Dynamic Crack Propagation in Fiber Reinforced Composites

P. Lonetti, C. Caruso, A. Manna
DYNAMIC FRACTURE MECHANICS

MONOLITIC MATERIALS

- Crack Branching phenomena
- Crack speeds are limited
- Unknown path of the crack

COMPOSITE STRUCTURES

- High crack speed
- Crack constrained along the interfaces


DYNAMIC CRACK GROWTH MODELING

Cohesive modeling
- Interface elements are introduced at the crack region
- Damaged constitutive relationship is required

Fracture Mechanics approaches
- Static analyses:
  (the time dependence is neglected “a priori”)
- Steady state crack growth approaches:
  (Moving reference system with the tip, crack tip speed is constant)
- Unsteady models:
  Full Time dependence, inertial forces, loading rate,....
Node release technique

Gradual release of the nodal forces behind the crack tip

Virtual crack closure methods

The ERR is evaluated by the mutual work at the crack tip and behind the crack tip

Moving mesh methodology

The nodes are moved to predict changes of the geometry produced by the crack motion
AIM OF THE WORK

Propose a generalized modeling based on Fracture mechanics and moving mesh methodology to predict the dynamic behavior of composite laminated structures

SUMMARY

Review the main equations of the ALE formulation in view of the Dynamic Fracture Mechanics approach

Evaluate the specialized expressions of the ERR by the use of the decomposition methodology of the J-integral and propose a proper mixed mode crack toughness criterion

Develop the finite element implementation. Propose validation by means of comparisons with experimental data and a parametric study to analyze dynamic crack behavior (i.e. crack arrest phenomena, allowable tip speeds and rate dependence of the interfacial crack growth)
The Reference configuration is fixed and independent of any placement of the material body
**BASICS OF MOVING MESH STRATEGY: ARBITRARY-LAGRANGIAN EULERIAN FORMULATION**

**Physical quantities:**

\[ \dot{\mathbf{y}} = \frac{d}{dt} \varphi(\mathbf{X}, t), \quad \mathbf{X}' = \frac{d}{dt} \chi(\mathbf{r}, t) \]

"Material"  "Referential"

\[ \dot{f} = f' - \mathbf{X}' \frac{d}{d\mathbf{X}} f(\mathbf{X}, t) \]

Time derivative rule

**Physical fields in ALE formulation**

\[ \ddot{\mathbf{u}} = \mathbf{u}'' - 2 \nabla_x \mathbf{u}' \cdot \mathbf{X}' - \nabla_x \mathbf{u} \mathbf{X}'' + \nabla_x \left( \nabla_x \mathbf{u} \right) \mathbf{X}' \mathbf{X}' + \nabla_x \mathbf{u} \nabla_x \mathbf{X}' \mathbf{X}' \]

"Material accel."

\[ \nabla_{\mathbf{X}} \mathbf{u} = \nabla_{\hat{x}} \mathbf{u} \mathbf{J}^{-1} \]

"Grad. transform."

\[ \det \mathbf{J} \neq 0 \]

"one-to-one relationship"
Multi-layer Modeling

2D Kinematic formulation

The laminate is divided into \( n \) mathematical layer representing the stacking sequence.

Compatibility equations LMM:

\[
\Delta u_i = u_{i+1} - u_i = 0, \quad \Delta v_i = v_{i+1} - v_i = 0,
\]

“undelaminated interfaces”

\[
\Delta v_i = v_{i+1} - v_i \geq 0,
\]

“delaminated interfaces”
Governing Equations: “Principle of d’Alembert”

\[ \sum_{i=1}^{n} \int_{V_i} \sigma \delta \nabla u \, dV + \sum_{i=1}^{n} \int_{V_i} \rho \bar{\varepsilon} \delta \bar{u} \, dV = \sum_{i=1}^{n} \int_{\Omega_i} t \delta u \, dA + \sum_{i=1}^{n} \int_{V_i} \bar{f} \delta u \, dV \]

