COMSOL® Analysis for Duct Acoustic

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Introduction

- Thermoacoustic or combustion instabilities constitute a major issue in several types of applications from aerospace propulsion systems to boilers and radiant heaters.

- They cause structural damaging, hardware melting, high noise, and overall systems failure.

- NASA Jet Propulsion Laboratory’s experiment on 1956. “A Mechanism for High-Frequency Oscillation in Ramjet Combustors and Afterburners” [ref.1]

- The goal is to perform Computational Aeroacoustics “CAA” simulation via COMSOL® Multiphysics, to predict the frequencies and mode shapes of the excited instabilities.

1- Dan E. Rogers and Frank E. Marble, “A mechanism for high-frequency oscillation in ramjet combustors and afterburners”, the American Rocket Society, 1956.

Figure 1: NASA space shuttle was powered by solid rocket propellants.
Thermoacoustic Instabilities

- When heat release oscillations and pressure acoustic waves are in phase, they are considered to be “coupled”. This causes the acoustic pressure mode to get excited and amplified.

- In other words, the heat release from the combustion must be released when the acoustic pressure wave near or at its maximum amplitude (antinode).

- Pressure acoustic oscillations effect dynamic processes, hence the feedback loop is created.

- These instabilities are generally characterized by two distinct frequencies which are: low pressure frequency (rumble), and high pressure frequency (screech).

- Mathematical expression: *Rayleigh’s criterion*

\[
\int_0^\tau \int_0^V p'(x, t) q' \, dv \, dt \geq \int_0^\tau \int_0^V \Phi(x, t) \, dv \, dt \quad \text{(eq.1)}
\]

- \( p' = \text{pressure oscillation} \)
- \( q' = \text{heat release oscillation} \)
- \( \Phi = \text{energy losses} \)
- \( \tau = \text{oscillation period} \)
- \( V = \text{combustor volume} \)
Duct Acoustic and Boundary Conditions

- Pressure standing waves:
  - node (minimum frequency amplitude) at open boundary
  - antinode (maximum frequency amplitude) at closed boundary
- The longitudinal standing waves inside a duct with simple geometry can be calculated by using the following equations:
  1. Close-close & open-open boundaries:
     - the fundamental longitudinal mode is ½ wavelength.
     \[ f(\text{any # of harmonic}) = \frac{\text{any # of harmonic} \times c}{2L} \] (eq.2)
     \[ c = \text{speed of sound} = \sqrt{\frac{R}{T \gamma}} \]
     \[ L = \text{length of the duct} \]
  2. Close-open boundaries:
     - the fundamental longitudinal mode is ¼ wavelength
     \[ f(\text{any # of harmonic}) = \frac{\text{any # of harmonic} \times c}{4L} \] (eq.3)
- Pressure transverse modes of a 2D rectangular duct can be calculated via this equation:
  \[ f_{m,n} = \left( \frac{c}{2\pi} \right) \sqrt{(1 - M^2)\left\{ \left( \frac{m\pi}{H} \right)^2 + \left( \frac{n\pi}{W} \right)^2 \right\}} \] (eq.4)

  \[ H = \text{height of the duct} \]
  \[ W = \text{width of the duct} \]
  \[ M = \text{Mach number} \]
  \[ (m,n) = \text{modes of order} \]
Experimental Duct and Conditions

- The experiment was carried out in a small combustion duct of rectangular cross section.
- The combustor was 1 in. by 4 in. rectangular cross section and extended 24 ½ in. in length beyond the end of the wedge-shaped flameholder (3 ½ in. long), as shown diagram below.
- The average temperature was 250 F = 394.26 K.
- Atmospheric pressure = 1 atm = 101.325 kPa.
- The pressure gage was located at the center of the duct, 11 in. away from the inlet.
COMSOL Simulation

• In order predict the frequencies and mode shapes of the excited instabilities:
  - Geometric characteristics of the system
  - Average temperature distribution

• Pressure Acoustic, Frequency Domain was used.
  - the interface solves the Helmholtz equation
    \[ \nabla \ast \left( \frac{1}{\rho_c} (\nabla p_t - q_d) - \frac{k_{eq}^2 \rho_t}{\rho_c} \right) = Q_m \text{ (eq.5)} \]
    \[ p_t = p + p_b \]
    \[ k_{eq}^2 = \left( \frac{\omega}{c_v} \right)^2 \]
    \[ \omega = 2\pi f \]

  - Pressure solved for

• The assumption made is that the duct is “acoustically” closed why?
  1- at the inlet we have converging nozzle with contraction ratio of 28/1.
  2- the end of the duct is connected to an exhaust, which does not contribute on the acoustic field.
  3- calculation and simulations values matched the experimental value.

