Modeling Energy Harvesting From Membrane Vibrations in COMSOL

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Outline

- Importance of Energy Harvesting
- Prestressed Membrane Structures
- Challenges in Energy Harvesting
- Transducer Materials
- Estimate of Harvested Energy
- Optimal Prestress and Transducer Locations
Importance of Energy Harvesting

- Recycling energy vs. expending energy
- Useful for multiple applications
- Increases autonomy
- Reach inaccessible locations
Why Membranes?

- Possibility of large amplitude vibrations

- Natural frequencies and mode shapes can be tuned by changing the prestress
Prestressed Membrane Structures

- Prestress is applied to ultra-lightweight membrane structures to keep them in desired shape and provide stiffness

Bat-Wing Micro Air Vehicles

Inflatable Space Antenna

Roof of Denver International Airport
Challenges in Energy Harvesting

- Maximize ability to support large strains
- Maximize power output
- Place transducers at points of high deformation
Transducer Materials

- Piezoceramics
  - Do not support large strains
  - Produce high voltages and useable power

- Electroactive polymers (i.e. ionic polymers)
  - Supports large strains
  - Do not produce useable amounts of power

- Flexible piezomaterials (PVDF, macro-fiber composites, etc.)
  - Designed to accommodate large strains while sustaining high piezoelectric constants
Transducer Materials

- Macro-fiber composites was found to have the highest piezoelectric coefficient for transverse stresses

- PVDF support highest strain limit, but would not produce higher output
Modeling Challenges

- Find relationships between membrane deformation and inputs
- Find relationships between electric field and deformation
- Finite element approach more suitable for nonlinear problem
- For MSC/NASTRAN use “thermal-piezo analogy”
- ANSYS/ABAQUS, can use piezoelectric elements directly
Governing Equation of a Membrane with Transducer

- Governing equations are determined from the following condition:

\[ \delta \Pi = 0 \]

\[ \Pi = \int_{t_1}^{t_2} (T - U + \int p \, wdA) \, dt \]

\[ T = \frac{1}{2} \left[ \int_{v_E} \rho^E v^2 \, dv + \int_{v_S} \rho^S v^2 \, dv \right] = \text{Kinetic Energy} \]

\[ U = \frac{1}{2} \left[ \int_{v_E} \sigma_r^T \epsilon d v_E + \int_{v_S} \sigma_r^T \epsilon d v_s + \int_{v_E} \sigma_r^T \epsilon d v_E \right] - \int_{v_E} D^T E d v_E = \text{Strain Energy} \]

- Substitution of constitutive relations and strain-displacement relationship into \( \Pi \) yields governing differential equations (nonlinear coupled PDEs)
Transducer Materials

- Analyzed the response of the membrane to find out the electric field generated in the transducer.
- The non-linear governing differential equations were solved using the Adomian decomposition method.
- The analysis was performed for different load cases for different transducer locations.
Modeling Challenges

Contour Map Showing the Electric Field That Can Be Harvested at Each Location

\[ P_1(x, y, t) = P_0 \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi y}{L} \right) \]

\[ P_2(x, y, t) = P_0 \sin \left( \frac{2\pi x}{L} \right) \sin \left( \frac{\pi y}{L} \right) \]
Governing Equation of a Membrane with Transducer

\[ P_3(x, y, t) = P_0 \sin\left(\frac{2\pi x}{L}\right) \sin\left(\frac{2\pi y}{L}\right) \]

\[ P_4(x, y, t) = P_0 \sin\left(\frac{3\pi x}{L}\right) \sin\left(\frac{2\pi y}{L}\right) \]

Contour Map Showing the Electric Field That Can Be Harvested at Each Location
Solution of the Equations

\[ P_5(x, y, t) = P_0 \sin \left( \frac{3\pi x}{L} \right) \sin \left( \frac{2\pi y}{L} \right) \]

\[ P_6(x, y, t) = P_0 \sin \left( \frac{3\pi x}{L} \right) \sin \left( \frac{2\pi y}{L} \right) \]

Contour Map Showing the Electric Field That Can Be Harvested at Each Location
Challenges in COMSOL

- Keys to modeling prestressed membranes are understanding the following:
  - Large deformations must be accounted for
  - Membranes have no bending stiffness, while COMSOL only has shell and solid elements, unlike ABAQUS
  - Prestress increases system stiffness, and thus alters eigenfrequencies
Challenges in COMSOL

- Alternating between COMSOL 3.5 and 4.0 for computational analysis
- Modeling prestress in membrane was difficult to adapt for COMSOL
- Keeping the prestress and having it interact with the piezoelectric patch more difficult
- Simple problem of analyzing the natural frequency of a plain square membrane successful
Simple Membrane Comparison

- Below are the results from a comparison made between literature\(^1\) and COMSOL for a simple membrane
- Prestress applied to .2 x .2 x .0001 m Kapton membrane in static step, then eigenfrequency analysis performed (1st mode shown)

\(^1\) S. Kukathasan and S. Pellegrino, Vibration of Prestressed Membrane Reflectors, ESA Contractor Report

<table>
<thead>
<tr>
<th>Prestress (N/m)</th>
<th>COMSOL Frequency</th>
<th>Analytical Frequency</th>
<th>ABAQUS Frequency</th>
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<td>50</td>
<td>87.87 Hz</td>
<td>88.95 Hz</td>
<td>88.67 Hz</td>
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</tbody>
</table>
Conclusions

- Energy harvesting from membranes requires heterogeneous materials and multi-physics.
- Many methods/software require sequential analysis and additional intermediate processing.
- Success has been obtained using analytical methods (Adomian) + FEM tools.
- Barriers still exist in implementing the solution in multi-physics software, e.g. COMSOL.
Overview of the Adomian Decomposition Method

- Consider the differential equation

\[ LY + NY + RY = g(t) \]

\[ Y = Y_0 - L^{-1} NY - L^{-1} RY \]

where \( Y_0 = Y(0) + tY'(0) + L^{-1} g(t) \)

- The general solution is assumed to be of the form

\[ Y = \sum_{n=0}^{\infty} Y_n \]

where \( Y_n = -L^{-1} NY_{n-1} - L^{-1} RY_{n-1} \)

\[ Y_0 = Y(0) + tY'(0) + L^{-1} g(t) \]

\[ A_0 = f(Y_0) \]

\[ Y_1 = -L^{-1} RY_0 - L^{-1} NA_0 \]

\[ A_1 = Y_1 \frac{d}{dY_0} f(Y_0) \]
Limitations

- Let us write \( S_n = \sum_{i=0}^{n} Y_n \)

- \( \lim_{n \to \infty} S_n = Y \)

- The solution \( Y \) converges if 

\[ \exists \alpha \leq 1, \quad \| Y_{k+1} \| < \alpha \| Y_k \|, \quad \forall k \in NU \{0\} \]