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Transport Phenomena and Shrinkage Modeling During Convective Drying of Vegetables

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Outline of the talk

Part 1

Formulation of a transport model describing the simultaneous transfer of momentum, heat and mass occurring in a convective drier where hot dry air flows, under turbulent conditions, around a food sample

The proposed model does not rely on the specification of interfacial heat and mass transfer coefficients and, therefore, represents a general tool capable of describing the behavior of real driers over a wide range of process and fluid-dynamic conditions.

Part 2

Formulation of a transport model describing the non-isotropic shrinkage deformation undergoing simultaneous heat and mass transfer (stress-strain analysis coupled to momentum, heat and mass transfer in a time-dependent deformed mesh)

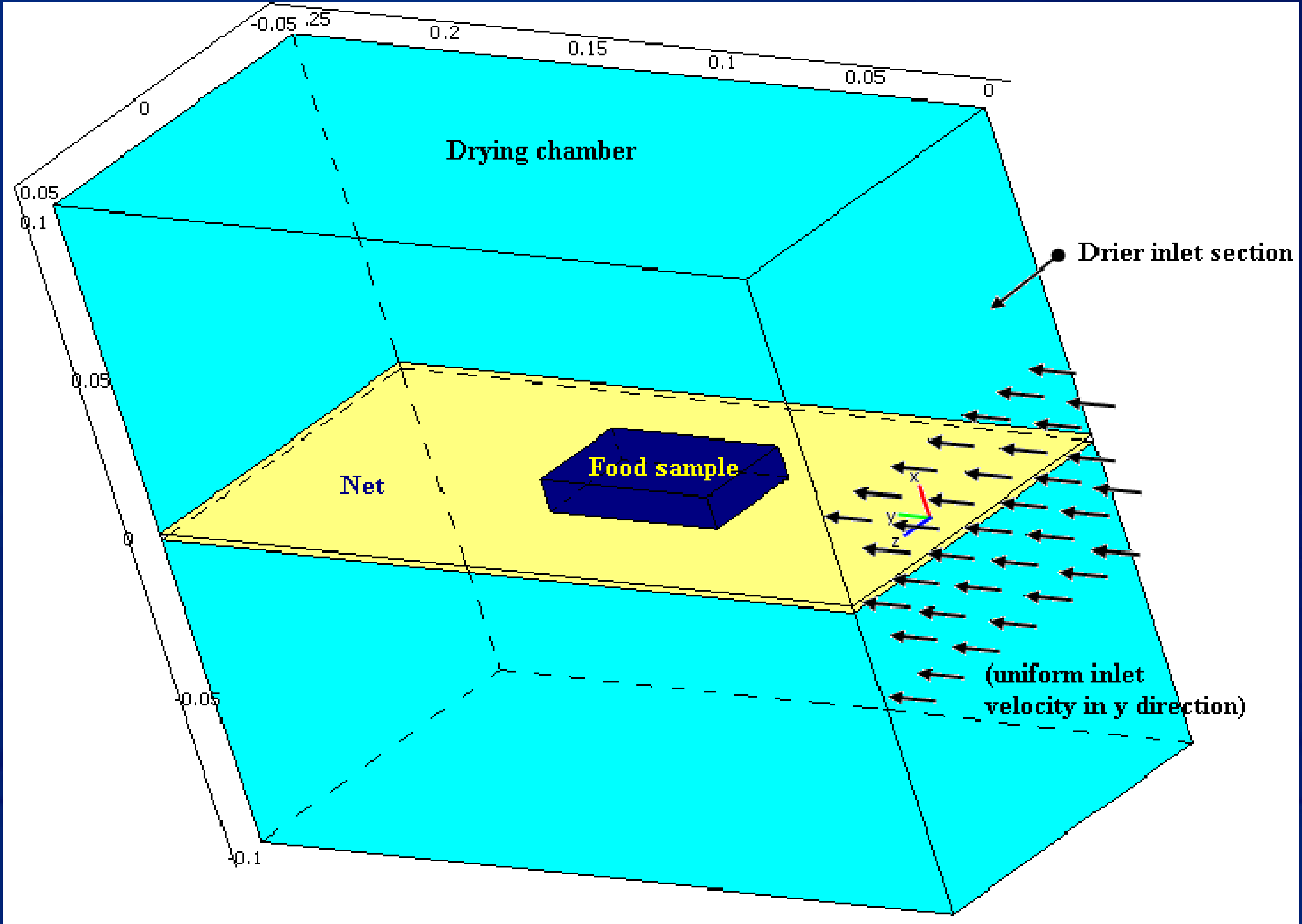
In a previous paper (*Journal of Food Engineering 87 (2008) 541–553*), a model analyzing the simultaneous transfer of momentum (for air only), and of heat and mass (for both air and food) occurring in a convective drier was presented.

It is intended, with the present work, to improve the accuracy of the previous model :

- Reformulation of unsteady-state momentum balance in terms of the k-ω model that behaves better than k-ε model, for instance close to the walls
- Besides liquid water and energy conservation, also the transport of vapor in food matter has been accounted for
- Shrinkage deformation was modeled by virtual work principle coupled to heat and mass balances written with reference to a deformed mesh (Arbitrary Lagrangian-Eulerian – ALE – description)

Part 1- 2/7

System schematization



Formulation of the transport model in the food (undeformed mesh)

Conservation equations for liquid water and vapor

$$\partial C_w / \partial t + \underline{\nabla} \cdot \left(- D_w \underline{\nabla} C_w \right) + \dot{I} = 0$$

$$\partial C_v / \partial t + \underline{\nabla} \cdot \left(- D_v \underline{\nabla} C_v \right) - \dot{I} = 0$$

Energy conservation

$$\rho_s C_{p_s} \partial T / \partial t - \underline{\nabla} \cdot \left(k \ \underline{\nabla} T \right) + \lambda \cdot \dot{I} = 0$$

Main hypotheses:

- Vapor and liquid water are in phase equilibrium at any time
- Convective transport is neglegible

The above equations have been coupled, by a proper set of boundary conditions, expressing the continuity at food/air interfaces, to the conservation equations in the air.

