The Full-System Approach for Elastohydrodynamic Lubrication

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Abstract: A ball is in contact with a plane, and a lubricant separates the two surfaces to decrease friction during their relative motion. To avoid wear, the lubricant film thickness should be higher than the surface roughness. The goal of this paper is to show how it is possible to solve efficiently the problem of elastohydrodynamics lubrication with Comsol Multiphysics, using a PDE on boundary to implement the Reynolds equation.

Keywords: Elastohydrodynamics, lubrication, Reynolds equation, tribology.

1. Introduction

The good performance of mechanical systems, bringing into play solids in relative motion, is conditioned by an adequate design of the connections, and thus of the contacts, potentially generating friction and wear. The usual (and very old [1]) technology to control these phenomena is lubrication. This one makes it possible to intercalate a medium (generally fluid, but sometimes solid or pasty) between the solids in contact, that is ready to support a load (normal with the contact) and to adapt the difference in speed (tangential) of surfaces, thus avoiding a direct interaction of the solids which would generate degradations [2]. For the fifty last years, remarkable progress has been made in order to build predictive models of the lubricated contacts. This was made possible due to the numerical resolution (more and more powerful) of the equations of lubrication whose icon is the Reynolds equation, derived from the Navier-Stokes equations in the case of a very thin lubricant film. In this approach, the physical parameters (pressure, viscosity, density) are considered constant across the lubricant thickness. If the resolution of only the Reynolds equation makes it possible to find the distribution of pressure in the fluid in the case of conform geometries (like the hydrodynamic bearings) [3], it is necessary to solve the elastic deformation of the solid surfaces in the case of non-conform contacts (as in the cam system, in ball bearings, etc.) where pressure, concentrated on a very weak contact area, reached the order of the gigapascal [4]. Traditional resolution methods of this lubrication regime, known as elastohydrodynamic lubrication (EHL), can be found in [5] and [6]. Since recently an original method based on finite elements with Comsol Multiphysics was developed in LaMCoS ([7], [8]). It allows a strong coupling between the deformation of the solid surfaces and the flow of lubricant computed by the Reynolds equation, on a boundary of the elastic solids (where the contact occurs).

2. Model

The calculation of film thickness and pressure distributions is done by solving simultaneously the two physics involved in EHL – hydrodynamics and linear elasticity – using a finite element analysis. This method will not be detailed here (see references [7] and [8] for details) but the equations solved will be recalled. Hertz theory for a ball-on-disc dry contact predicts a contact radius a and a maximum pressure p_H such that:

$$a = \left(\frac{3LR_B\left(E_B(1-\upsilon_D{}^2)+E_D\left(1-\nu_B{}^2\right)\right)}{4E_BE_D} \right)^{1/3}$$
$$\text{and } \; p_H = \frac{3L}{2\pi a^2} \qquad (1)$$

where R_B is the ball radius, L the applied normal load, (E_B, E_D) and (υ_B, υ_D) the Young modulus and Poisson ratio of the spherical-end and plane solid respectively. In the present EHL model, the elastic deformation due to contact pressure is computed over an equivalent cubic solid with an edge 60a long, as represented in Figure 1. By using equivalent material properties for this solid (expressed later in this section), the total deformation of both solids B and D are calculated. The hydrodynamic problem (Reynolds equation) is solved over a part of the upper surface of the cube (a square of edge 6*a* long). These dimensions have been established as the smallest ones above which the accuracy of the results remains unchanged.

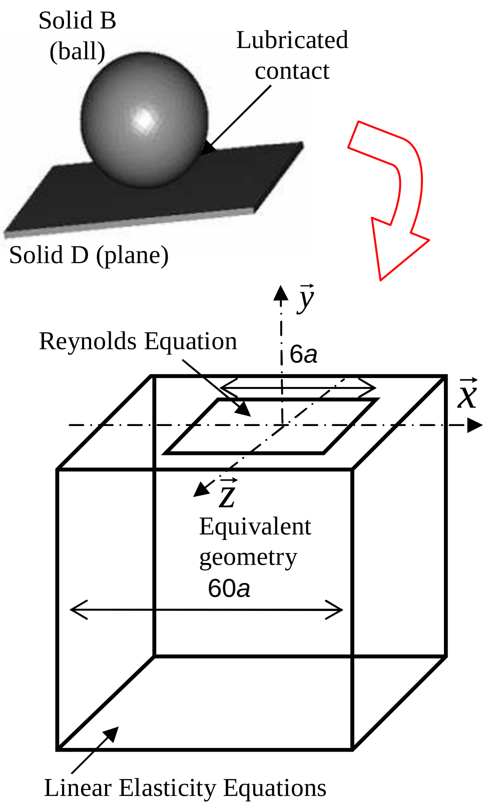

Figure 1. The geometries for the elastic deformation problem and, on one boundary, the lubricated contact The caption should be centered underneath the figure and set in 9-point font.

