Numerical Study of Membrane Polarization for a Network of Connected Pores

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Introduction: the chargeability of earth materials is essentially an electrochemical effect affected by many factors such as the grain size, the types of the present minerals, the type and mobility of ions within the pore fluids, etc. If ground is chargeable, it responds as if resistivity was a complex quantity. In other words, a phase shift occurs as ground responds to an oscillating input current.

One type of polarization called membrane polarization occurs when pore space narrows to within several boundary layer thicknesses, where upon application of an electrical potential, a ion-selective membrane is created causing surplus and deficiency of both cations and anions at each ends of pore throat. Upon removal of



electrical potential, discharging is measured in time or frequency domain (Figure 1).

System of Equations: the governing equations for charge transport in pore space according to Marshall and Madden (1959) are the continuity equations for the concentrations of cations and anions and Poisson's equation for the potential U (Equations 1-3). These partial differential equations (PDEs) are coupled and solved using equation-based modeling in COMSOL Multiphysics[®] to numerically model more realistic pore structures.

Results: in the following, the spectral behavior of different pore structures are modeled and compared.

Pore Sequence Model: comparing the phase spectra of three different pore sequences, which shows a shift of characteristic frequencies to higher values whereas the phase magnitudes have decreased with increasing the number of pores (Figure 2). Pore Network Model: comparing the phase spectra of a simple cylindrical model with those of differnet networks, which shows a reduction in the phase maxima by mol/m^3 increasing the veriety of the pores (Figure 3). ×10⁻⁸

$$i\omega c_p = D_p \Delta c_p + \nabla \left[\mu_p c_{p0} \nabla u \right] \quad (1)$$

$$i\omega c_n = D_n \Delta c_n + \nabla [\mu_n c_{n0} \nabla u] \quad (2)$$

$$\Delta u = \frac{F}{\varepsilon} (c_n - c_p) \tag{3}$$



Figure 1. Membrane polarization effect.

 c_p, c_n : cation and anion concentrations, D_p, D_n : diffusion coefficients, μ_p, μ_n : ion mobilities, u: electrical potential, F: Faraday's constant, ε : permittivity.



Conclusions: COMSOL[®] has made it possible to numerically model the membrane polarization effect at the pore scale in frequency domain. The influence of the pore sizes and structures on the spectral behavior of sequence or network of pores has been studied. **Figure 2**. Excess ion concentraion mol/m^3 for different pore sequences.



References:

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