Internal work

External work

\[ \sum_{i=1}^{n} \int_{V_i} \sigma \delta \nabla u \, dV = \sum_{i=1}^{n} \int_{V_i} \mathcal{C} \left( \nabla_r u J^{-1} \right) \delta \left( \nabla_r u J^{-1} \right) \det(J) \, dV_r \quad : \text{Jacobian} \]

\[ \sum_{i=1}^{n} \int_{V_i} \rho \bar{\varepsilon} \delta \bar{u} \, dV = \sum_{i=1}^{n} \int_{V_i} \rho \left[ u'' - 2 \nabla_r u' \cdot J^{-1} \cdot X' - \left( \nabla_r u' J^{-1} \right) \cdot X'' + \nabla_r \left( \nabla_r u' J^{-1} \right) J^{-1} X' X' + \nabla_r u' J^{-1} \cdot \left( \nabla_r X' J^{-1} \right) X' \right] \delta \bar{u} \det(J) \, dV_r \]

\[ \sum_{i=1}^{n} \int_{\Omega_i} \bar{t} \delta u \, dA + \sum_{i=1}^{n} \int_{V_i} \bar{f} \delta u \, dV = \sum_{i=1}^{n} \int_{\Omega_i} t \delta u \det(J) \, d\Omega_r + \sum_{i=1}^{n} \int_{V_i} \bar{f} \delta u \det(J) \, dV_r \]
ERR RATE EVALUATION: J-INTEGRAL APPROACH


INDEX:

INTRODUCTION

MOTIVATIONS

ALE MODEL

FORMULATION

FE MODEL

RESULTS

CONCLUSIONS

Expressions of the ERR

\[ J = \lim_{\varepsilon \to 0} \left[ \int_{\Omega} \left( (W + K) n_1 - t \frac{\partial u}{\partial X} \right) ds \right] \]

\[ J = \left[ \int_{\partial \Omega} \left( (W + K) n_1 - t \frac{\partial u}{\partial X} \right) ds + \int_{\Omega} \left[ \rho \left( \ddot{u} - f \right) \nabla u - \rho \ddot{u} \nabla u \right] dA \right] \]

“Path independent”

(Nishioka, T, 2001)

Decomposition of the ERR into symmetric and antisymmetric fields

\[ J_I = G_I = \left[ \int_{\partial \Omega} \left( (W^S + K^S) n_1 - \sigma^S_{ij} n_j \frac{\partial u^S}{\partial X} \right) ds + \int_{\Omega} \left[ \rho \left( \ddot{u}^S - f^S \right) \nabla u^S - \rho \ddot{u}^S \nabla u^S \right] dA, \]

\[ J_{II} = G_{II} = \left[ \int_{\partial \Omega} \left( (W^{AS} + K^{AS}) n_1 - \sigma^{AS}_{ij} n_j \frac{\partial u^{AS}}{\partial X} \right) ds + \int_{\Omega} \left[ \rho \left( \ddot{u}^{AS} - f^{AS} \right) \nabla u^{AS} - \rho \ddot{u}^{AS} \nabla u^{AS} \right] dA, \]
DYNAMIC CRACK PROPAGATION ANALYSIS: GROWTH CRITERION

Crack growth criterion

\[ G_D = \frac{G_0}{1 - \left( \frac{c_t}{V_R} \right)^m} \]

Material parameter

“Critical value of the ERR”
(Freund, 1990; Ravi-Chandar, 2004)

\[ c \rightarrow V_R \quad G_D(c_t) = \infty \]

“Rayleigh wave speed”

\[ c \rightarrow 0 \quad G_D(c_t) = G_0(0) \]

“initiation value”

1) Mixed mode crack growth criterion

\[ g_f = \frac{G_I}{G_{ID}(c_t)} + \frac{G_{II}}{G_{IID}(c_t)} - 1 \leq 0 \]

Material parameter

\[ G_{ID}(c_t) = \frac{G_{0I}}{1 - \left( \frac{c_t}{V_R} \right)^m} \]
\[ G_{IID}(c_t) = \frac{G_{0II}}{1 - \left( \frac{c_t}{V_R} \right)^m} \]
ALE formulation to describe mesh motion

\[ \nabla^2 X_1 = 0, \quad \nabla^2 X_2 = 0. \]

\[ \Delta X_1 = X_1 - r_1 \quad \Delta X_2 = X_2 - r_2 \]

"Mesh displacements of nodes
Should be regular"

