The Collection Efficiency of Particles on a Ribbon in a Turbulent Flow

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Abstract: The collection efficiency of aerosol particles on a ribbon in a turbulent flow is analyzed using COMSOL Multiphysics. The flow field is solved using Chemical Engineering module and particle tracing plots are obtained using equations of motion including Khan and Richardson drag force. A MATLAB script is used to count the captured particles on the ribbon and determine the capture efficiency with varying Stokes number. The model results are compared with experimental data from literature. This type of analysis can potentially be extended to other collector geometries and can be useful for studying particle capture systems.

Keywords: aerosol particles, capture, turbulent flow, particle tracing

1. Introduction

The study of the motion of aerosol particles in air is very interesting in terms of understanding air quality and control of nuisance dust in the ambient air. Especially interesting is the case where a turbulent flow is used to capture unwanted particles. Capture media of several diverse geometric shapes have been used in the past to investigate capture efficiency of particles, and we want to better understand the capability of COMSOL Multiphysics in this regard. COMSOL’s excellent post processing capability for tracing particle trajectories in a flow seems very useful in this application, and in this paper, we present some validating results on a simple geometry where experimental data from literature is available (ref 1) for comparison with finite element results.

We consider the simple case of a long, horizontal, thin ribbon placed with its surface perpendicular to the air flow in a vertical air flow channel (Figure 1). The flow channel has a rectangular cross section and extends parallel to the length of the ribbon. The inlet air velocity $u_0$ is high enough so that the flow is turbulent (Reynolds number $Re$ is higher than 5000). Particle-laden air is introduced vertically from the bottom of the channel at a uniform controlled velocity. In the experiment described in reference 1, involatile dyed liquid dibutyl phthalate droplets were generated well away from the inlet and introduced into the inlet with a stream of air. The size of droplets was varied from 20 to 40 micron and air velocity was varied from 2.2 to 6.2 m/s. Various shapes of obstacles were placed in a chamber of 0.20 m square and 0.30 m high. A steel ribbon was placed 0.05 m away from the bottom entrance of the test chamber. An air sheath surrounding the inlet and outlet regulated the flow to be free of edge effects as evidenced from measured velocity profile and droplet count (ref 1). The amount of collected droplets on the ribbon was experimentally determined by a simple gravimetric method.

![Figure 1](image_url)

**Figure 1.** Schematic of the cross section of the test chamber in the vertical flow channel containing the ribbon. The ribbon and the chamber walls extend both ways perpendicular to the plane of the page.

Due to the inertia of moving particles they can not always follow the streamlines of air near an obstacle, a ribbon, for example. Some particles will impact the ribbon and be captured...
by it. The rest of the particles, which can follow the streamlines, will exit the channel from the outlet with air.

The aerosol collection efficiency, $E$, of an obstacle in an air flow, is defined as the ratio between the number of particles of uniform size impacting on the obstacle and the number of particles which would have passed through its projected area if the obstacle were absent.

2. COMSOL Model

The ribbon-in-the-long-channel can be modeled with a simplifying 2-D symmetric geometry shown in Figure 2, where only one half of the channel is considered. The dimensions of the channel are as same as that of the test chamber described in reference 1: 0.10 m wide and 0.30 m high. The ribbon is $3 \times 10^{-3}$ m thick and 0.019 m wide.

![Figure 2. 2-D geometry and boundary conditions used in the COMSOL model.](image)

Turbulent flow analysis in Chemical Engineering module (COMSOL v3.5a) was used for developing the flow field in the channel with a thin ribbon in place by solving the Navier Stokes equations. We used stationary $\kappa$-$\epsilon$ turbulence model with the simplifying 2-D geometry shown in Figure 2. The following boundary conditions were used in analyzing the flow. The bottom inlet: velocity condition, $u_0 = 4.2$ m/s; the right side: logarithmic wall condition; outlet on top: pressure = 0 and viscous stress-free condition; the left vertical side: symmetry boundaries; and the ribbon represented by logarithmic wall condition.

Triangular meshes with adaptive mesh generation were used in the $\kappa$-$\epsilon$ turbulence model. Final solution had 140k degrees of freedom with 16,455 elements.

After obtaining the flow field solution with COMSOL, the solution was exported to MATLAB. Using a script, particle tracing was carried out for various sizes of particles thereby varying the Stokes number, which is a dimensionless number, defined below. One hundred particles of a particular size were released from the bottom boundary (inlet of the channel) equally distributed along the projected width of the ribbon on that boundary. The termination of a particle trajectory was read off by “endpts” command and particle impaction was determined by checking it against the vertical location of the ribbon. Once the end point of a trajectory reached the vertical position of the ribbon, the particle is considered to have impacted the ribbon and collected on it. The number of impacted particles was thus counted and collection efficiency was simply expressed as a percentage with respect to the total original number of released particles. The total number of particles is 100 in this case, hence the collection efficiency is the count of the number of impacts it self.

