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# The Full-System Approach for Elastohydrodynamic Lubrication

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## Outline

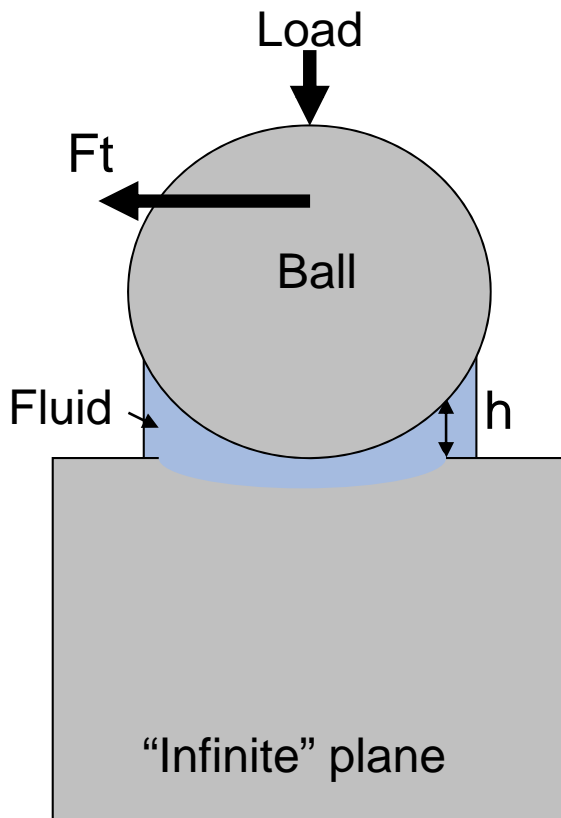
Problematic : Elastohydrodynamic lubrication

- Isothermal analysis & Newtonian lubricant  
*(a basic model)*
- Thermal analysis & non-Newtonian lubricant  
*(an advanced model)*

## Problematic

A lubricated ball-on-plane contact → EHL

- $S \sim 0.1 \text{ mm}^2$
- $\eta \uparrow \uparrow$  (piezo-viscous)
- $P \sim 10^9 \text{ Pa}$
- $u_3 \sim$  film thickness



Objectives :

- $h$  (Film thickness)  
→ Wear & lifetime prediction
- $F_t$  (Friction force)  
→ Energy loss prediction

## At least 2 Physics

# Isothermal Newtonian Model

complexity level:

## ■ Equivalent elasticity deformations

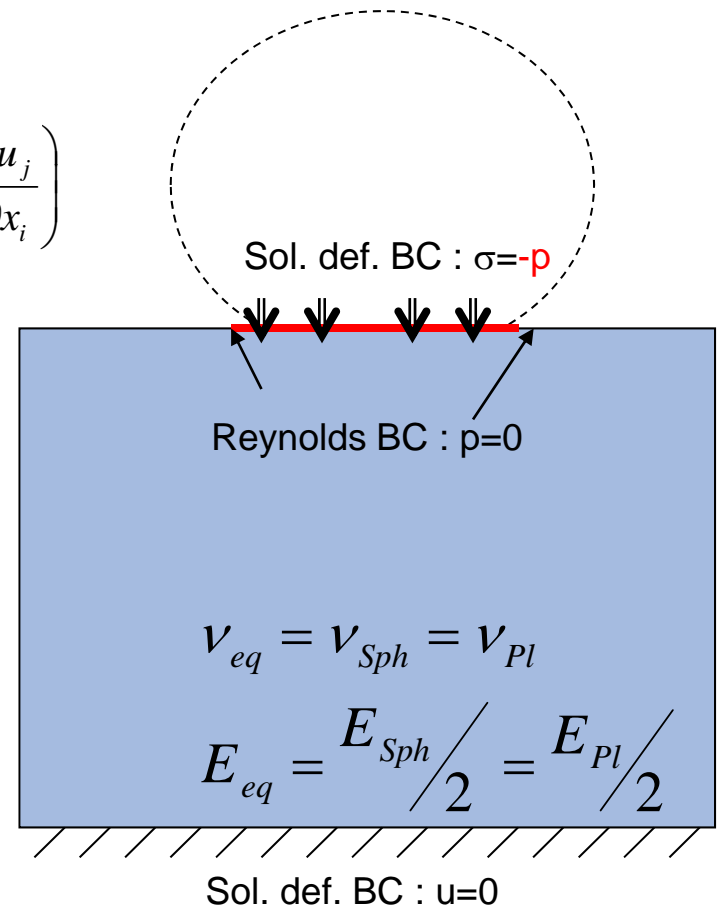
$$\sigma_{ij} = \frac{E_{eq}}{1+\nu_{eq}} \left( \varepsilon_{ij} + \frac{\nu_{eq}}{1-2\nu_{eq}} \varepsilon_{kk} \delta_{ij} \right) \quad \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