No heat/mass transfer coefficient is, therefore, needed; the proposed approach is useful when food shape changes irregularly with time (shrinkage)

Formulation of the transport model in the air (undeformed mesh)

Momentum balance **and continuity equation** **k-ω model (Wilcox)**

$$\partial \rho_a / \partial t + \underline{\nabla} \cdot \rho_a \underline{u} = 0$$

$$\rho_a \, \partial \underline{u} / \partial t + \rho_a \underline{u} \cdot \underline{\nabla} \underline{u} = \underline{\nabla} \cdot \left[- p \underline{I} + (\eta_a + \eta_t) \left(\underline{\nabla} \underline{u} + (\underline{\nabla} \underline{u})^T - (2/3)(\underline{\nabla} \cdot \underline{u}) \underline{I} \right) - (2/3) \rho_a k \underline{I} \right]$$

where

$$\eta_t = \rho_a \, k / \omega$$

Turbulent kinetic energy

$$\rho_a \, \partial k / \partial t + \rho_a \underline{u} \cdot \underline{\nabla} k = \underline{\nabla} \cdot [(\eta_a + \sigma_k \eta_t)(\underline{\nabla} k)] + \eta_t P(\underline{u}) - (2 \rho_a k / 3)(\underline{\nabla} \cdot \underline{u}) - \beta_k \rho_a k \omega$$

Dissipation per unit turbulent kinetic energy

$$\rho_a \, \partial \omega / \partial t + \rho_a \underline{u} \cdot \underline{\nabla} \omega = \underline{\nabla} \cdot [(\eta_a + \sigma_\omega \eta_t)(\underline{\nabla} \omega)] + (\alpha \omega / k)[\eta_t P(\underline{u}) - (2 \rho_a k / 3)(\underline{\nabla} \cdot \underline{u})] - \beta \rho_a \omega^2 / k$$

Mass balance referred to vapor and Energy conservation

$$\partial C_2 / \partial t + \underline{\nabla} \cdot \left(- D_a \underline{\nabla} C_2 \right) + \underline{u} \bullet \underline{\nabla} C_2 = 0$$

$$\rho_a C_{pa} \, \partial T_2 / \partial t - \underline{\nabla} \cdot \left(k_a \underline{\nabla} T_2 \right) + \rho_a C_{pa} \underline{u} \cdot \underline{\nabla} T_2 = 0$$

Part 1- 5/7

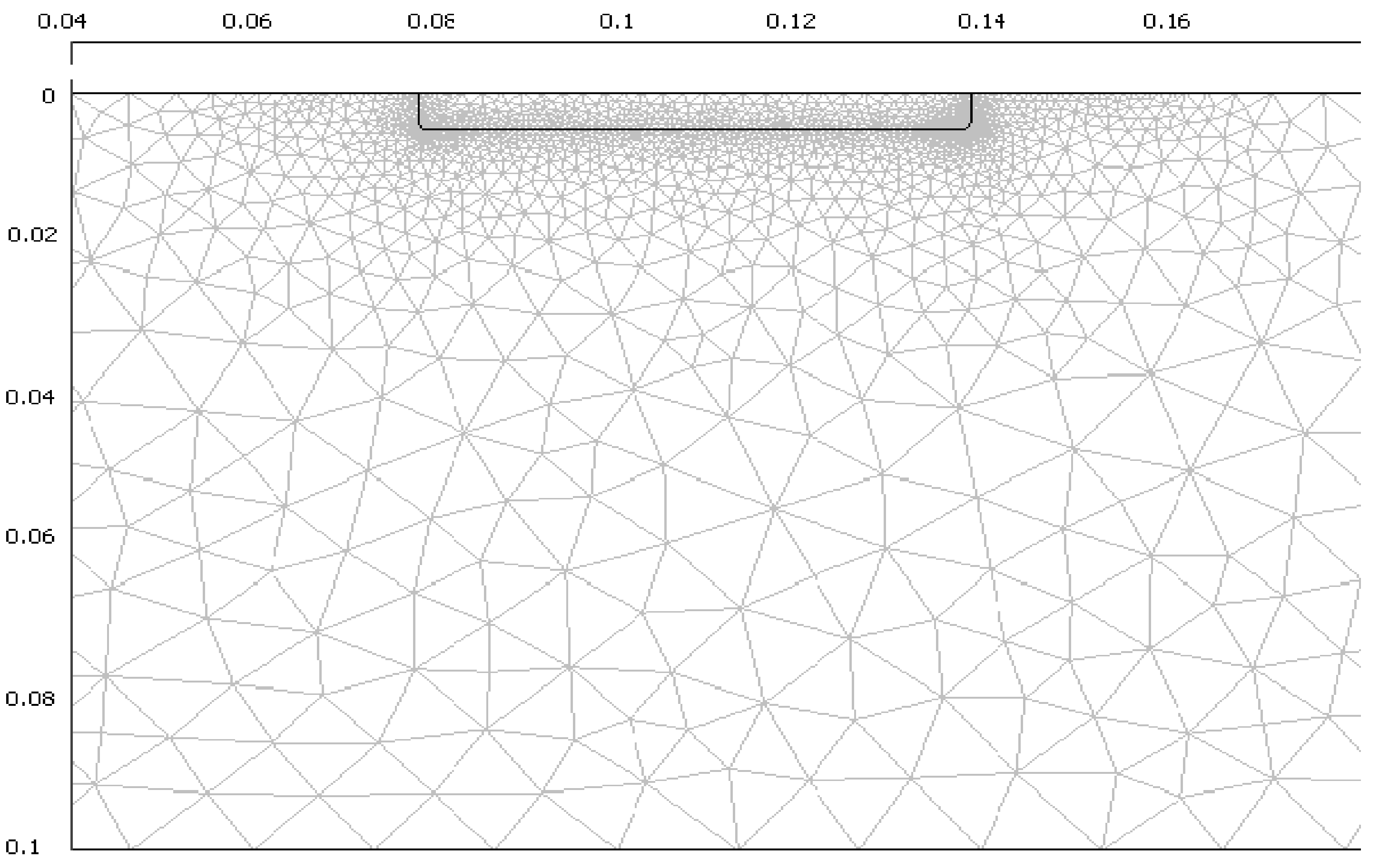
System of PDEs solved by FEM (Comsol Multiphysics 3.4).

Total number of 12950 triangular finite elements leading to about 112500 DOFs.

The mesh consisted of 6611 and 6339 elements within the food and air domains.