- <u>The hydrodynamic problem</u>

Assuming fully-flooded conditions, laminar flow, Newtonian rheology, smooth surfaces and isothermal, steady-state regime, the classical Reynolds equation can be written as a function of pressure *p* as:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial z}\right)-\frac{\partial(U_m\rho h)}{\partial x}=0 \quad (2)$$

with :

- h the film thickness, expressed as:

$$h(x,z)=h_0+\frac{x^2}{2R_B}+\frac{z^2}{2R_B}-u_2(x,z) \qquad (3)$$

where $u_2(x,z)$ is the displacement of the equivalent solid in the *y*-direction,

- ρ the density, varying with pressure according to the Dowson-Higginson relationship:

$$\rho(p)=\rho_R\frac{0.59\times10^9+1.34p}{0.59\times10^9+p} \qquad (4)$$

where ρ_R is the reference density at ambient pressure,

- η the viscosity, varying according to Roelands equation as:

$$\eta(p)=\eta_R\exp\left((\ln(\eta_R)+9.67)\left(-1+\left(1+\frac{p}{P_R}\right)^{z_R}\right)\right) \quad (5)$$

where η_R is the reference viscosity, P_R = $1.96\text{x}10^8$ Pa and $z_R=\frac{\alpha P_R}{\ln(\eta_R)+9.67}$,

- U_m being the mean entrainment velocity :

$$U_m=\frac{U_B+U_D}{2} \qquad (6)$$

with U_B and U_D respectively the velocities of the two solids : the ball and the plane.

Zero pressure boundary conditions are applied at the edges of the contact domain.

- <u>Elastic deformation</u>

The elastic deformation is calculated over an equivalent solid with Young modulus E_{eq} and Poisson ratio υ_{eq} being a composition of both solids B and D characteristics. When solids B and D are both made of the same material, the equivalent solid characteristics are simply:

$$\left.\begin{array}{l} E_{eq} = \dfrac{E_B}{2} = \dfrac{E_D}{2} \\ \upsilon_{eq} = \upsilon_B = \upsilon_D \end{array}\right\} \qquad (7)$$

The equations solved in the cubic volume represented in Figure 1 are:

$$\sigma_{ij} = \frac{E_{eq}}{1+\upsilon_{eq}}\left(\varepsilon_{ij} + \frac{\upsilon_{eq}}{1-2\upsilon_{eq}}\varepsilon_{kk}\delta_{ij}\right) \quad (8)$$

with

- σ_{ij} representing the nine components of the stress tensor,
- $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ representing the strain matrix,

(u_1, u_2, u_3) being the displacements in the three directions of space $(x_1, x_2, x_3) = (x, y, z)$.

Zero displacement boundary condition is applied to the bottom surface of the domain. In the contact region (where p satisfies Reynolds equation), the condition $\sigma_{22}\left(=\sigma_{yy}\right) = -p$ is applied. Zero stress boundary condition is applied to the other boundaries.

- Load equilibrium

The external load applied to the contact is totally supported by the lubricant film. Therefore, the equilibrium of forces requires that the total pressure generated in the contact domain S balances the external applied load L:

$$\iint_S p(x,z)dS = L \qquad (9)$$

This equation is satisfied by adjusting h_0, the constant parameter of the film thickness equation. This way, knowing the geometry, the materials and fluid characteristics, the load and velocities applied to the contact then the unknowns (the pressure field inside the contact, h_0 and the deformation of the surfaces, and thus the film thickness) can be determined by solving the complete system of equations formed by (2), (8) and (9).

3. Tips for Comsol implementation

The implementation into Comsol Multiphysics environment is easy for the elastic problem with the classical *Structure Mechanics* box. As the Reynolds equation is not supported directly by Comsol, it has to be added with the *PDE mode* box. The Reynolds equation acts on the 2D surface (boundary) of the 3D deformable solid, so a *weak form on boundary* as to be specified.

Lagrange quadratic elements for the elastic problem and Lagrange quintic elements for the Reynolds problem are chosen. The mesh can be extremely coarse for the elastic problem, but it has to be smaller than a tenth of the Hertzian contact radius *a* (see eq. (1)) to solve the Reynolds equation in the contact area..

For high pressure (as it is classically the case in EHL), stabilization techniques (SUPG, GLS and Isotropic Diffusion) have to be implemented (see [7]).