## ■ Navier Stokes → Reynolds

$$\nabla \left( \frac{\rho h^3}{12u_e \eta} \nabla p \right) = \frac{\partial(\rho h)}{\partial x}$$

with :  $\rho = \rho(p, \dots)$ ,  $\eta = \eta(p, \dots)$  and

$$h(x, y) = h_0 + \frac{x^2}{2R} + \frac{y^2}{2R} - u_3(x, y)$$



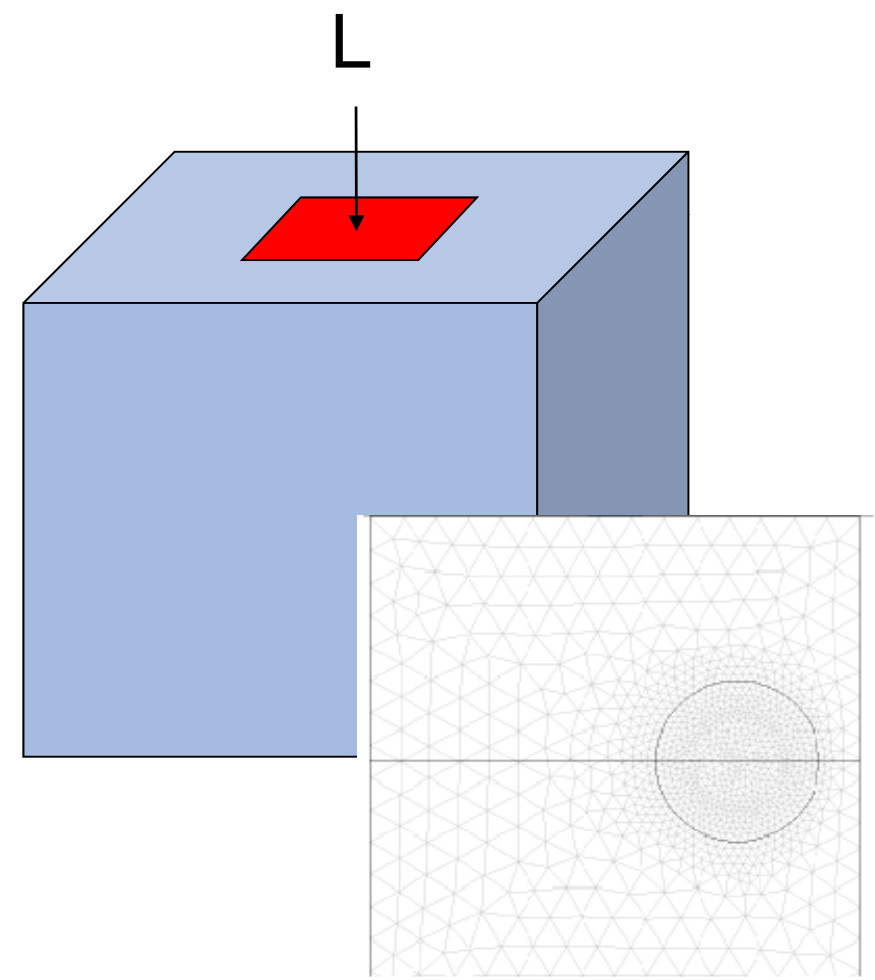
# Building the model

complexity level: complexity level:

- 1 geometry
- 2 physics
  - Solid, stress-strain module
  - PDE, Weak form on Boundary module
- 1 global equation  
+ 1 integration coupling variable



$$\iint_S p(x, y) dS = L$$



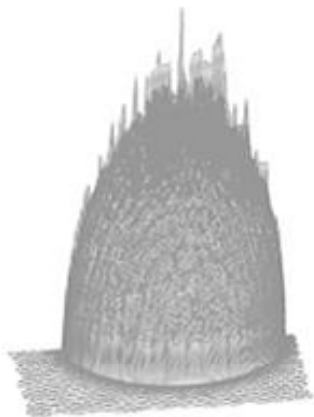
## Difficulties

complexity level: complexity level:

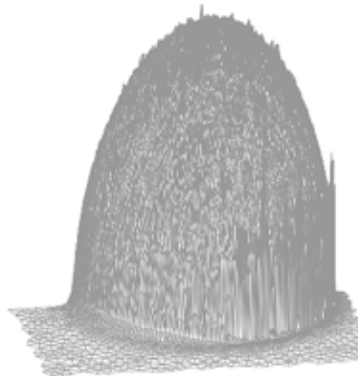
### ■ Stabilization techniques

$p \uparrow \rightarrow \eta \uparrow \uparrow \rightarrow$  Diffusion  $\ll$  Convection  $\rightarrow$  Galerkin fails  $\rightarrow$  SUPG, GLS and/or ID

