Fracture Toughness Evaluation for magnetostrictive problem using COMSOL-Multiphysics

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Introduction

- **Magnetostriiction**: Changes in geometrical shape due to alignment of dipole in a magnetic environment
Introduction

• **Applications:**
  - ferromagnetic and magneto-ceramic materials
  - Nuclear reactors
  - Aerospace
  - Electrical motors,
  - Transducers etc,
  - Electro-magnetic devices
Introduction

Presence of crack

- The state of stress near the crack tip become complex
- Needs to be characterize properly, it will require
  1. A new integral
  2. The numerical validation (COMSOL Multiphysics)

Formulation new integral

- Concept of Conservation of Energy is used to derive the integral
Introduction

- The Rice’s Integral has been modified for magneto-thermo-elastic environment.
- The integral is derived for generalized 3D geometry.

Numerical Validation

- Two physics is used: AC/DC and structural physics
- Rectangular cracked beam is used under magnetic environment
- Integral has been calculated
Integral Formulation

Equation of motion \[ \sigma_{ij,j} + F_i = \rho \ddot{u}_i \]
Integral Formulation

Using divergence theorem, re-arranging the terms we get

\[ \int_{\Gamma_1 + \Gamma_c} T_i \frac{du_i}{dl} d\Gamma + \iint_{A_i} F_i \frac{du_i}{dl} dA = \iint_{A_i} \rho \ddot{u}_i \frac{du_i}{dl} dA + \iint_{A_i} \sigma_{ij} \frac{d\varepsilon_{ij}}{dl} dA + J^u \]

Adding and subtracting the term \( \int_{\Gamma_2} T_i \frac{du_i}{dl} d\Gamma \) we get

\[ \int_{\Gamma_2} T_i \frac{du_i}{dl} d\Gamma + \int_{-\Gamma_2} T_i \frac{du_i}{dl} d\Gamma + \iint_{A_i} F_i \frac{du_i}{dl} dA = \iint_{A_i} \rho \dddot{u}_i \frac{du_i}{dl} dA + \iint_{A_i} \sigma_{ij} \frac{d\varepsilon_{ij}}{dl} dA + J^u - \int_{\Gamma_1 + \Gamma_c} T_i \frac{du_i}{dl} d\Gamma \]
Integral Formulation

\[ J^u = \int_{\Gamma_1+\Gamma_c-\Gamma_2} T_i \frac{d u_i}{d l} \, d \Gamma + \int_{\Gamma_2} T_i \frac{d u_i}{d l} \, d \Gamma + \iint_{A_1} \left[ (F_i - \rho \ddot{u}_i) \frac{du_i}{dl} - \left( \sigma_{ij} \frac{d \varepsilon_{ij}}{dl} \right) \right] dA \]

\[ J^u = \iint_{A_1} \left( \sigma_{ij} \frac{d u_i}{d l} \right)_{, j} \, dA + \int_{\Gamma_2} T_i \frac{d u_i}{d l} \, d \Gamma + \iint_{A_1} \left[ (F_i - \rho \ddot{u}_i) \frac{du_i}{dl} - \left( \sigma_{ij} \frac{d \varepsilon_{ij}}{dl} \right) \right] dA \]

\[ J^u = \iint_{A_1} \left[ (\sigma_{ij} \frac{d u_i}{d l})_{, j} + (F_i - \rho \ddot{u}_i) \frac{du_i}{dl} - \sigma_{ij} \frac{d \varepsilon_{ij}}{dl} \right] dA + \int_{\Gamma_2} T_i \frac{d u_i}{d l} \, d \Gamma \]