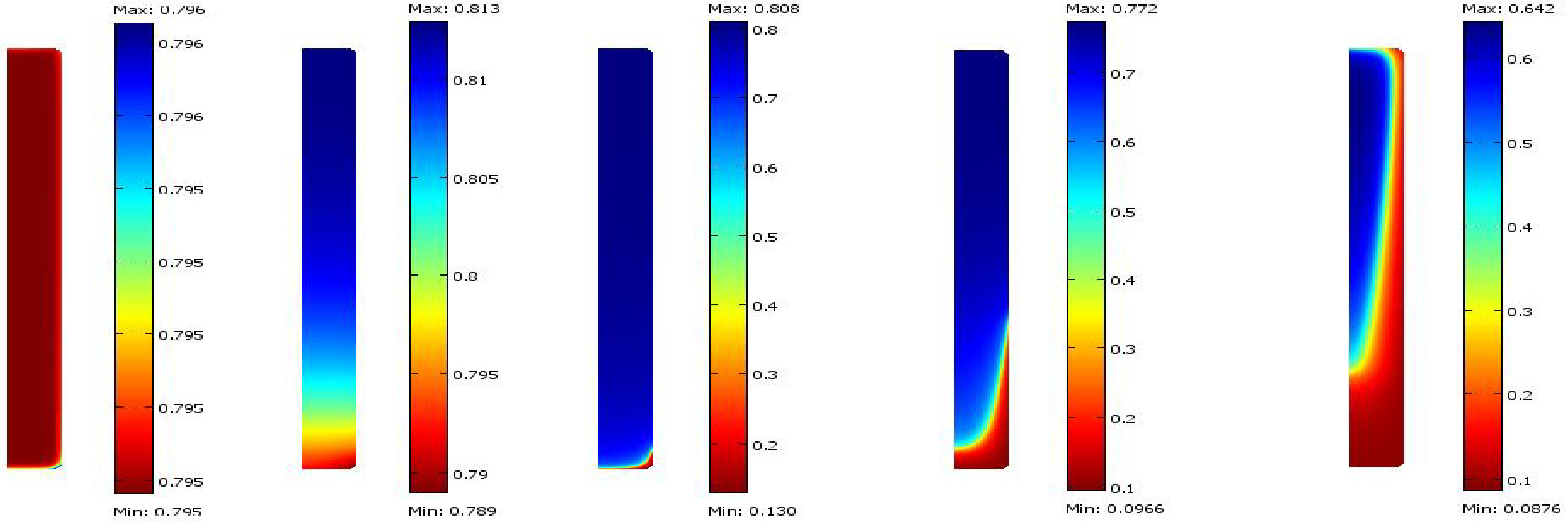
The considered mesh provided a good spatial resolution and the solution was independent on the grid size even with further refinements.

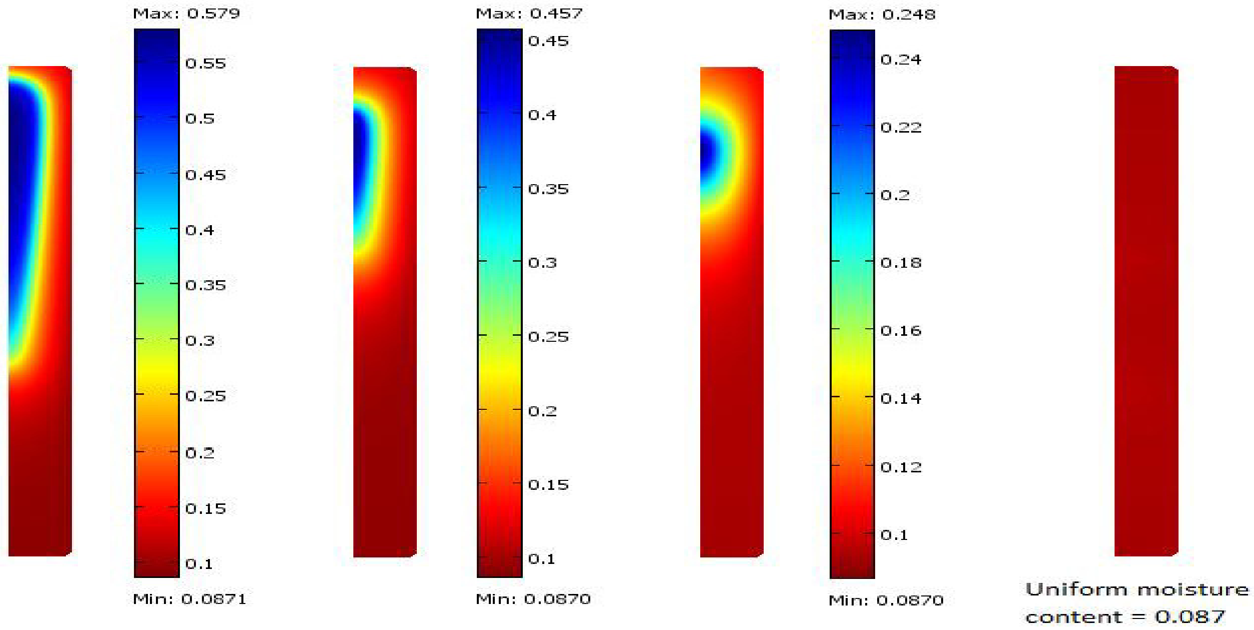
Lagrange finite elements of order two were chosen for all the variables.



Part 1- 6/7

Time evolution of potato moisture content (on a wet basis)





Wet bulb temperature: 343 K;