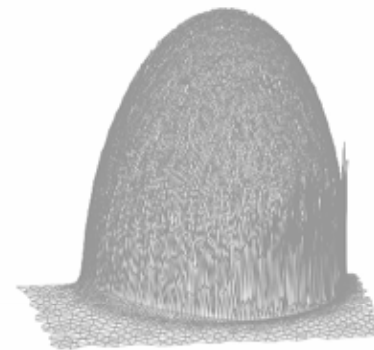
Standard Galerkin



GLS / SUPG



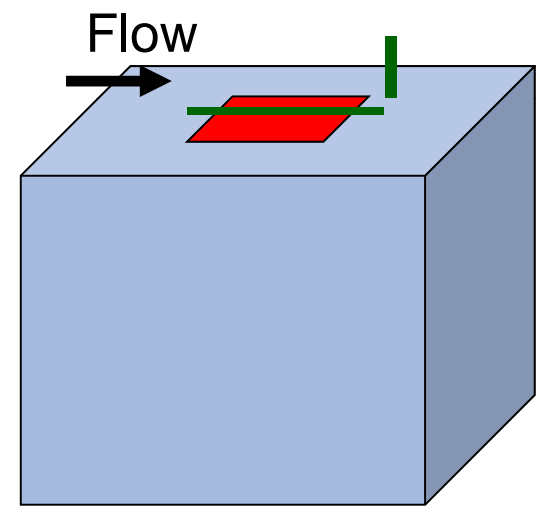
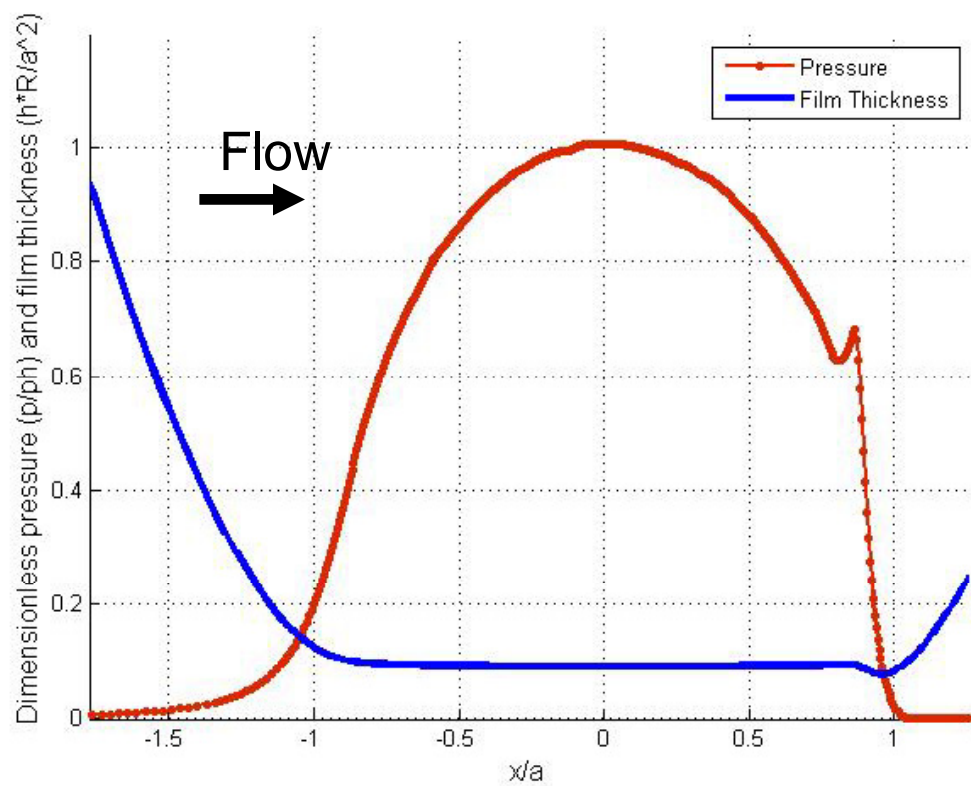
GLS / SUPG + ID



**See :** A.C. Galeão et al. App. num. math., vol. **48**, pp. 205 – 222, 2004

# Classical results

complexity level:



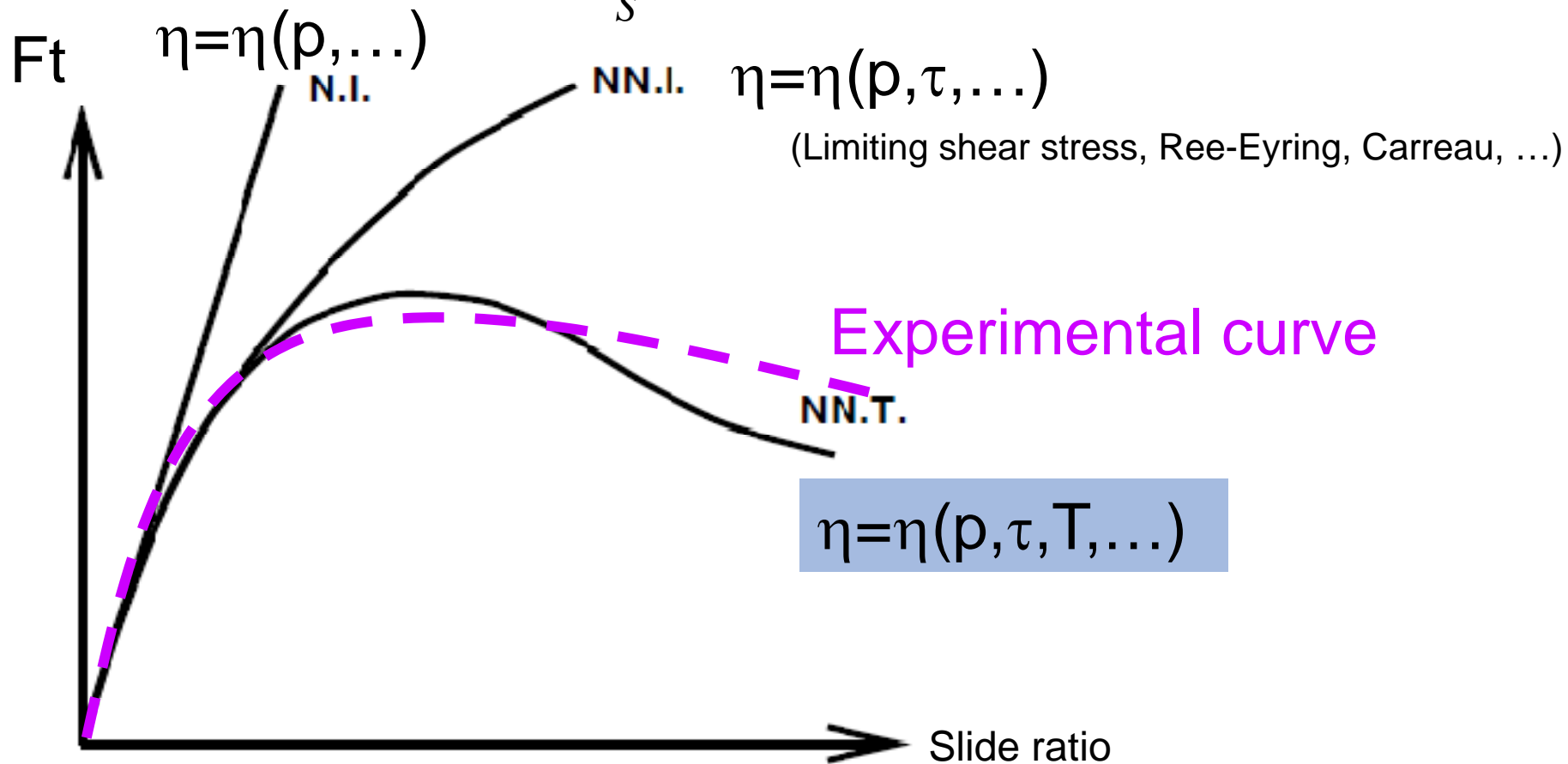
Qualitative validation

Pressure distribution in the fluid and film thickness distribution along the line

**Friction**

$$F_t = \int_S \tau_{xy} dS$$

complexity level:





# Thermal non-Newtonian Model

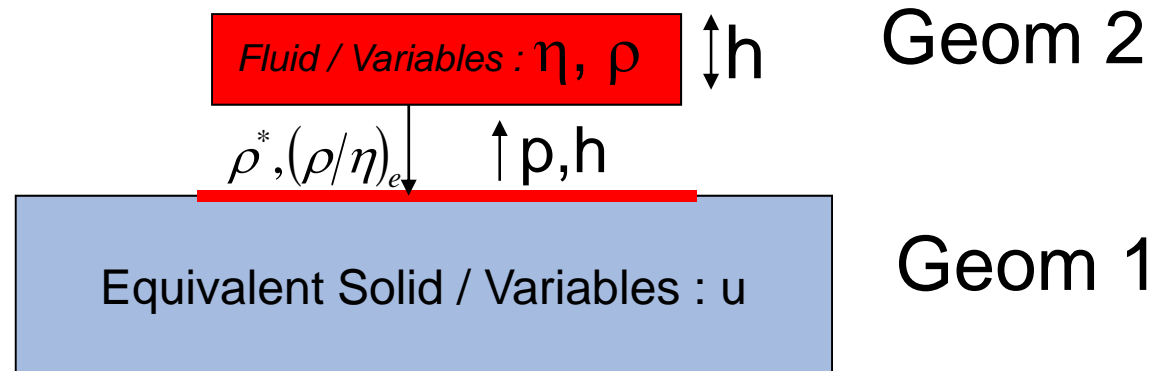
complexity level: 

- $\eta = \eta(\rho, \tau, T, \dots)$  &  $\rho = \rho(T, \dots)$
- A thermal generalized Reynolds equation

$$\frac{\partial}{\partial x} \left[ \left( \frac{\rho}{\eta} \right)_e h^3 \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \frac{\rho}{\eta} \right)_e h^3 \frac{\partial p}{\partial y} \right] = 12 \frac{\partial}{\partial x} (\rho^* U_m h)$$