Terms under parenthesis vanishes and the integral may be written as
Integral Formulation

\[ J^u = \int_{\Gamma_2} T_i \frac{d u_i}{d l} d \Gamma \]

\[ x_1 = X_1 \cos \theta_0 + X_2 \sin \theta_0 - l \]
\[ x_2 = -X_1 \sin \theta_0 + X_2 \cos \theta_0 \]
Integral Formulation

\[X_1 = x_1 \cos \theta_0 - x_2 \cos \theta_0 + l \cos \theta_0\]
\[X_2 = x_1 \sin \theta_0 + x_2 \cos \theta_0 + l \sin \theta_0\]

\[u_i(X_1, X_2, l) = u_i(x_1 \cos \theta_0 + x_2 \sin \theta_0 - l, -x_1 \sin \theta_0 + x_2 \cos \theta_0, l)\]

\[\frac{du_i}{dl} = \frac{\partial u_i}{\partial l} + \frac{\partial u_i}{\partial x_i} \frac{dx_i}{dl} = \frac{\partial u_i}{\partial l} + \frac{\partial u_i}{\partial x_1} \frac{dx_1}{dl} + \frac{\partial u_i}{\partial x_2} \frac{dx_2}{dl}\]

\[\frac{\partial u_i}{\partial x_1} \frac{dx_1}{dl} = -\frac{\partial u_i}{\partial x_1} ; \quad \frac{\partial x_2}{\partial l} = 0\]

\[\frac{du_i}{dl} = \frac{\partial u_i}{\partial l} - \frac{\partial u_i}{\partial x_1}\]
Integral Formulation

\[ J^u = \int_{\Gamma_2} T_i \left( \frac{\partial u_i}{\partial l} - \frac{\partial u_i}{\partial x_1} \right) d\Gamma \]

Hence, the fracture process region is assumed to be constant in dimensions and moving along with the same speed as the crack tip, and hence, \( \frac{\partial u_i}{\partial l} = 0 \) holds in \( \Gamma_2 \)

\[ J^u = -\int_{\Gamma_2} T_i \left( \frac{\partial u_i}{\partial x_1} \right) d\Gamma \]
Integral Formulation

\[
\frac{\partial u_i}{\partial x_1} = \cos \theta_0 \frac{\partial u_i}{\partial X_1} + \sin \theta_0 \frac{\partial u_i}{\partial X_2}
\]

\[
J^u = - \int_{\Gamma_2} T_i \left( \cos \theta_0 \frac{\partial u_i}{\partial X_1} + \sin \theta_0 \frac{\partial u_i}{\partial X_2} \right) d\Gamma
\]

Further simplifying we get

\[
J^u_k = - \int_{\Gamma_2} T_i \frac{\partial u_i}{\partial X_k} d\Gamma
\]
Path Independence of Integral

\[
\overline{J}_k^u = - \int_{\Gamma_1 + \Gamma_c} T_i \frac{\partial u_i}{\partial X_k} d\Gamma + M_k(A)
\]

\[
\overline{J}_k^u - J_k^u = - \int_{\Gamma_1 + \Gamma_c} T_i \frac{\partial u_i}{\partial X_k} d\Gamma + M_k(A) + \int_{\Gamma_2} T_i \frac{\partial u_i}{\partial X_k} d\Gamma
\]

\[
\overline{J}_k^u - J_k^u = - \int_{\Gamma_1 + \Gamma_c - \Gamma_2} T_i \frac{\partial u_i}{\partial X_k} d\Gamma + M_k(A)
\]

\[
\overline{J}_k^u - J_k^u = - \int_{\Gamma_t} T_i \frac{\partial u_i}{\partial X_k} d\Gamma + M_k(A)
\]
Path Independence of Integral

For path independence of integral

\[ M_k(A) = \int_{\Gamma_t} T_i \frac{\partial u_i}{\partial X_k} d\Gamma = \iint_A (\sigma_{ij} \frac{\partial u_i}{\partial X_k})_j dA \]

Therefore,

\[ \bar{J}^u_k = J^u_k = \iint_{A_i} (\sigma_{ij} \frac{\partial u_i}{\partial X_k})_j dA - \int_{\Gamma_1 + \Gamma_c} T_i \frac{\partial u_i}{\partial X_k} d\Gamma \]

\[ J^u_k = \iint_{A_1} \left( \rho \ddot{u}_i - F_i \right) \frac{\partial u_i}{\partial X_k} dA + \iint_{A_1} (\sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial X_k}) dA - \int_{\Gamma_1 + \Gamma_c} T_i \frac{\partial u_i}{\partial X_k} d\Gamma \]
Path Independence of Integral

\[ \int \int_{A_1} \left( \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial X_k} \right) dA = \int \int_{A_1} \sigma_{ij} \left( \frac{\partial \varepsilon^e_{ij}}{\partial X_k} + \frac{\partial \varepsilon^m_{ij}}{\partial X_k} \right) dA \]