3. Equations

A moving particle in an air flow experiences a drag force from the air movement. We used the Khan and Richardson force (ref. 2) for the drag force. Additionally, gravitational and buoyancy forces act on the particle as well. For the size range of particles considered in this study the gravitational and buoyancy force can even be neglected.

The governing equation for the motion of a particle, then, is given by

$$m_p \frac{du}{dt} = F_{KR} - F_B$$  \hspace{1cm} (1)
where \( m_p \) is the mass of the particle given by

\[
m_p = \frac{4}{3} \pi r_p^3 \rho_p \quad (r_p \text{ and } \rho_p \text{ are the radius and density of particle, respectively}).
\]

\( u_i \) is the component of velocity in the \( i \)-th direction; \( F_{KR} \) is the Khan & Richardson force given by

\[
F_{KR} = \pi r_p^2 \rho (u - u_p)^2 [1.84 Re_p^{-0.31} + 0.293 Re_p^{0.06}]^{-3.45}
\]

and \( Re_p \) is the particle Reynolds number given by

\[
Re_p = \frac{(u - u_p) 2r_p \rho_p}{\mu} ;
\]

and \( F_B \), the buoyancy force, is given by

\[
F_B = \frac{4}{3} \pi r_p^3 (\rho_p - \rho) g .
\]

The dimensionless Stokes number \( Stk \) is given by

\[
Stk = \frac{\rho_p u_0 d_p^2}{18 \mu h}
\]

where \( d_p \) is the diameter of the particle, \( \mu \) is the viscosity of air, \( u_0 \) is the air velocity at the inlet, and \( h \) is the width of the ribbon. We vary the particle diameter \( d_p \) for varying the dimensionless Stokes number.

Equation (1) is integrated in MATLAB to determine the particle trajectory in the velocity field obtained from solving the \( \kappa-\varepsilon \) turbulence model. Table 1 lists all important parameters used in the model.

**4. Results**

**4.1 Flow Field**

Figure 3 shows the velocity field in the test chamber in the presence of the ribbon. Also shown is the stream line plot where undisturbed flow occurs along the right wall as expected while beyond the ribbon a region of circulation is observed.

**Figure 3.** Velocity field and streamlines in the channel.

**4.2 Particle Capture Efficiency**

Figure 4 shows the trajectories of 10 particles released at the inlet of the channel for different Stokes numbers. The collection efficiency is observed to increase with increasing Stokes number. It is also observed that at smaller Stokes numbers particles are more dispersed in the outlet area.

We are using the initial velocity of the released particles to be the same as that of the air flow since in the experimental set up, the particles were released far away from the inlet allowing them to come to equilibrium with the flow.

Figure 5 shows the calculated particle collection efficiency as a function of Stokes number. The calculation is compared with the experimentally obtained mean curve from the reference 1.

### Table 1: Parameters used in the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of air ( \rho )</td>
<td>1.2</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Viscosity of air ( \mu )</td>
<td>(1.8 \times 10^{-5})</td>
<td>Pa.s</td>
</tr>
<tr>
<td>Density of butyl phthalate particles ( \rho_p )</td>
<td>1050</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Range of radius of particles ( partr )</td>
<td>10 – 130</td>
<td>micron</td>
</tr>
<tr>
<td>Velocity of free-air ( u_0 )</td>
<td>4.2</td>
<td>m/s</td>
</tr>
</tbody>
</table>
The scatter in the experimental data is as high as 20%, as indicated in the reference 1. Figure 5 shows that the calculated curve agrees with the experimental curve within 20% experimental error. The COMSOL calculation yields the traditional s-shaped curve for the collection efficiency. There is a small constant shift on the Stokes number axis indicating a systematic error, most likely, in the estimation of the $Stk$.

![Particle trajectories at different Stokes numbers; $Stk = 0.006, 0.1, 0.4$ and 10.](image)

**Figure 4.** Particle trajectories at different Stokes numbers; $Stk = 0.006, 0.1, 0.4$ and 10.

**Figure 5.** Calculated particle collection efficiency as a function of Stokes number compared with experimental data from Ref. 1. The error bar shows the 20% experimental error.

### 5. Conclusions

The COMSOL Multiphysics was very useful in calculating the particle collection efficiency of a ribbon as described. The calculated $E$ vs. Stokes number curve agrees well with the experimentally observed curve within 20% experimental error. The MATLAB script of the present work can be extended to analyzing other interesting geometries such as discs, cylinders and spheres.

### 6. References


### 7. Acknowledgements

We acknowledge the assistance of Markus Jonsson of COMSOL with MATLAB/COMSOL interface.