Mechanical Damage Models for Concrete
From Classical Mazars’ model to fully integrated multiaxial regularized methods

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Outline

- Introduction
- Mazars’ damage model
- External material vs built-in implementation
- Regularization
- $\mu$ damage model
- Conclusions
- Application case
Introduction

• Concrete structures
  – Damage mechanics
  – Quasi-brittle behaviour
  – Cracking representation
  – Various loading conditions
  – Load bearing capacity and post-peak behaviour
  – Accurate representation through a simple isotropic model

Final goal:
Develop concrete mechanics model in the Comsol interface that allows its coupling with other processes, such as chemical degradation for durability assessment, or moisture and heat transport.
Mazars’ damage model

- Damage mechanics theory [1]
  - Based on scalar damage variable affecting directly the stiffness tensor
    \[ E = E_0 \cdot (1 - d) \]
  - Stress – strain non-linear relation
    \[ \sigma = f(d) \cdot \varepsilon \]
    \[ d = f(\varepsilon) \]

- Mazars’ formulation [2]
  - \( d \) is isotropic and scalar
  - Different laws for compression or tension stress state
  - Strain maximum values

\[ Kachanov \, LM, \, 1958. \, Isv. \, Akad. \, Nauk. \, SSR, \, 8, \, 26–31. \]
\[ Mazars \, J, \, 1986. \, Engineering \, Fracture \, Mech., \, 25(5–6), \, 729-737. \]

Uniaxial cyclic loading test. Test representation, damage variable evolution and deformation (left); time evolution of the top face displacement in m (cyclic) and damage variable (monotonous increasing) on the right.
External material vs. built-in implementation

**External material model**
- Comsol post by Ed Gonzalez (2015) [1]
  - Any constitutive model can be programmed

**Built-in implementation**
- 2 History variables storage
  - Domain Ordinary Differential Equations
  - Specific solver configuration

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2 additional degrees of freedom

Segregated step + Previous solution node

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- **Advantages**
  - Fully coupling with other constitutive models
  - Fully coupling with other physics
  - Variables availability (pre/post-process)
  - Easier adjustment or reformulation (compilation avoided)

- **Drawback**
  - Increased model complexity (DOF’s and solvers)

Results of the verification (uniaxial compression) test in terms of stress-strain curves using two different damage model implementations.

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Regularization method – Gradient enhanced formulation

- Implicit gradient formulation [1 ; 2]:
  - Implemented as a Helmholtz differential equation

\[
\ddot{\varepsilon} - l^2 \nabla^2 \varepsilon = \ddot{\varepsilon}
\]

- Local equivalent strain \( \dot{\varepsilon} \)
- Non-local equiv. strain \( \ddot{\varepsilon} \)
- Characteristic length \( l \) (m)

- Three-point bending tests of notched and unnotched concrete beams modelled with the regularized and non-regularized models

Results of the Comsol model for different mesh refinements (left), results from [3] (right).

Results of the Comsol regularized damage model for different mesh refinements.

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Mechanical parameters and damage model parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notched</th>
<th>Unnotched</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_0 ) (GPa)</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_0 )</td>
<td>1.2 \times 10^{-4}</td>
<td>0.9 \times 10^{-4}</td>
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<tr>
<td>( \varepsilon_f )</td>
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<td>0.003</td>
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<tr>
<td>( A_c )</td>
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<td></td>
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<tr>
<td>( B_c )</td>
<td>1500</td>
<td></td>
</tr>
<tr>
<td>( l ) (mm)</td>
<td>0.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

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Regularization method – Gradient enhanced formulation

- Comparison with results presented in [1]
  - Damage variable evolution

Evolution of mechanical damage as a function of the imposed displacement in the top face center point

**μ** damage model

- Improvements of the formulation presented in [1]:
  - Two principal models:
    - Cracking in tensile state
    - Crushing in compressive state
  - Good representation of cyclic loading paths
  - Behaviour under biaxial compression
  - Behaviour under triaxial (EA) compression

![Biaxial loading tests from [2], model results for classical and μ damage models](image)

![Triaxial test modelled in [1] (blue) and Comsol results (red)](image)

Conclusions

- The damage models presented:
  - Are completely built-in Comsol interface
  - Overcome mesh dependency problems
  - Represent mechanical behaviour under diverse loading paths
  - Are validated with several experimental tests
  - Can be coupled with other constitutive models and/or processes

The goal of implementing a robust damage model to be coupled with other physical and chemical processes has been successfully achieved.

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This work has received funding from the European Union's Horizon 2020 Research and Training Programme of the European Atomic Energy Community (H2020-NFRP-2014/2015) under grant agreement n° 662147 (CEBAMA) and from the Swedish Nuclear Fuel and Waste Management Company (SKB), which are gratefully acknowledged.
Application case – Glaciation

- Deep geological repository for nuclear waste
Application case – Sulphate attack (HCM)

- The concrete damage model presented:
  - Completely built-in Comsol interface

Changes in transport properties due to mechanical effects

- Log$_{10}$ (Def$^{m2/s}$)
- Log$_{10}$ (K$^{m/s}$)

Effect on material

- Mechanical damage (1)
- Ettringite precipitation (v.f.)
- Thaumasite precipitation (v.f.)
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