Air velocity: 1 m/s

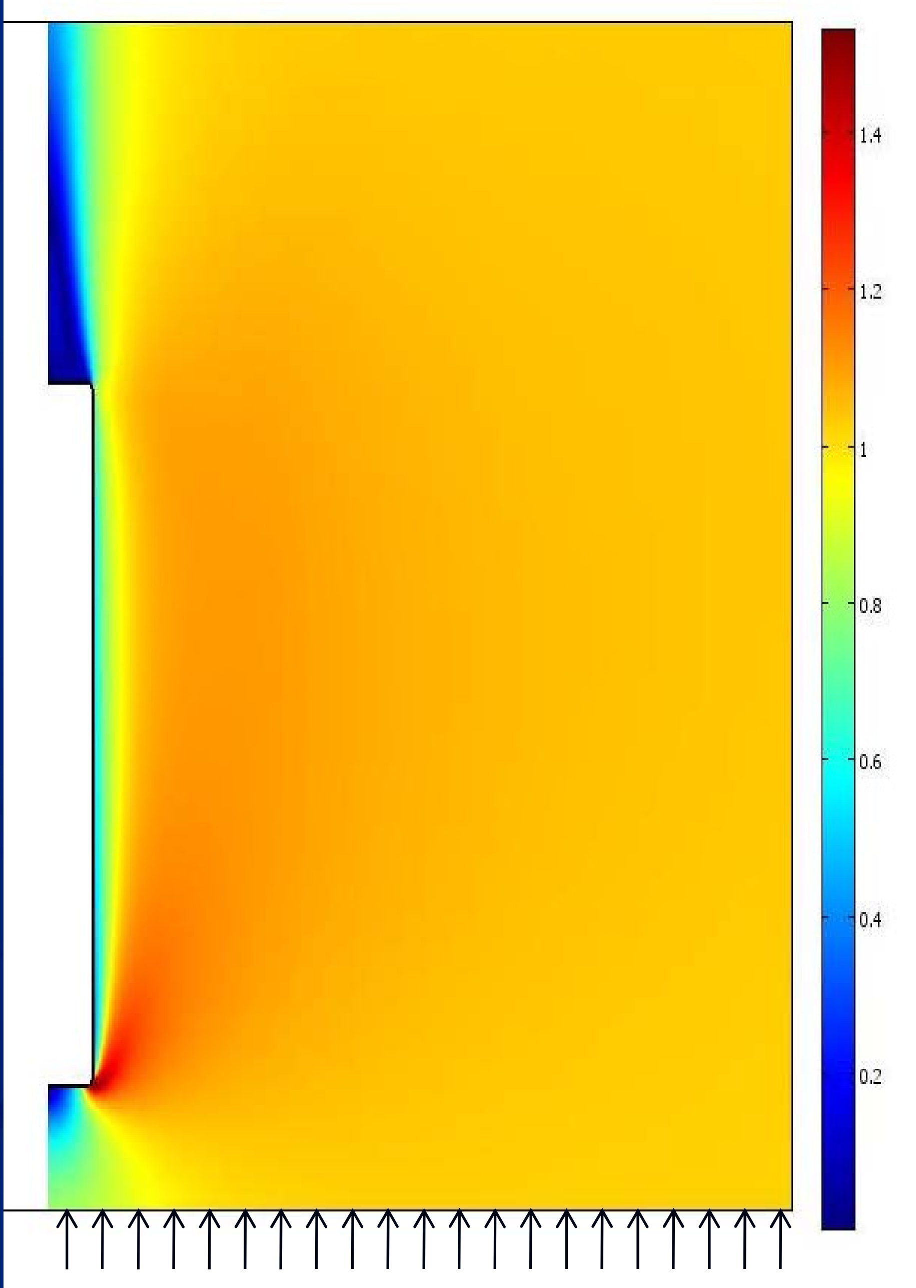
Air absolute humidity: 62.4 g_w/m^3

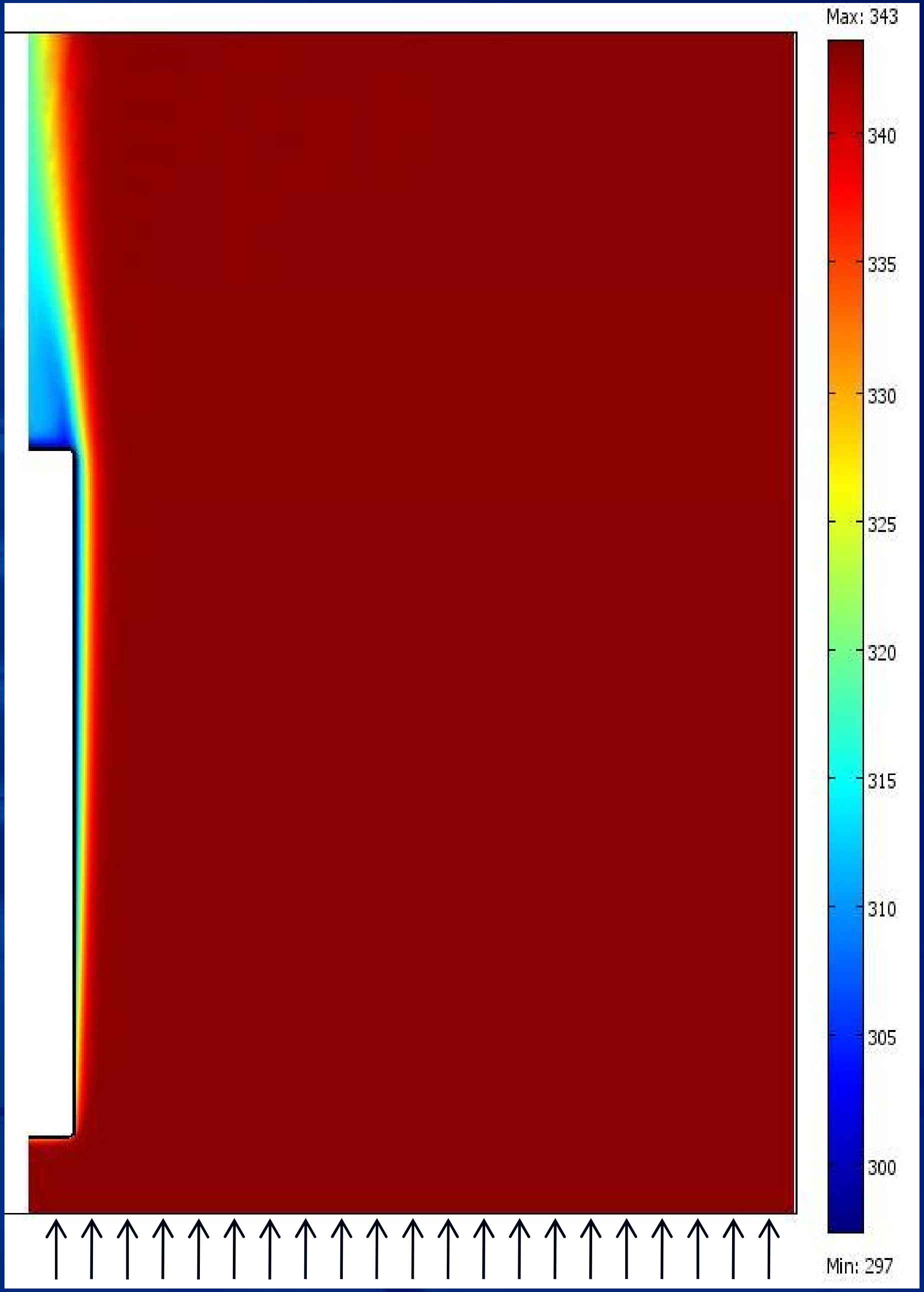
Food Initial temperature : 283 K

Initial moisture content of the food (on a wet basis) : 0.795 Kg_w/Kg

Part 1- 5/7

Velocity field and temperature profile close to food sample (t = 6 min)





Part 2- 1/5

Modeling of food shrinkage I

Local total strains $\{d\varepsilon\}$ are a function of changes in mechanical strains $\{d\varepsilon_s\}$ (constrained deformation due to food mechanical properties, i.e. elasto-plasticity) and in shrinkage strains $\{d\varepsilon_0\}$ (the sum of a free deformation due to moisture loss)

$$\{d\varepsilon\}=\{d\varepsilon_s\}+\{d\varepsilon_0\}$$

Total strain $\{d\varepsilon\}$ *is* a function of total displacement $\{dU\}$

$$\{d\varepsilon\}=[A]\{dU\}$$

Changes in stresses $\{d\sigma\}$ are function of changes in mechanical strains $\{d\varepsilon_s\}$

$$\{d\sigma\}=[D]\{d\varepsilon_s\}$$

Where [D] is the stress-strain matrix containing the Young Modulus.