Magnetostriction is defined empirically

\[ \varepsilon^m_{ij} = \frac{3}{2} \frac{\lambda_s}{M^2_s} M^2(t,x) \]

Simplified expression is

\[ J^u_k = \int_{\Gamma_1 + \Gamma_c} \left\{ W^e n_k - T_i \frac{\partial u_i}{\partial X_k} \right\} d\Gamma + \int \int_{A_1} \left( \rho \dddot{u}_i - F_i \right) \frac{\partial u_i}{\partial X_k} dA + \int \int_{A_1} \frac{3\lambda_s \sigma_{ij}}{M^2_s} M \frac{\partial M}{\partial X_k} dA \]

\[ J^u_k = (J^u_k)_{2D} \quad \text{for } (k = 1, 2) \]
Using contour integral method, the three dimensional integral has been simplified as

\[
\left( J^u_k \right)_{3D} = \int_{\Gamma_1 + \Gamma_c} \left\{ W^n_e n_1 - T_i \frac{\partial u_i}{\partial X_1} \right\} d\Gamma - \iint_{A_t} (\sigma_{i3} u_{i,3} )_3 \ dA_1 \\
+ \iiint_{A_t} (\rho \ddot{u}_i - F_i) \frac{\partial u_i}{\partial X_1} \ dA_1 + \iint_{A_t} \sigma_{ij} \frac{3\lambda_s}{M_s^2} M \frac{\partial M}{\partial X_k} \ dA
\]

In the absence of body forces and material inertia

\[
\left( J^u_k \right)_{3D} = \int_{\Gamma_1 + \Gamma_c} \left\{ W^n_e n_1 - T_i \frac{\partial u_i}{\partial X_1} \right\} d\Gamma - \iint_{A_t} (\sigma_{i3} u_{i,3} )_3 \ dA_1 + \iint_{A_t} \sigma_{ij} \frac{3\lambda_s}{M_s^2} M \frac{\partial M}{\partial X_k} \ dA
\]
Numerical validation

Integration contours

The arrangement of steel housing, helical coil and the cracked specimen (sectional view)
Numerical validation

Model contains 92210 tetrahedral elements and 13296 hexahedral elements

Convergent Meshed model
Numerical validation

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus $(E)$</td>
<td>$60 \times 10^9$ Pa</td>
<td>Saturated Magnetostriction $(\lambda_s)$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Density $(\rho)$</td>
<td>7870 kg/m$^3$</td>
<td>Saturation magnetization $(M_s)$</td>
<td>$15 \times 10^5$ A/m</td>
</tr>
<tr>
<td>Poisson’s ratio $(\nu)$</td>
<td>0.3</td>
<td>Effective domain density $(a)$</td>
<td>7000 A/m</td>
</tr>
</tbody>
</table>

The Langevin function [2,3]

\[
\frac{M}{M_s} = \left[ \coth \left( \frac{H_e}{a} \right) - \left( \frac{a}{H_e} \right) \right] \\
\]

$H_e$ is the effective magnetic field and $a$ is domain density constant with dimensions of magnetic field.

From Magnetostriction Model [1]

\[
\lambda(t, x) = \sum_{i=0}^{\infty} \gamma_i M^{2i}(t, x)
\]
Numerical validation
Numerical Results

\[
\frac{(J_k)^{3D}}{(J_k)^{3D}_{sat}}
\]

Current Density (A/m\(^2\))
Fracture Toughness

As per ASTM E1921

\[ K_{lc} = \sqrt{\frac{(J_{kc}^u)_{3D}}{(1-v^2)}} \cdot \frac{E}{E} \]

- Only magnetic load is not sufficient
- Integral value saturate at saturated Magnetization
- Peak load applied to attain fracture toughness in magnetic field
Conclusion

- A conservation path independent integral \( (J_k^u)_{3D} \) for a straight crack has been proposed to have the physical meaning of energy release rate.

- The magnetization saturates after increasing current density.

- The integral \( (J_k^u)_{3D} \) value saturates at saturated magnetization.
References

Thank you