Boundary conditions

\( (\Delta X_1 = 0, \Delta X_2 = 0) \) on \( \Omega_1 \cup \Omega_2 \),
\( \Delta X_2 = 0 \) on \( \Omega_3 \cup \Omega_4 \),
\[ \Delta X'_1 = 0 \iff g_f < 0 \text{ on } \Omega, \]
\[ \Delta X'_1 = c_i \iff g_f \geq 0 \text{ on } \Omega, \]
\( \Delta X'_2 = 0 \) on \( \Omega \),
\( \Delta X_1(0) = 0, \Delta X_2(0) = 0, \Delta \dot{X}_1(0) = 0, \Delta \dot{X}_2(0) = 0 \)

Mesh regularization technique

"Winslow Smoothing method"

Minimize the mesh warping

Example: DCB scheme
VARIATIONAL FORMULATION AND FE IMPLEMENTATION

Weak forms: coupled equations for the ALE and PS formulations:

\[ \sum_{i=1}^{n} \int_{V_i} C \left( \nabla_x u \right) \delta \left( \nabla_x u \right) \det(J) \, dV_r + \sum_{i=1}^{n} \int_{V_i} \rho \left[ u'' - 2 \nabla_x u' \cdot J^{-1} \cdot X' - \left( \nabla_x u \cdot J^{-1} \right) \cdot X'' \right] \delta u \, dV_r \]

\[ + \nabla_x \left( \nabla_x u \cdot J^{-1} \right) \cdot J^{-1} \cdot X' \cdot X' + \nabla_x u \cdot J^{-1} \cdot \left( \nabla_x X' \cdot J^{-1} \right) \cdot X' \delta u \, dV_r \]

\[ = \sum_{i=1}^{n} \int_{\Omega_i} t \delta u \, d\Omega_r + \sum_{i=1}^{n} \int_{V_i} f \delta u \, dV_r \]

\[ \int_{V_r} \left( \nabla_x \Delta XJ^{-1} \right) \cdot \left( \nabla_x wJ^{-1} \right) \det(J) \, dV_r + \int_{\Omega_r} \left[ \delta \lambda \left( X' - \zeta_0 \right) \dot{X} + \lambda \delta X \ddot{X} \right] J^{-1} \, ds = 0, \]

Explicit equations for PS+ALE
Implicit equation the crack growth

Crack growth criterion
FE approximation by “Comsol Multiphysics”:

- Quadratic Lagrangian interpolation functions for displacements, velocity and acceleration fields
- Quadratic Lagrangian interpolation functions for mesh points displacements

**FE equations**

\[ \sum_{i=1}^{n} M_i \dddot{\tilde{U}}_i + \sum_{i=1}^{n} C_i \ddot{\tilde{U}}_i + \sum_{i=1}^{n} (K_i + K_{0i} + K_{1i} + K_{2i}) \tilde{U}_i + \sum_{i=1}^{n} T_i + \sum_{i=1}^{n} P_i = 0 \]

\[ \widetilde{W} \cdot \Delta \dddot{X} + \widetilde{Q} \cdot \Delta \ddot{X} + L = 0, \]

**Solution Procedure**

- Implicit time integration scheme based on variable-step-size backward differentiation formula

**Non Linear Equations System**

**CHECK**

**MESH ELEMENTS QUALITY**

**Iterative-incremental Solving procedure**
RESULTS: VALIDATION OF THE STRUCTURAL MODEL

INDEX:
- INTRODUCTION
- MOTIVATIONS
- ALE MODEL
- FORMULATION
- FE MODEL
- RESULTS
- CONCLUSIONS

DCB mode I loading scheme

Comparisons with experimental data

AS 3501-6 Graphite/Epoxy

Experimental data

Proposed model

$\frac{a/L}{0.367, h/B=0.1, m=0.5, G_0/G_{st}=0.3, \alpha=0.1 \text{mm/s}}$

Comparisons with experimental data

DCB mode I loading scheme

AS 3501-6 Graphite/Epoxy

$\frac{E_d E_c}{(G_0BL)}$

$\frac{c_t/c_{sh}}{t \alpha/L}$

$\frac{0.1/c_{sh}}{0.1}$

$\frac{0.01/c_t}{0.01}$

0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0

0.0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6

0.0 0.3 0.6 0.9 1.2 1.5 0.0 0.3 0.6 0.9 1.2 1.5

0.1 0.2 0.3 0.4

a/L=0.367, h/B=0.1, m=0.5, G_0/G_{st}=0.3, \alpha=0.1 \text{mm/s}
RESULTS: EFFECT OF THE LOADING RATE