Yang, H., Sakai, N. and Watanabe, M., Drying model with non-isotropic shrinkage deformation undergoing simultaneous heat and mass transfer, *Drying Technology*, Vol 19 (7), pages 1441-1460 (2001).

Part 2- 1/5

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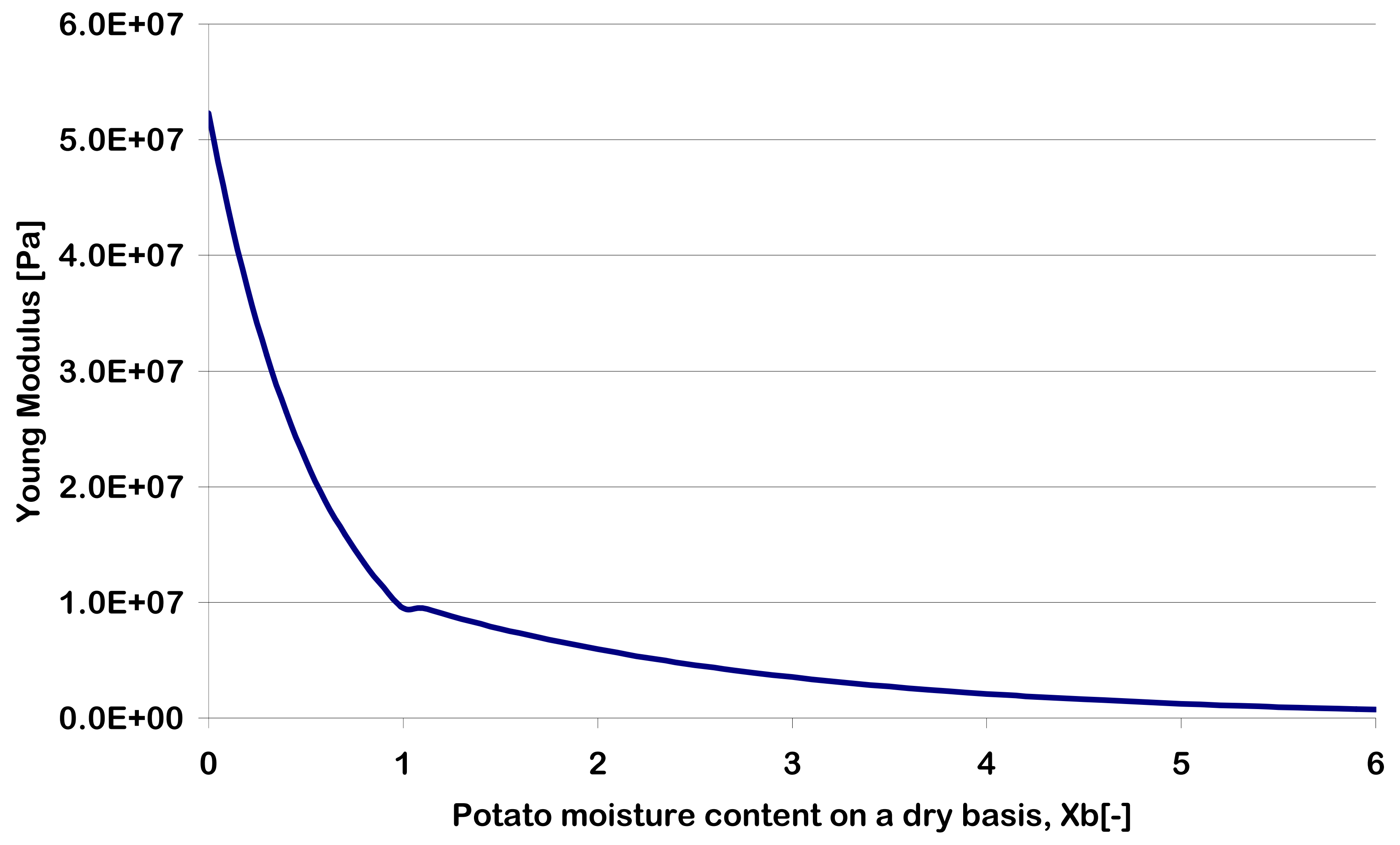


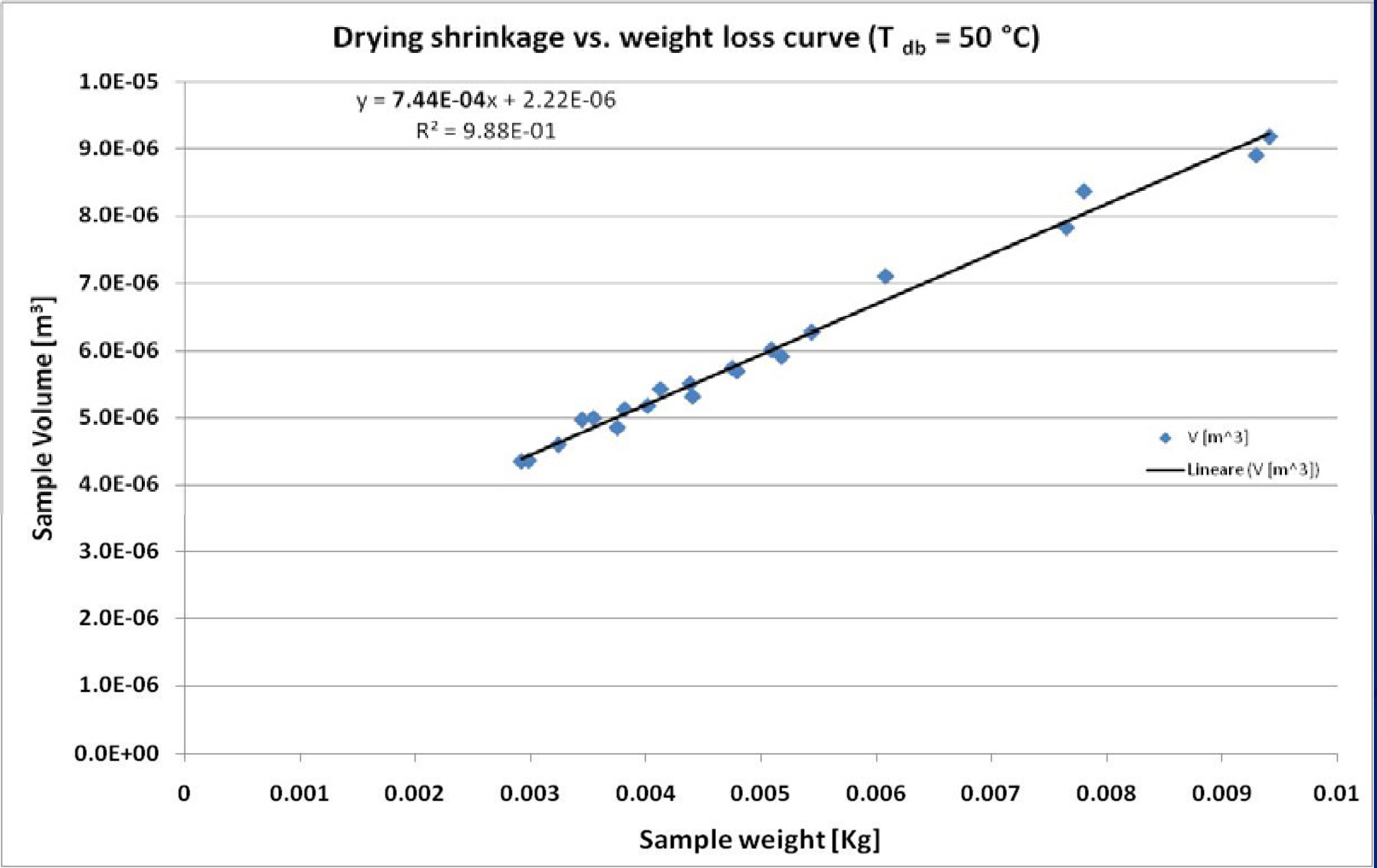
Yang, H., Sakai, N. and Watanabe, M., Drying model with non-isotropic shrinkage deformation undergoing simultaneous heat and mass transfer, *Drying Technology*, Vol 19 (7), pages 1441-1460 (2001).