$$\rho^* = \int_0^h f_1(\rho, \eta) dz$$

$$\left( \frac{\rho}{\eta} \right)_e = \int_0^h f_2(\rho, \eta) dz$$



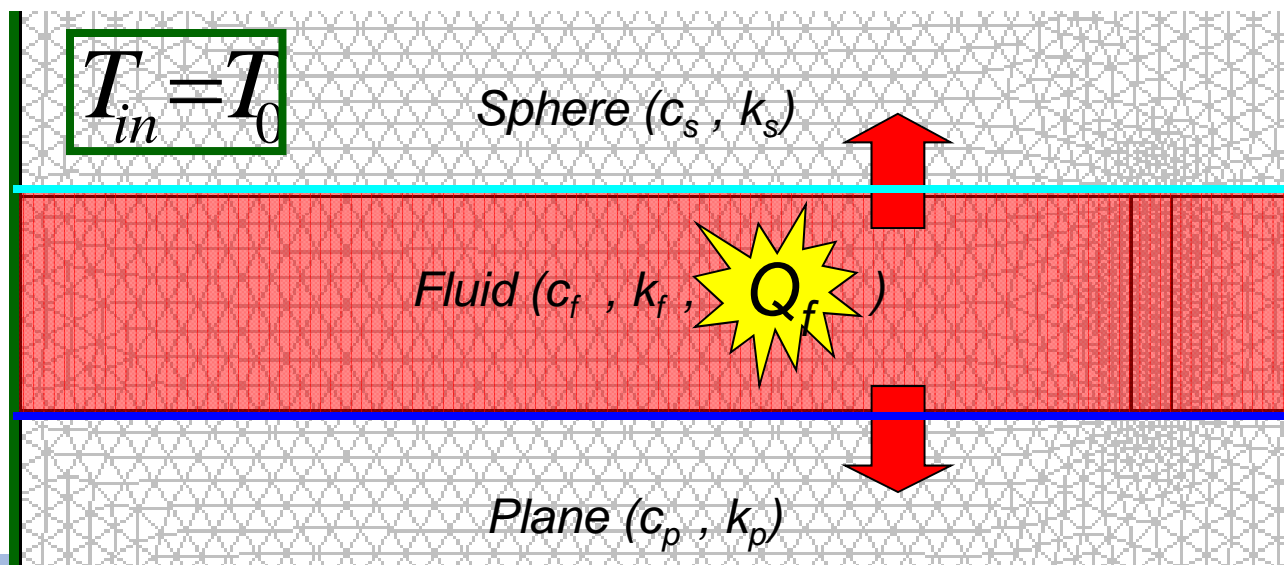
# Thermal non-Newtonian Model

complexity level:

- The 3 energy equations (convection/conduction)

$$-\nabla(k\nabla T) + \rho c \vec{U} \nabla T = Q \quad Q_{fluid} = Q_{compression} + Q_{shear}$$

- In Comsol: A new physic added to Geom. 2 :  
**Convection and Conduction Module**



$$k_f \frac{\partial T}{\partial z} \Big|_{z=h^-} = k_s \frac{\partial T}{\partial z} \Big|_{z=h^+}$$

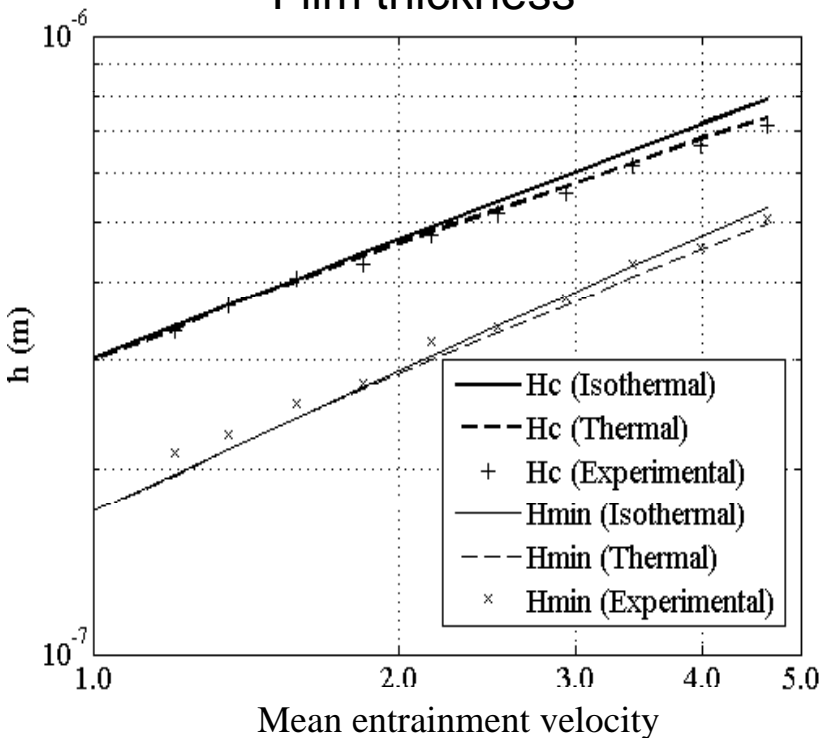
$$k_f \frac{\partial T}{\partial z} \Big|_{z=0^+} = k_p \frac{\partial T}{\partial z} \Big|_{z=0^-}$$

