Magnetic Control of Deformation of a Ferrofluid Droplet in Simple Shear Flow

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INTRODUCTION: A detailed investigation of the effect of uniform magnetic field on the deformation of the ferrofluid droplet in a two dimensional (2D) simple shear flow by means of numerical simulation is presented here. In this case, the magnetic field is applied perpendicular to the flow field domain.

COMPUTATIONAL METHODS: The conservative level set method is used to track the dynamic interface of the droplet where the level set function is advected by the velocity field:

\[
\frac{d\phi}{dt} + u \cdot \nabla \phi = \gamma \nabla \cdot \left( \epsilon \nabla \phi - \phi (1 - \phi) \frac{\nabla \phi}{|\nabla \phi|} \right)
\]

Being treated as a single phase flow, the different properties of the flow domain are related to \( \phi \) through the following equations:

\[
\begin{align*}
\rho &= \rho_c + (\rho_d - \rho_c)\phi; \quad \eta = \eta_c + (\eta_d - \eta_c)\phi \\
\mu &= \mu_c + (\mu_d - \mu_c)\phi; \quad \chi = \chi_c + (\chi_d - \chi_c)\phi
\end{align*}
\]

The flow field under the effect of uniform magnetic field can be governed by the continuity and momentum equations:

\[
\nabla \cdot u = 0
\]

\[
\rho \left( \frac{\partial u}{\partial t} + u \cdot \nabla u \right) = -\nabla p + \nabla \cdot \mathbf{T} + F_\sigma + F_m
\]

where, the surface tension force, \( F_\sigma \) can be defined as:

\[
F_\sigma = \nabla \cdot [\sigma (\mathbf{I} + (-\mathbf{n}\mathbf{n}^T))\delta]
\]

and magnetic force, \( F_m \) can be calculated as:

\[
F_m = \nabla \cdot \mathbf{T}_m = \nabla \cdot (\mu \mathbf{H} \mathbf{H}^T - \frac{\mu}{2} \mathbf{H}^2 \mathbf{I})
\]

The magneto-static Maxwell equation can be written as:

\[
\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = 0, \quad \nabla \cdot (\mu \nabla \phi) = 0
\]

\[
\mathbf{M} = \chi \mathbf{H} \quad \text{and} \quad \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi) \mathbf{H}
\]

RESULTS: The effect of different shear flow rates and magnetic field strengths on the deformation of the ferrofluid droplet deformation

CONCLUSIONS: At a low shear rate (\( \text{Ca} \leq 0.02 \)), the deformation of the ferrofluid droplet is controlled by the magnetic field effect due to its dominant nature over shear flow effect while for the high shear rate (\( \text{Ca} \geq 0.25 \)), the deformation of the ferrofluid droplet is predominantly determined by the shear flow although the magnetic field has a considerable effect at higher strengths.

REFERENCES:
1. COMSOL, “CFD Module Application Library Manual.”

Figure 1. Schematic illustration of a ferrofluid droplet suspended in another viscous fluid in a simple shear flow under a uniform magnetic field, \( \mathbf{H}_0 \).

Figure 2. Effect of a perpendicular magnetic field on the deformation, \( D \) of the ferrofluid droplet against time.

Figure 3. Outline of the droplet shape at steady state.

Figure 4. Velocity and magnetic field for \( \text{Ca} = 0.0194, \text{Re} = 0.0015 \) at \( \alpha = 90^\circ \).
(a) \( B_\text{m} = 1.4549 \), (b) \( B_\text{m} = 2.8515 \), (c) \( B_\text{m} = 4.7138 \).

Figure 5. Velocity and magnetic field for \( \text{Ca} = 0.2333, \text{Re} = 0.018 \) at \( \alpha = 90^\circ \).
(a) \( B_\text{m} = 1.4549 \), (b) \( B_\text{m} = 2.8515 \), (c) \( B_\text{m} = 4.7138 \).

Dimensionless Groups:
- \( \text{Re} = \frac{\rho u \Delta y}{\eta_c} \)
- \( \text{Ca} = \frac{\eta_c B_0 \Delta y}{\sigma} \)
- \( B_\text{m} = \frac{R_0 \mu_0 B_0^2}{2 \sigma} \)