Part 2- 2/5

Modeling of food shrinkage II

It is necessary to express the free drying shrinkage strains $\{d\varepsilon_0\}$.

Hypothesis: the free deformation due to moisture loss was taken as proportional to the water content variation, through a constant (the hydrous compressibility factor) that has been estimated from the experimental data showing drying shrinkage vs. weight loss



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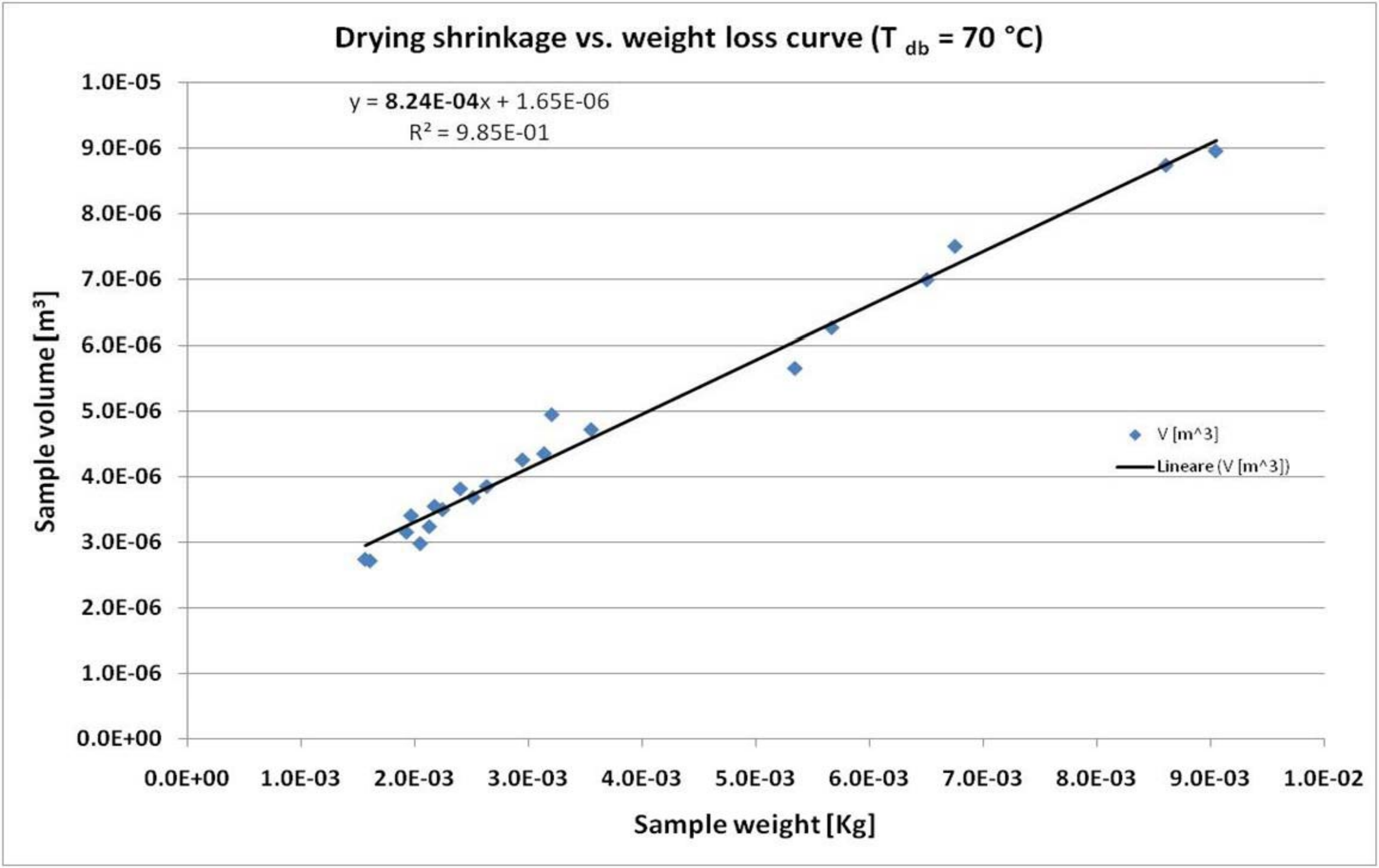


Part 2- 3/5

Modeling of food shrinkage III

Virtual work principle applied to obtain the equilibrium equation. By assuming that zero body and surface forces are applied to food, it can be written:

$$\int_V \delta \{d\varepsilon\}^T \{d\sigma\} dV = 0$$

As far as the boundary conditions are concerned, one side of the food rests on the drier net (fixed position) whereas the other three are free to move

The transport equations for both air and food, together with the virtual work principle have been written referring to a time dependent deformed mesh that accounts for food volume variation due to water evaporation.

