A NOVEL FEM METHOD FOR PREDICTING THERMOACOUSTIC COMBUSTION INSTABILITY

G. Campa
S.M. Camporeale

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Combustion Instability

Improvement of gas turbine performance

Efficiency

Emissions

annular combustors

fuel premix

Larger tendency to the creation of thermoacoustic instability

- excessive noise
- mechanical failures
- flame detaching and misfiring
- production of emissions

Heat released oscillations

Acoustic Oscillations
Objectives

• Introduction of a mathematical model able to study acoustics and heat released oscillations;

• Resolution of this mathematical model through FEM technique in order to identify the instability conditions of the system examined;

• Demonstrate the capability of COMSOL Multiphysics to accurately solve eigenvalue problems in the analysis of combustion instability applied to complex geometries.
Mathematical Model

With regards to a compressible and viscous fluid, the conservation equations of mass, momentum and energy can be written as follows:

\[
\frac{D\rho}{Dt} + \rho \nabla \mathbf{u} = 0 \\
\rho \frac{D \mathbf{u}}{Dt} = -\nabla p + \frac{\partial \sigma_{i,j}}{\partial x_j} \mathbf{e}_i
\]

\[
\rho \frac{D}{Dt} \left( e + \frac{1}{2} u^2 \right) = -\nabla \cdot (p \mathbf{u}) + q + \nabla \cdot (k \nabla T) + \frac{\partial}{\partial x_j} \left( \sigma_{i,j} u_i \right)
\]

When a fluid is considered non viscous, a gas perfect and mean flow negligible, the linear forms of the previous equation brings to the inhomogeneous wave equation:

\[
\frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} - \rho \nabla \cdot \left( \frac{1}{\rho} \nabla p' \right) = \frac{\gamma - 1}{c^2} \frac{\partial q'}{\partial t}
\]

Heat release rate per volume unit [W/m^3].
In COMSOL Multiphysics the *Acoustics Module* is the module adopted in the present analysis.

The application mode used, *Pressure Acoustics*, is able to find the complex eigenfrequencies of the system.
Subdomain Settings

Air has been described introducing:

- *Fluid density*;
- *Speed of sound*.

Heat release has been described introducing:

- *Monopole source* in agreement with the heat release law imposed.
Boundary Settings

Solid walls are modelled with *Sound Hard Boundary*.

Inlet & Outlet wall are modelled with one of the following conditions:

- *Sound Hard Boundary*;
- *Sound Soft Boundary*;
- *Normal Acceleration*.

The right condition is defined in agreement with the examined test.
Preliminary Tests on Linear Combustion Chamber

\[ Q'(t) = -\left[ \beta \frac{\overline{\rho} \overline{c}^2}{(\gamma - 1)} \right] u_1(t - \tau) \]
Analytical solution is available:

\[
\tan\left(\frac{\omega}{c}b\right) \tan\left(\frac{\omega}{c}(l-b)\right) = 1 - \beta \exp(-i\omega \tau)
\]
Tests on Linear Combustor with variation of section

\[ \frac{\hat{Q}}{Q} = -k \frac{\hat{m}_i}{m_i} \exp(-i\omega \tau) \]

Time delay
Axial Mode
Pressure Wave
Tests on Annular Combustion Chamber

\[ \frac{\hat{Q}}{Q} = -k \frac{\dot{m}_i}{m_i} \exp(-i \omega \tau) \]
Unstable for $\tau > 0.002$ s

Unstable for $\tau < 0.002$ s and $\tau > 0.004$ s

Unstable for $\tau = 0.001$ s, 0.003 s, 0.005 s

Unstable for $0.003$ s $< \tau < 0.006$ s
Conclusions

• The mathematical model introduced has been successfully solved;
• COMSOL Multiphysics provides sufficient level of accuracy in the identification of stable and unstable eigenmodes;
• The FEM analysis has been successfully applied to different kinds of heat release law and different boundary conditions;
• The present approach is appropriate to treat complex geometry;
• COMSOL Multiphysics provides a commercially available tool that can analyze combustion instability problem.
Further Applications

The method can be applied to analyze the effects of:

• Passive damping devices;
• Geometry of the system;
• Flame response functions;
• Transfer function matrices of the burners.
Thank you for attention

Questions?