# Thermal non-Newtonian Model

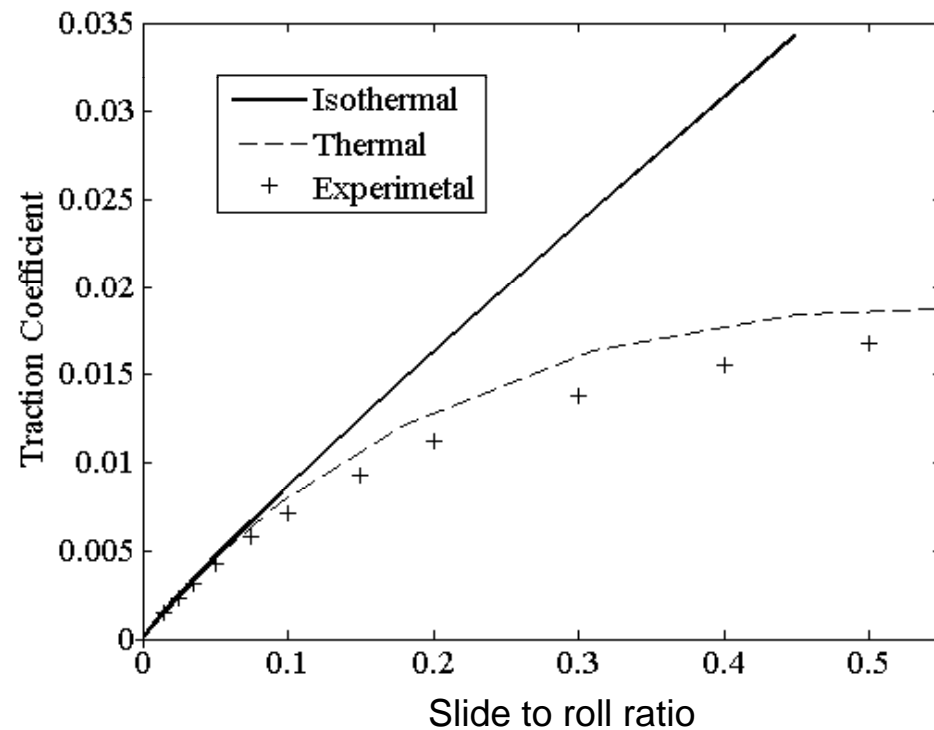
complexity level:

## Results

Film thickness



Friction



**See :** W. Habchi et al. Tribology Letters, vol. 30, n° 1, pp. 41 – 52, 2008

Thank you for your attention

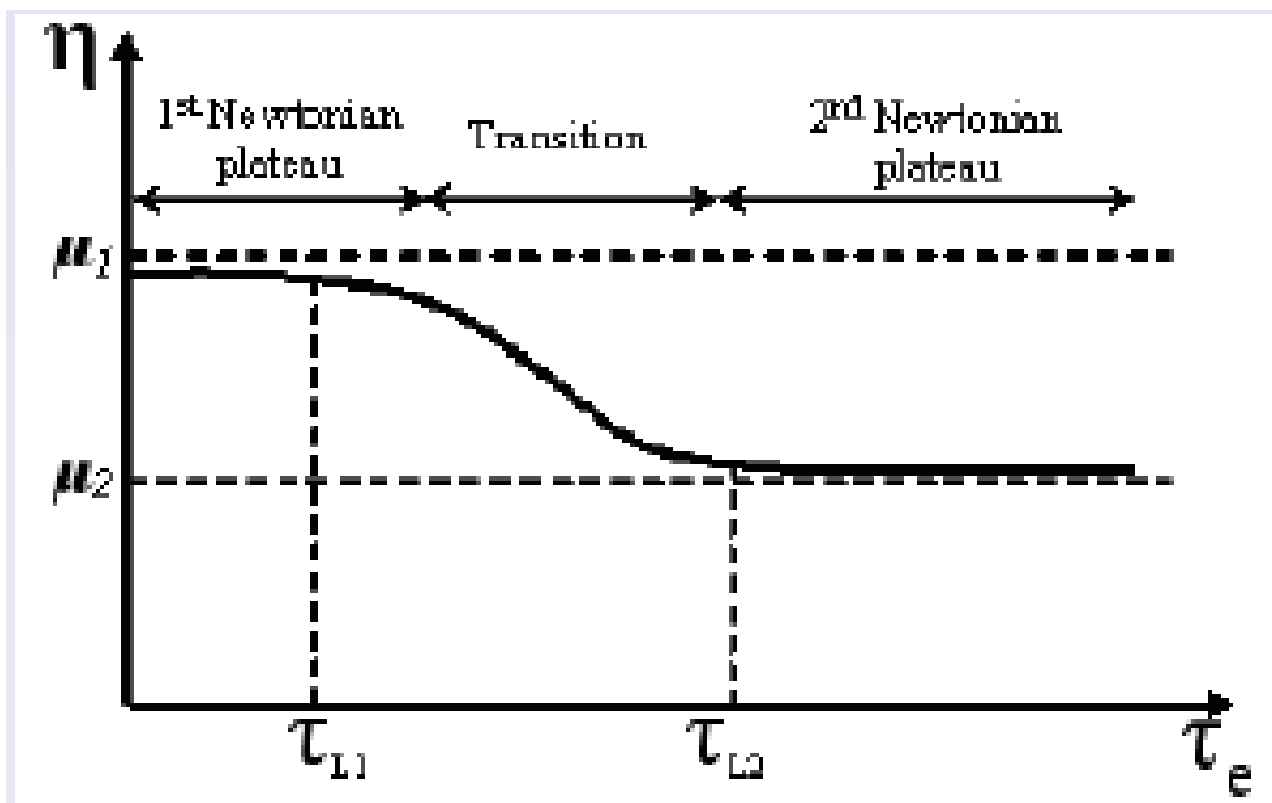
**Questions ?**

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*(our comsol model file will be on the proceeding cd)*



