Numerical Homogenization of Viscoelastic Composites with Piezoelectric Fibers

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Layout

○ Introduction

○ Theoretical Formulation

○ Numerical Analysis

○ Conclusion
Introduction

Piezoelectric effects:

Electrodes

Strain
(Displacement)

Piezoelectric material

Strain
(Stress)

Direct Effect
(Sensing)

Voltage
(Electric field)

Converse Effect
(Actuation)

Voltage
(Current)
Introduction

Real

Assumed

Representative Unit Cell
Introduction

Matrix
- Square
- Linear Visco-elastic

Fiber
- Circular
- Linear Piezo-electric
- Poled in Fiber direction

Piezo-Visco Composite
Theoretical Formulation

Linear Piezoelectric Constitutive Equations:

**e-form** (used in FEA):

\[
\begin{align*}
T &= C^E S - e^t E \\
D &= e S + \varepsilon^S E
\end{align*}
\]
Theoretical Formulation

Linear Piezoelectric Constitutive Equations:

Stress

\[ T = \mathbf{C}^E \mathbf{S} - \mathbf{e}^t \mathbf{E} \]

Strain

\[ \mathbf{D} = \mathbf{eS} + \mathbf{\varepsilon}^S \mathbf{E} \]

Elastic Coefficients
At constant Electric Field

Electric Field

Permittivity
At constant strain

Stress Piezoelectric Coupling Coefficients

Electrical Displacement

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Theoretical Formulation

Linear Isotropic Viscoelastic Material:

\[ E(w) = E(\xi(w)) + iE(\eta(w)) \]

\[ h = \frac{E(\xi(w))}{E(\eta(w))} \quad \Rightarrow \quad E(w) = E(\xi(w))(1 + ih) \]
Theoretical Formulation

Homogenization:

\[
\bar{T}_{ij} = \frac{1}{V} \int T_{ij} dV \\
\bar{D}_{ij} = \frac{1}{V} \int D_{ij} dV \\
\bar{S}_{ij} = \frac{1}{V} \int S_{ij} dV \\
\bar{E}_{ij} = \frac{1}{V} \int E_{ij} dV
\]

\[
\bar{T} = \hat{C}(w) \bar{S} - \hat{e}(w)^T \bar{E} \\
\bar{D} = \hat{e}(w) \bar{S} + \hat{e}(w) \bar{E}
\]
Numerical Analysis

Geometry of representative volume element:
Numerical Analysis

- Viscoelastic material is modeled using:

\[ M_0(s) = D_0 + \frac{D_1 n!}{s^n} \]

- \( D_0 \) is the initial elastic compliance,
- \( D_1 \) and \( n \) are experimentally determined parameters, and 
- \( s = i\omega \) with \( \omega \) denoting the frequency.

- The Young modulus \( E(s) \) is taken as the inverse of \( M_0(s) \)

- Piezoelectric Material used: PZT-7A
Numerical Analysis

Frequency-dependent viscoelastic material implementation in COMSOL

Youngs modulus in terms of frequency
Numerical Analysis
Numerical Analysis

Periodicity Condition
Numerical Analysis

Deformed Viscoelastic material under normal and shear load

Figure 3: Deformed unit cell of viscoelastic material
Numerical Analysis

Sample Mechanical Calculation:
- Apply normal and shear on the cell cross section (all potentials = 0)

\[
C_{11} = \frac{T_{11}}{S_{11}}
\]

\[
C_{66} = -\frac{T_{11}}{S_{12}}
\]

Figure 4: Deformed unit cell of viscoelastic matrix reinforced with PZT fiber
Numerical Analysis

Sample Piezoelectric Calculation

- Apply potential difference across fiber (all displacements = 0)

\[ \hat{e}_{31} = \frac{T_{11}}{E_3} \]

**Figure 4:** Deformed unit cell of viscoelastic matrix reinforced with PZT fiber
Results

Figure 5. Real and Loss Modulus LaRC-SI with respect to frequency for different boundary condition.
Results

Figure 6. Effective storage and loss elastic modulus ($C_{11}$) for a viscoelectroelastic composite for different volume fraction.

PZT fiber reinforced in viscoelastic matrix.
Figure 7. Effective storage and loss piezoelectric modulus ($e_{31}$) for a viscoelectroelastic composite for different volume fraction.
Results

PZT fiber reinforced in viscoelastic matrix with different fiber cross section.

Figure. Effective storage and loss piezoelectric modulus ($e_{31}$) for a viscoelectroelastic composite.

Major to minor axis ratio of the ellipse is $r_p$. 
Conclusions

- The results show that the material properties strongly depend on:
  - Frequency-dependent viscoelastic properties
  - Piezoelectric fiber volume fraction

- COMSOL directly calculates the frequency-dependent properties with no need for customized functions/subroutines for material properties or periodic BCs