Numerical Modelling of Viscous Damping for Acoustic Resonances of Suspended Microparticles

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Why acoustics and particles?

The particle motion can be controlled by acoustic forces.

1D focussing of glass particles  

focussing of alumina discs  

2D focussing  

rotation of particle clumps  
Motivation: At high frequencies particle resonances can be excited.

The acoustic radiation force acting on a PMMA particle. Multiple resonances can be observed.

Motivation: At high frequencies particle resonances can be excited

Fig. The acoustic radiation force acting on a PMMA particle. Multiple resonances can be observed.

Viscosity is neglected!

Former results were obtained without considering the viscous damping!

**Fig. 4.** $Y_p$ vs $ka$ relation for stainless steel spheres in water. Solid curve: calculated by Eq. 29. Broken curve: calculated for rigid stainless steel spheres. $O$: measured.
Former results were obtained without considering the viscous damping!

Influence of viscosity on the force amplitude is neglected! Unfortunately, including the damping is computationally expensive...
Project goals

Understanding the influence of the viscosity on:

- The stored energy,
- the acoustic radiation force,

for an acoustically excited particle close to resonance.

Three different FEM (Comsol) models are used:

- **Inviscid Model**: Radiation losses, computationally cheap
- **Viscous Model**: Radiation & viscous losses, computationally expensive
- **Loss Factor Model**: Radiation & viscous losses, computationally cheap
Setup: Particle placed in standing wave

- 10 μm radius polystyrene particle
- One-dimensional standing wave.
- Placed between pressure and velocity node.
FEM Setup

- Simple symmetrical setup
- Viscous boundary layer requires many mesh elements
Particle vibrations for the inviscid fluid
Particle vibrations for the inviscid fluid

Particle in Standing Wave

Stored Energy [J]

Frequency [MHz]

Acoustic Radiation Force [N]

Frequency [MHz]
Multiple resonances can be observed
Viscosity adds damping and mass to the vibration mode.
Mode Shapes for the inviscid case

Displacement amplitude

Pressure amplitude

Particle in Standing Wave

Frequency [MHz]
Viscous boundary layer

Velocity amplitude

Displacement amplitude

pressure
The viscous boundary layer (BL)

Adds **mass** and **damping** to the vibration.

The added **mass** and **damping** can be approximated analytically if:

- BL thickness smaller than wavelength, $\delta << \lambda$.
- BL thickness smaller than body curvature, $\delta << r$.

Fig. The tangential velocity profile in the viscous boundary layer for a moving wall and resting external fluid.
The added mass and loss factor

**Added Mass:**

The particle density $\rho_p$ is changed to

$$\rho^* = \rho_p (1 + \phi_\rho)$$

with

$$\phi_\rho = \frac{\delta \rho_f}{4E_{\text{kin}}} \int_S <v_{\text{Diff}}^2> dS.$$

**Loss factor [1]:**

The damping is included in the complex youngs modulus $E^*$:

$$E^* = E_p (1 + i \phi_v)$$

with

$$\phi_v = \frac{\delta \rho_f}{4E_{\text{strain}}} \int_S <v_{\text{Diff}}^2> dS.$$

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**Algorithm**

1. Inviscid simulation.
2. Acquire $v_{\text{Diff}}^2$, $T_{\text{kin}}$ and $E_{\text{str}}$.
3. Calculate $\phi_\rho$ and $\phi_v$.
3. Simulate again with **added mass** and **loss factor**.

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The results for the approximate model
The results for the approximate model

Particle in Standing Wave

The added mass and loss factor

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Outlook

- Investigate thermal losses.
- Calculate the acoustic radiation force between multiple particles.
- Incorporate acoustic streaming.
Thank you!
Scattered power
Motivation: Calculating the particle trajectories at high frequencies

One-cell-per-(acoustic)-well