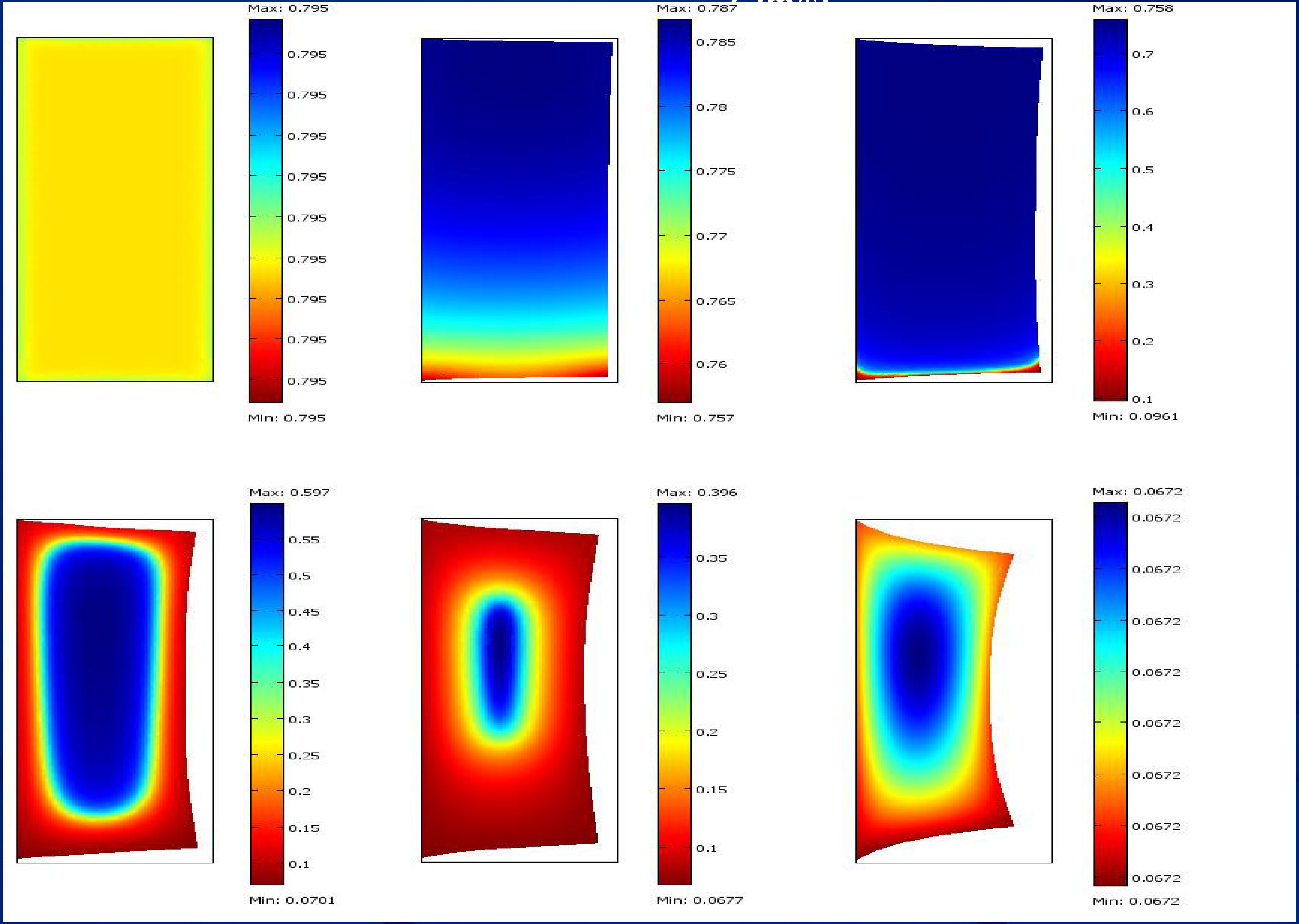
An Arbitrary Lagrangian-Eulerian (ALE) description, implemented by Comsol Multiphysics has been adopted.

The motion of the deformed mesh is modeled using Laplace smoothing.

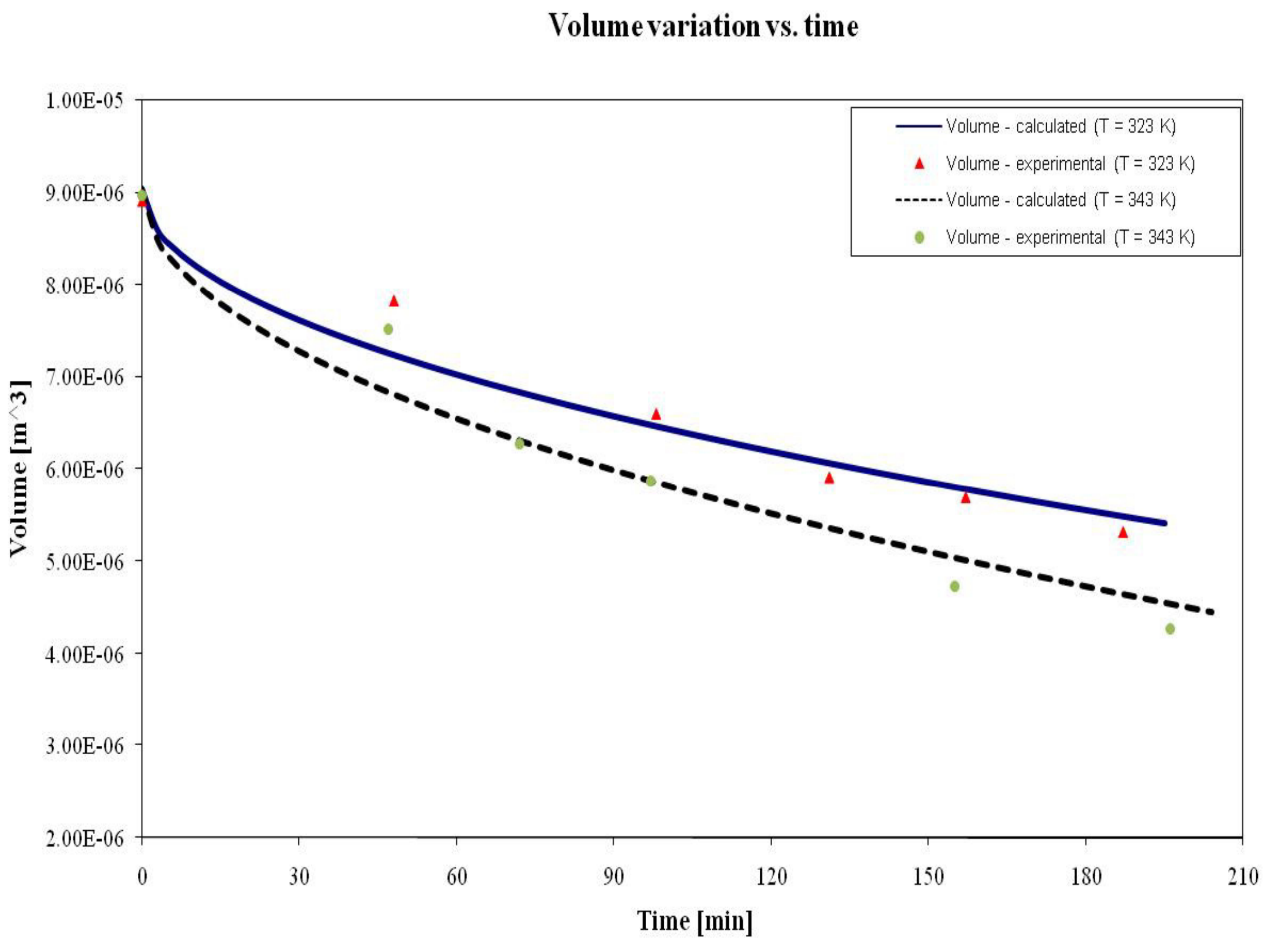
The boundary conditions control the displacement of the moving mesh with respect to the initial geometry. At the boundaries of food sample, this displacement is the same as the structural deformation. At the exterior boundaries of the fluid domain, it is set to zero in all directions.

