Getting State-Space Models from FEM Simulations

Jos van Schijndel
Overview

• Background on my work

• State-Space models, WHAT and WHY

• How to get them from FEM

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \]
Overview

• Background on my work

• State-Space models, WHAT and WHY

• How to get them from FEM
Assistant professor
Since 1998 COMSOL® User

Entrepreneur
Since 2015
The Built Environment is Multiscale

<table>
<thead>
<tr>
<th>Material</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>m</td>
</tr>
<tr>
<td>Building</td>
<td>10 m</td>
</tr>
<tr>
<td>Urban Area</td>
<td>1 km</td>
</tr>
</tbody>
</table>
Physics of the Built Environment

Scale level [mm]

Material ~ mm

Material Physics
• Durability
• Energy
Physics of the Built Environment

Scale level [m]

Construction ~ m

Construction Physics
• Safety
• Durability
• Energy
Physics of the Built Environment

Scale level [10 m]

Building ~ 10 m

Building Physics
• Indoor Climate (T,RH,v, Pollutant)
• Building systems
• Health
• Energy
Physics of the Built Environment

Scale level [km]

Urban Area ~ km

Urban Physics

- Urban Climate (Pollutant, Wind)
- Urban Systems
- Aquifer
- Energy
# Modeling the Built Environment

## Physics and Scales

<table>
<thead>
<tr>
<th>Physics Scales</th>
<th>Heat</th>
<th>Moisture</th>
<th>Air</th>
<th>Pollutant</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>~ mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~ m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~ 10m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~ km</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
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What are State-Space models?

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\vdots \\
\frac{dx_n}{dt}
\end{bmatrix} =
\begin{bmatrix}
\Delta & \cdots & \Delta \\
\vdots & \ddots & \vdots \\
\Delta & \cdots & \Delta
\end{bmatrix}
\begin{bmatrix}
x_1 \\
\vdots \\
x_n
\end{bmatrix} +
\begin{bmatrix}
\Delta \\
\vdots \\
\Delta
\end{bmatrix} u(t)
\]

\[
x = Ax + Bu(t)
\]

\[
y = Cx + Du(t)
\]

\[
y = [\Delta \cdots \Delta]
\begin{bmatrix}
x_1 \\
\vdots \\
x_n
\end{bmatrix} + [\Delta]
\begin{bmatrix}
u(t)
\end{bmatrix}
\]
Why are State-Space models so handy?

\[
A(1,1) = \frac{-1}{R_1C_1} - \frac{1}{R_2C_1}; \\
A(1,2) = \frac{1}{R_2C_1}; \\
A(2,1) = \frac{1}{R_2C_2}; \\
A(2,2) = \frac{-1}{R_2C_2} - \frac{1}{R_3C_2}; \\
A(2,3) = \frac{1}{R_3C_2}; \\
A(3,2) = \frac{1}{R_3C_3}; \\
A(3,3) = \frac{-1}{R_3C_3} - \frac{1}{R_4C_3}; \\
A(3,4) = \frac{1}{R_4C_3}; \\
A(4,3) = \frac{1}{R_4C_4}; \\
A(4,4) = \frac{-1}{R_4C_4}; \\
\]

%B calc
B(1,1) = \frac{1}{R_1C_1}; \\
B(2,1) = 0; \\
B(3,1) = 0; \\
B(4,1) = 0; \\
%C calc 
C = [0 0 0 1]; \\
%D calc 
D = 0;
Why are State-Space models so handy?

\[ \begin{array}{cccc}
-8.8889e-04 & 5.8586e-04 & 0 & 0 \\
2.9293e-04 & -5.8586e-04 & 2.9293e-04 & 0 \\
0 & 5.8586e-04 & -6.7910e-04 & 9.3240e-05 \\
0 & 0 & 0.0501 & -0.0501 \\
\end{array} \]

\[ \begin{array}{c}
3.0303e-04 \\
\hline \\
0 \\
\hline \\
0 \\
\hline \\
0 \\
\end{array} \]

\[ C=[0 \ 0 \ 0 \ 1]; \]

\[ D=0; \]
Why are State-Space models so handy?

• Compact notation of dynamic systems

• The availability of high quality public domain solvers for state-space systems (Octave, Python, R, …)

• These State-Space model solvers are extremely efficient in simulating dynamic responses.
Overview

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State Space systems from FEM

(1) Creating a full state-space matrix system using specific COMSOL functionality and including reduced order systems

(2) Using identification techniques for example the MatLab identification Toolbox to fit SS systems

(3) Creating a lumped parameter SS model from first principles, where parameters have a physical meaning.
State Space systems from FEM
COMSOL MatLab functionality

%Extract full SS model
M2 = mphstate(model,'soll','out',{'A' 'B' 'C'
'D' 'x0'},...
    'input','modl.varl', 'output',
    'modl.doml');

%Create system in MatLab
sys2= ss(M2.A,M2.B,M2.C,M2.D);

%Simulate full SS
y2=lsim(sys2,u,t,M2.x0);

%Reduce order
Options = balredOptions();
sys2Reduced2 = balred(sys2,8,Options);

%Simulate reduced SS
y3=lsim(sys2Reduced2,u,t);
State Space systems from FEM Identification MatLab Toolbox
State Space systems from FEM

First principles

\[ T_e(t) - R_1 \]

\[ \begin{array}{c}
T_1 \\
C_1 \\
R_2 \\
C_2 \\
R_3 \\
C_3 \\
R_4 \\
C_4 \\
\end{array} \]

\[ A = \begin{bmatrix}
-8.8889 \times 10^{-4} & 5.8586 \times 10^{-4} & 0 & 0 \\
2.9293 \times 10^{-4} & -5.8586 \times 10^{-4} & 2.9293 \times 10^{-4} & 0 \\
0 & 5.8586 \times 10^{-4} & -6.7910 \times 10^{-4} & 9.3240 \times 10^{-5} \\
0 & 0 & 0.0501 & -0.0501 \\
\end{bmatrix} \]

\[ B = \begin{bmatrix}
3.0303 \times 10^{-4} \\
0 \\
0 \\
0 \\
\end{bmatrix} \]

\[ C = [0 \ 0 \ 0 \ 1]; \]

\[ D = 0; \]
State Space systems from FEM

Results
State Space systems from FEM

Results

Time=0 s  Surface: Temperature (degC)
State Space systems from FEM

Results
State Space systems from FEM

Conclusions

• All three approaches: are capable of significantly reduce computation duration time without loss of accuracy.

• Comparing the three approaches from a physical point of view, the lumped parameter model is preferable
  • because its parameters (state-space matrices) have a physical meaning
  • therefore parameters studies can be done without the necessity to simulate the FEM model over and over again.

• Finally, notice that no general conclusions can be obtained from this rather limited study.