# Carreau law



# Weak Form

Boundary Settings - Weak Form, Boundary (reynolds)

Boundaries Groups

Group selection

- rest
- reynolds

Name: reynolds

New Delete

Weak Init Element Color

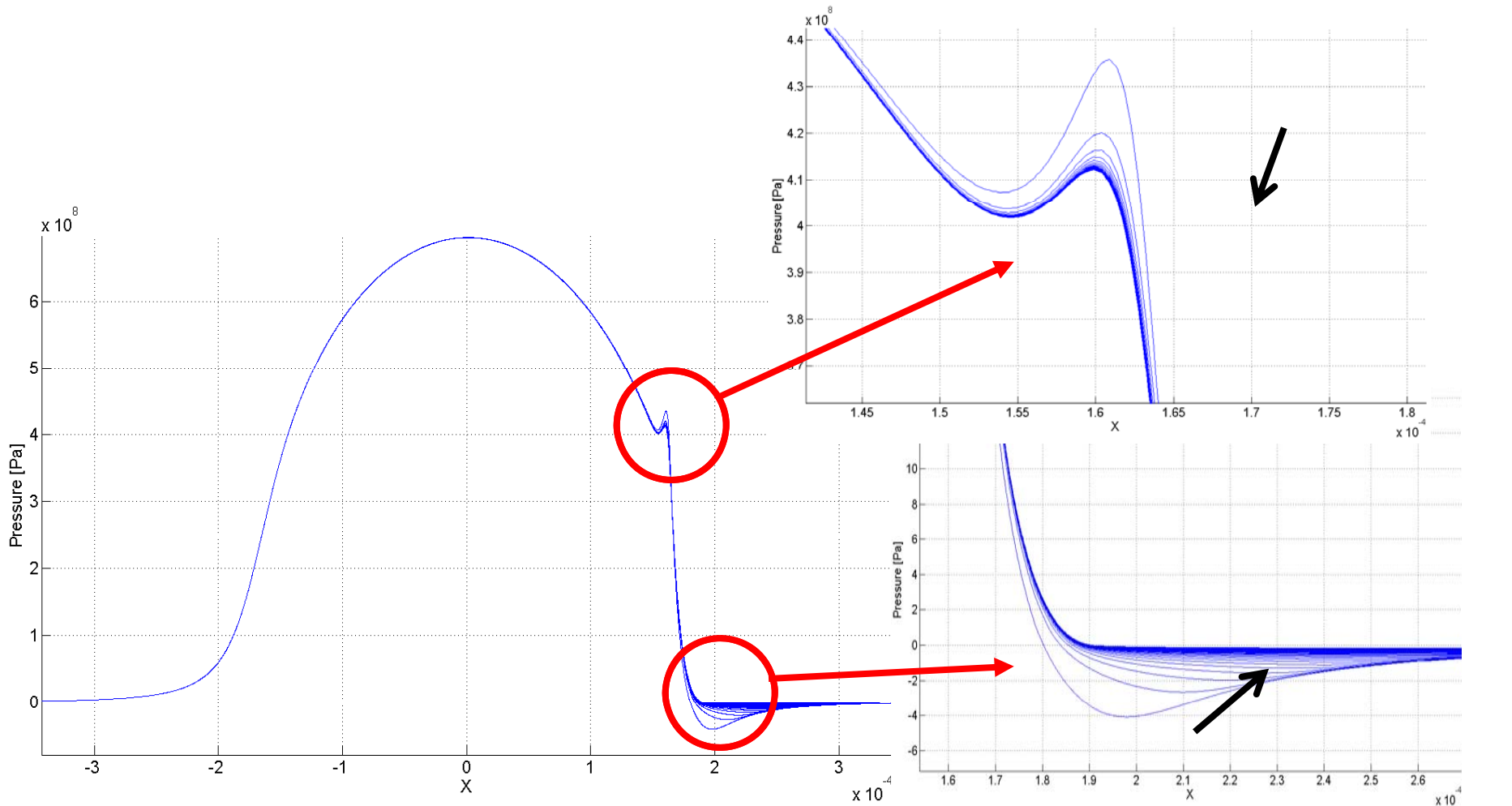
Weak terms	Weak term
weak	$\text{epsAdim} * (\text{pTx} * \text{test}(\text{pTx}) + \text{pTy} * \text{test}(\text{pTy})) - \nu \text{mx} * \rho \text{Adim} * \text{hAdim} * \text{test}(\text{pTx})$
dweak	$0$
constr	$+1 \text{e}6 * \text{h}^2 * \text{p} * (\text{p} < 0) * \text{test}(\text{p}) + \text{residual} * \text{tau} * (\text{betax} * \text{test}(\text{pTx}) + \text{betay} * \text{test}(\text{pTy}) - \text{epsAdim} * (\text{test}(\text{pTxx}) + \text{test}(\text{pTyy})))$
Constraint type:	Idea $+0.5 * \text{h} * \text{sqrt}(\text{betax}^2 + \text{betay}^2) * (\text{pTx} * \text{test}(\text{pTx}) + \text{pTy} * \text{test}(\text{pTy})) / \text{order}$
constrf	$0$ Constraint force