- DCB mode I loading scheme
- Influence of the loading rate
- Evolution of the crack tip speed

**INDEX:**
- INTRODUCTION
- MOTIVATIONS
- ALE MODEL
- FORMULATION
- FE MODEL
- RESULTS
- CONCLUSIONS

**DCB mode I loading scheme**

**Influence of the loading rate**

**Evolution of the crack tip speed**
DEFORMED SHAPE OF THE BEAM UNDER MODE I LOADING CONDITIONS

Horizontal displacement of the crack-tip front

t = 0.065 s

t = 0.12 s

t = 0.18 s

x direction

Department of Structural Engineering - University of Calabria -
DEFORMED SHAPES OF THE BEAM UNDER MIXED MODE LOADING CONDITIONS

TRIANGULAR MESH ELEMENTS

CRACK - TIP
RESULTS : MODE II ENF SCHEME

- **ENF mode II loading scheme**
- **Comparisons with experimental data**
- **S2/8553 Glass/Epoxy**
RESULTS: MODE II ENF SCHEME

INTRODUCTION

High amplifications in the ERR prediction

INDEX:

- INTRODUCTION
- MOTIVATIONS
- ALE MODEL
- FORMULATION
- FE MODEL
- RESULTS
- CONCLUSIONS

Formulation

FE Model

Results


\[
\frac{a}{L} = 0.25, \quad \frac{h}{B} = 0.213, \quad m = 1, \quad \frac{G_0}{G_{st}} = 0.52,
\]

\[
t_0 / T_i = 0.1, \quad t_0 / T_i = 0.4, \quad t_0 / T_i = 0.7, \quad t_0 / T_i = 1
\]

\[
t_0 / T_i = 0.5, \quad t_0 / T_i = 2
\]
RESULTS: MIXED MODE ANALYSIS

INDEX:
INTRODUCTION
MOTIVATIONS
ALE MODEL
FORMULATION
FE MODEL
RESULTS
CONCLUSIONS

Mesh tip discretization

AS 3501-6 Graphite/Epoxy

\[ \frac{\Delta X(a)}{L} \]

\[ \begin{align*}
\frac{t}{\alpha/L} & = 0.0, 5.0 \times 10^{-9}, 1.0 \times 10^{-8}, 1.5 \times 10^{-8}, 2.0 \times 10^{-8}, 2.5 \times 10^{-8} \\
\frac{\Delta X(a)}{L} & = 0.00, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40
\end{align*} \]

- Proposed model
- Experimental data

\[ a/L = 0.220, H/B = 0.485, \ h_1/h_2 = 0.66, \ m = 0.5, \ G_0/G_u = 0.526, \ \alpha = 0.025 \text{ mm/s} \]
RESULTS : MIXED MODE ANALYSIS

INDEX:
- INTRODUCTION
- MOTIVATIONS
- ALE MODEL
- FORMULATION
- FE MODEL
- RESULTS
- CONCLUSIONS

Crack arrest phenomenon

Loading curves

\[ u/u_{st} = \{1.2; 1.42; 1.74; 2.36; 3.98; 5.08; 7.12\} \]
\[ \text{Eq. (22), } g_f = \{1.5; 3.0; 5.0; 10; 30; 50; 100\} \]
RESULTS: MIXED MODE ANALYSIS

INDEX:
- INTRODUCTION
- MOTIVATIONS
- ALE MODEL
- FORMULATION
- FE MODEL
- RESULTS
- CONCLUSIONS

Time incrementation

Department of Structural Engineering - University of Calabria -
CONCLUDING REMARKS

- A delamination model for general loading conditions based on moving mesh methodology and fracture mechanics is proposed.

- New expressions of the ERR mode components based on the J-integral decomposition procedure.

- Comparisons with experimental data are proposed to validate the delamination modelling.

- The analyzed parametric study shows that delamination phenomena are quite influenced by the loading rate, inertial effects leading to high amplifications in the ERR prediction and the crack growth.