** Radial temperature distribution for varying shaft sizes (1ft – 3ft diameter) at 24hr.

Furthermore, numerical modeling helps to better understand the temperature distributions resulting from longitudinal heat loss at the ends of a drilled shaft. Figure 8 shows the temperature distribution at the interface between concrete and soil at the bottom of a drilled shaft. Model results like this have shown that the temperature distribution in these regions can be approximated by an inverse hyperbolic tangent function. These findings have been further reinforced by comparisons with field data. (Johnson, 2016)

![Figure 7](image)

**Figure 7.** Relationship between shaft size and temperature for a given radial measurement location.

![Figure 8](image)

**Figure 8.** Bottom of shaft temperature distribution – hyperbolic approximation based on model results.
Case Study

Temperature data collected from a 35ft deep drilled shaft with an upper diameter of 54in and lower diameter of 48in (step shaft) was used to verify the ability to use COMSOL® modeling to better assess the as-built shaft integrity. In this case, a two layer soil strata was encountered that was further complicated by the use of different construction techniques that changed the surrounding soil diffusivity characteristics. Figures 9 – 11 show the 3-D model, the temperature vs. time trace from one point in the shaft, and the resulting thermal profiles (modeled and measured). The measured thermal profile showed normal results, with both top and bottom roll-off conditions as well as a thermal transition at the bottom of casing corresponding to an expected change in shaft diameter. The change in soil diffusivity can also be seen at a depth of 12ft which again corresponds with the change in surrounding soil conditions and was not caused by an increase in shaft diameter.

**Figure 9.** 2-D axisymmetric model of drilled shaft with variations in size and soil strata.

**Figure 10.** Measured and modeled temperature vs. time curves near mid-length of shaft.

**Figure 11.** Measured and modeled temperature profile near peak of hydration.

**Conclusions**

The use of COMSOL® numerical modeling has been shown to be an effective tool in simulating concrete hydration behavior, specifically for the temperature distributions that result from cast-in-place concrete foundations.
References


Pauly, Nicole M. (2010). “Thermal Conductivity of Soils from the Analysis of Boring Logs.”


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