• No-flow simulation.

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Case 1                  | Close-close
Fund Longitudinal      | 279.817 Hz

Case 2                  | Close-open
Fund Longitudinal      | 139.909 Hz

Table 1: calculated values of the fundamental longitudinal mode by using equations 2 and 3.
Experimental Results

- The low-frequency oscillation was about 280 cps.
  - corresponds to the fundamental longitudinal mode (x-axis)

- Of particular interest the high-frequency oscillation mode that was about 3800 cps.
  - corresponds to an antisymmetric transverse mode across the 4 in. dimension of the duct
  - it is accompanied by vortices shed having the same frequency.

- The driving mechanism:
  - In the present case the vortex is off center in the duct, therefore the antisymmetric transverse mode is excited.

The transverse velocity waves associated with transverse pressure standing wave → Vortex formed and moved into the area behind the flameholder (hot zone) → Vortex contains combustible materials. → The combustion of these material in the hot zone generates pressure waves (heat release oscillation) → Pressure waves produced excites one of the natural acoustic mode of the duct, if they are in phase.

- Figure 6: The vortex shedding accompanied with the high-frequency oscillations “screech”. [ref.1]
- Figure 7: Both modes are unaffected by flow
- Figure 8: The shape of two modes were measured experimentally.
- Figure 9: Vortex formation due to transverse velocity waves.

The transverse velocity waves associated with transverse pressure standing wave
COMSOL Results

Figure 11: 3D plot of the high-frequency oscillation, which is around 3684 Hz. (antisymmetric transverse mode along the z-axis).

Figure 12: 3D plot of the low-frequency oscillation, which is around 279.57 Hz (the fundamental longitudinal mode of the duct).
Figure 13: A line was drawn along the x-axis to find the behavior of the modes in the longitudinal direction.

Figure 14: The acoustic pressure variations of the two modes along the duct.

Figure 15: The SPL of the two modes along the duct (x-axis).
Figure 16: This line represents the gage pressure, which was located normal to the surface in the z-axis and 11 in. away from the inlet along the x-axis.

Figure 17: The acoustic pressure field of the modes along the z-axis, where the pressure gage was located.

Figure 18: The SPL of modes along the z-axis. The SPL of the screech frequency is about 96 dB.
Figure 19: A third line was drawn along the y-axis to gain more information about the behavior of the modes. At the same location of the pressure gage 11 in. away from the inlet.

Figure 20: The acoustic pressure field of the modes along y-axis.

Figure 21: The SPL of the modes along y-axis.
Observations and Validations

- The simulations values were in a great agreement with the experimental values as shown in table 2.
- Both showed that low-frequency oscillation was the fundamental longitudinal mode and the high-frequency oscillation was the fundamental antisymmetric transvers mode along the z-axis.
- From COMSOL results we can see that the high-frequency mode is rapidly changing in the longitudinal direction (x-axis). While the low-frequency has no interference along z-axis.
- Both modes are constant along the y-axis.
- The low-frequency mode will probably be excited at the inlet or outlet. The high-frequency will be excited at several location of the central region of the duct.
- COMSOL simulation validated that the oscillation frequencies are not effected by flow.
- Both modes have ½ wavelength pattern but in different directions.

<table>
<thead>
<tr>
<th></th>
<th>Low-frequency oscillation</th>
<th>High-frequency oscillation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental results</td>
<td>285 cps</td>
<td>3800 cps</td>
</tr>
<tr>
<td>COMSOL results</td>
<td>279.57 Hz</td>
<td>3684 Hz</td>
</tr>
</tbody>
</table>

Table 2: comparison between experimental and COMSOL results.
COMSOL® App for Designing Afterburner Ducts

- COMSOL® CAA analysis provide more information about the behavior of excited instabilities inside the duct.
- COMSOL helps to estimate the location of the excitation.
- COMSOL Analysis can help to analyze the impacts of geometry and location of flameholders and fuel injectors on instabilities:
  1. By easily varying the dimensions of the duct.
  2. Suggesting the proper location of the flameholder and fuel injectors.
- Future work: Application software to design afterburners - saves time and reduces cost.

THANK YOU!