Due to the diverging surfaces at the contact exit, negative pressures may arise (cavitation zone). To avoid this non realistic solution, a penalty method can be used to enforce the negative pressures to zero, as described in [9].

The load equilibrium (eq. (9)) can be implemented a smart way with Comsol, using an Integration Coupling Variable to be equal to a given constant with the help of the *Global Equation* toolbox. The corresponding unknown is h_0, even if it does not appear explicitly in the equation. Be aware that this creates a null value on the matrix diagonal and prevents the use of iterative algorithms.

4. Results

This model allows the calculation of the pressure and lubricant film thickness distribution over the contact area. The technological interest is the comparison of the minimum film thickness and the roughness of the real surfaces in order to anticipate some potential direct interaction betweens the two surfaces, which can cause degradation of the system (wear, cracks propagation, etc.). A case study is proposed in this paper, which corresponds to the solids, lubricant and operating conditions described in Table 1.

Qualitative results are shown in Figure 2, where the pressure and film thickness profiles are plotted (with dimensionless values). In EHL, the pressure distribution is very close to the Hertzian distribution but for the continuous increase at the inlet and a pressure spike at the outlet of the contact. The corresponding film thickness exhibit a quite constant value near the center of the contact, and a local minimum value is found at the outlet.

Ball radius (*R*)	0.0127 m
Ball speed (U_B)	1 m/s
Plane speed (U_D)	1 m/s
Load (*L*)	20 N
Young modulus (ball and plane)(*E*)	210 GPa
Poisson ratio (υ)	0.3
Viscosity (ambient) (η_0)	0.012 Pa.s
Pressure-viscosity coefficient (α)	15 GPa^{-1}

Table 1. Calculation parameters (default values)

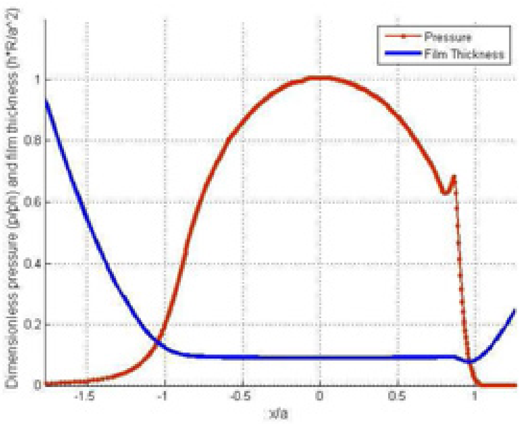

Figure 2. Film thickness and pressure distribution (dimensionless) along axis x.

For a quantitative comparison, we refer to Hamrock et al. [10] who predict central film thickness values as follows (these results were obtained with a curve fitting of finite-difference results in 1977):

$$h_c = 1.9149 R^{-1.67} \alpha^{0.53} \left(\frac{E}{1-\upsilon^2}\right)^{-0.073} L^{-0.067} \eta_0^{0.67} U_m^{0.67}$$

In order to discuss the thickness increase with the power 0.67 on the mean velocity, we plot in Figure 3 a set of value given by our Comsol model, in log/log scale. The linear curve fit in Figure 3 shows a slope of 0.686 which is very close to the results from the literature.

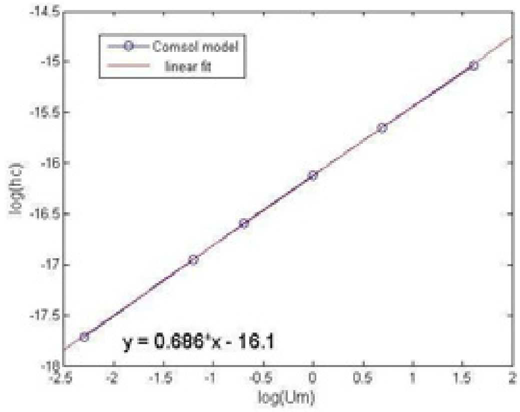

Figure 3. Parametric study of central film thickness (hc) value as a function of the mean velocity (Um), in log/log scale.

7. Conclusions

A new method is proposed in this paper in order to determine the lubricant film thickness between two solids in a lubricated contact. This parameter is of crucial importance to anticipate wear, fatigue, cracks, etc. Based on Comsol Multiphysics environment, the method benefits from the structural mechanics and PDE modes, as a non classical equation (Reynolds equation for lubrication) as to be solved on the boundary of the solids. The model is now able to easily include a wide range of physical effects like shear thinning of the lubricant or viscosity decrease due to friction heating.

8. References

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