OK Cancel Apply Help



# Penalty term for cavitation

complexity level: 



**See :** W. Habchi et al. Tribology Letters, vol. 30, n° 1, pp. 41 – 52, 2008

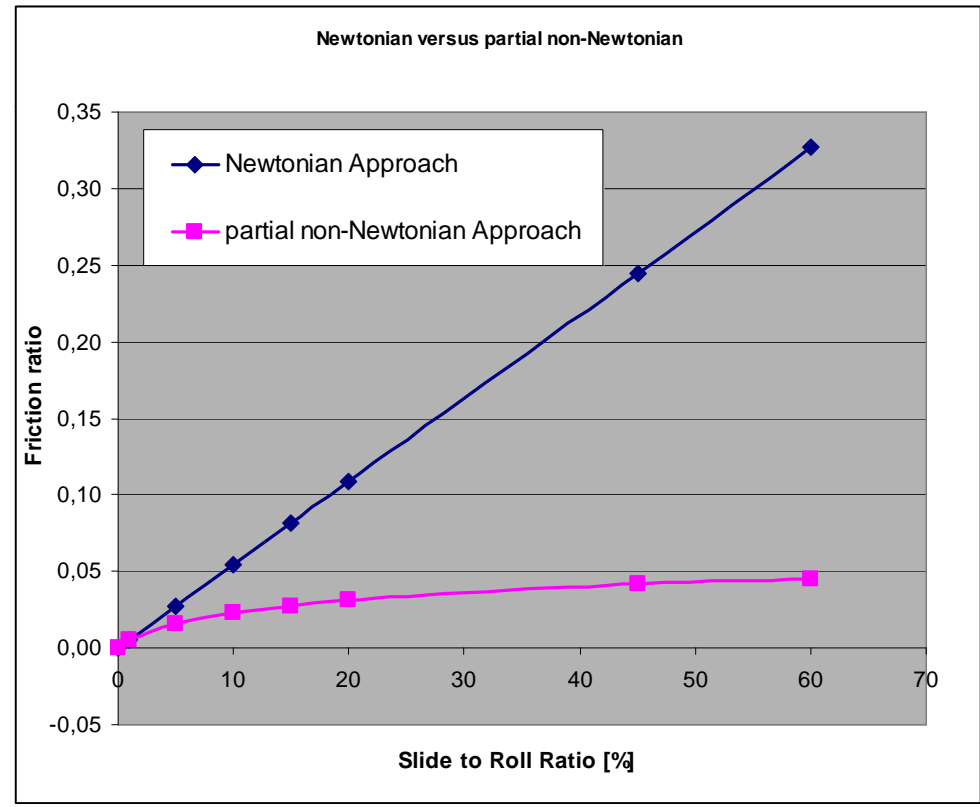
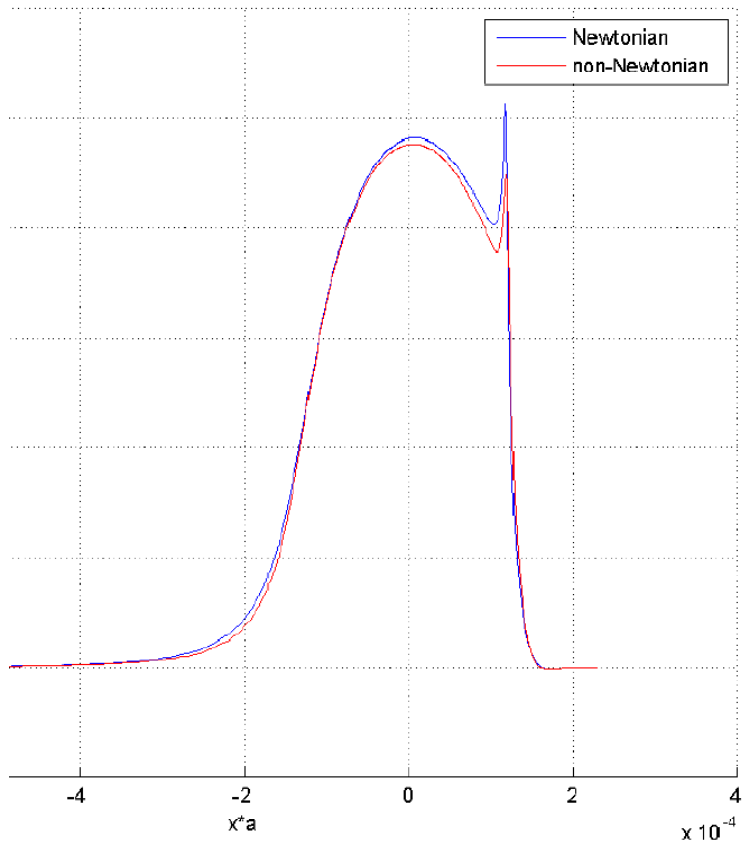


# Isothermal non-Newtonian Model

complexity level:

## More results

Pressure along the central line of the contact



**See :** B. Najji et al. ASME J. of Tribology, vol. 111, pp. 29 – 33, 1989