Part 2- 4/5

Time evolution of potato moisture content (on a wet basis) during drying accounting for shrinkage effect. (Air temperature of 50°C, air velocity of 2.2m/s)

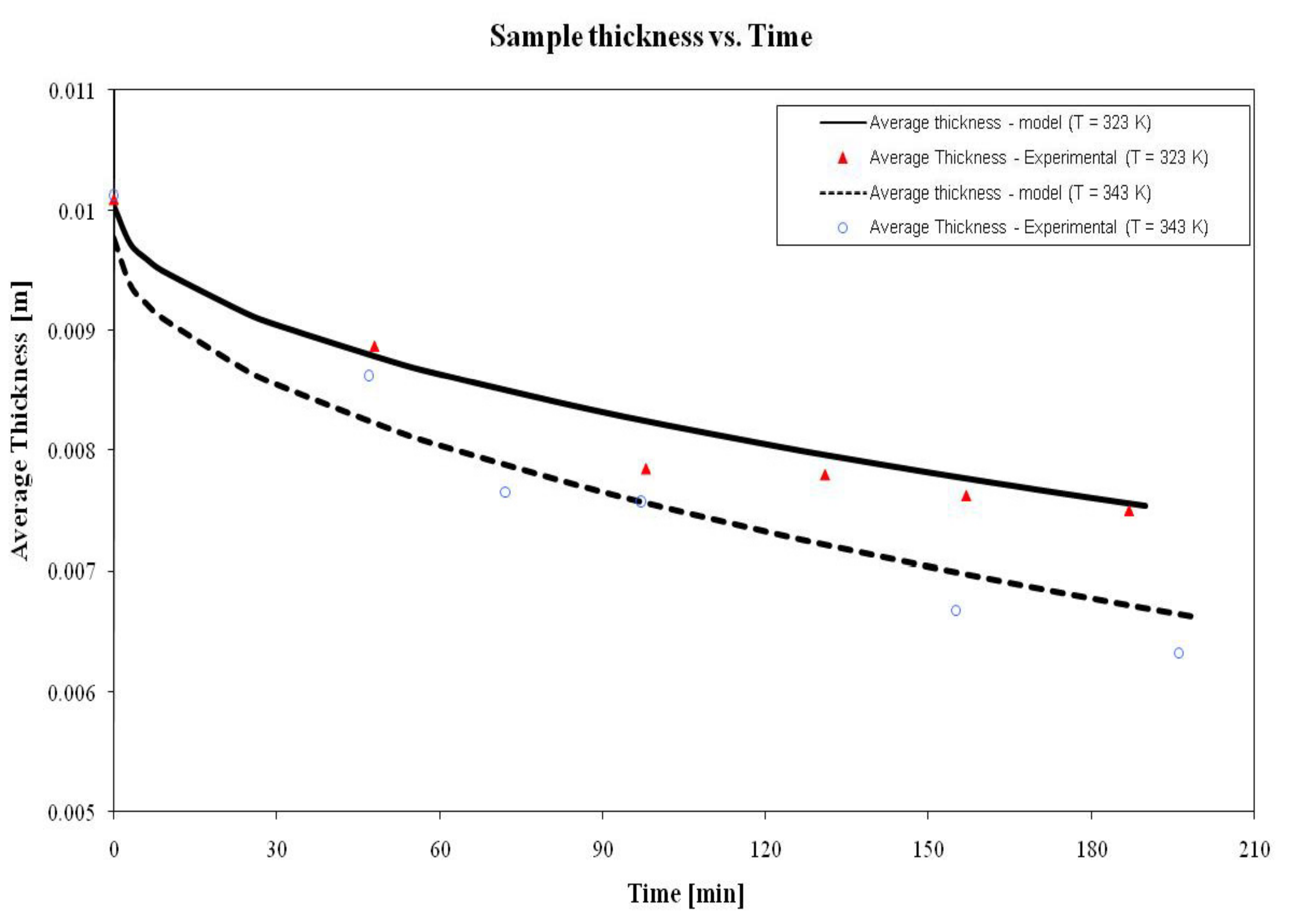


Comparison between measured and calculated dimensions



Part 2- 5/5

Comparison between measured and calculated dimensions



Conclusions and future developments

The transport phenomena involved in food drying have been analyzed. A general predictive model, i.e. not based on any semi-empirical correlation for estimating heat and mass fluxes at food-air interface, has been formulated.

The proposed model also predicted the spatial moisture profiles at all times, thus allowing detecting the regions within the food core, where high values of moisture content can promote microbial spoilage.

The model has been also improved to predict food shrinkage by coupling transport equations to virtual work principle, written with reference to a deformed mesh whose movement has been described by ALE method. The obtained results are promising.

It is intended to improve the transport model by considering the influence of convection inside the food .

Also shrinkage description needs to be improved, for instance by formulating a different assumption relating free deformation due to moisture loss to water content variation, or by taking into account the influence of body and surface forces applied to food. Extension to a 3D domain.

Thank you for your kind Attention

Any Question?