Implementation of the Perfectly Matched Layer to Determine the Quality Factor of Axisymmetric Resonators in COMSOL

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Introduction

- Accurate electromagnetic model is needed for various axisymmetric optical resonators such as micro-discs and micro-toroids.
- A COMSOL model for such resonators exists but without perfectly matched layer.
- Unwanted reflections from the computation wall reduces the accuracy of the model.
- Quality factor determination with high accuracy is important for certain applications.
Whispering Gallery Modes (WGM)

- In open cavities light circulates in the form of WGM
- WGM field does not occupy the whole cavity
- Portion of a WGM field lies outside the cavity
Previous Finite Element (FEM) Model

- Full vectorial model – No transverse approximation
- No PML or any other absorbing boundary condition
- Important parameters can be extracted for various cavity geometries:
  - Quality factors
  - Mode Volumes
  - Resonant Frequencies
  - Shapes of fundamental and higher order modes
- Estimated quality factor for the disc resonator is $1.31 \times 10^7 < Q < 3.82 \times 10^7$
  - Prior knowledge of the mode frequency is required
  - Quality factor for one mode at a time
  - Need to change the boundary conditions for each bound and recalculation of the model each time

Perfectly Matched Layer (PML)

- Modes of an open optical micro-cavity radiate into surroundings
- PML acts as an artificial boundary to truncate the computation domain
- PML as an anisotropic absorber – modification of the diagonal permittivity and permeability tensors of the absorber
PML: Mathematical Details

Oxborrow’s Master FEM Equation

\[
\int_V \left( \nabla \times \vec{H} \right) \epsilon^{-1} \left( \nabla \times \vec{H} \right) - \alpha \left( \nabla \cdot \vec{H} \right) \left( \nabla \cdot \vec{H} \right) + \epsilon^{2} \vec{H} \cdot \mu \cdot \frac{\partial^2 \vec{H}}{\partial t^2} \right) \, dV
\]

Modified FEM Master Equation

PML in cylindrical coordinates

\[
\begin{align*}
\vec{\epsilon} &= \epsilon \vec{\Lambda}, \quad \vec{\mu} = \mu \vec{\Lambda}, \\
\vec{\Lambda} &= \left( \frac{\vec{r}}{r} \right) \left( \frac{s_z}{s_r} \right) \hat{r} + \left( \frac{r}{\vec{r}} \right) \left( s_z s_r \right) \hat{\phi} + \left( \frac{r}{\vec{r}} \right) \left( \frac{s_r}{s_z} \right) \hat{z} \\
s_r &= \begin{cases} \\
1 & 0 \leq r \leq r_m \\
1 - jG \left( \frac{r - r_m}{t_p} \right)^2 & r > r_m \\
1 & z < z_{ml} \\
1 - jG \left( \frac{z_{ml} - z}{t_{zl}} \right)^2 & z_{ml} \leq z \leq z_{mu} \\
1 & z > z_{mu} \\
1 & \frac{(r - r_m)^3}{3t_r^2} & r > r_m \\
\end{cases}
\end{align*}
\]

Our FEM model

- Full vectorial model – No transverse approximation
- PML along the computation box
- Important parameters can be extracted for various cavity geometries accurately:
  - Quality factors
  - Mode Volumes
  - Resonant Frequencies
  - Shapes of fundamental and higher order modes
- Quality factor of the disc resonator with the PML is $1.60 \times 10^7$
  - No prior knowledge of the mode frequency is required
  - Quality factor for all modes simultaneously
  - One time execution of the model

**Fundamental TE mode of a 12 microns silica micro-sphere in air (False Colors)**
Results: silica microsphere

Quality factor of fundamental TE modes at 850nm

Quality factor of fundamental TM modes at 850nm
Conclusions

- Excellent agreement between the simulation and analytical results
- Third generation model - No transverse approximation and with PML
- Model is applicable to any axisymmetric micro-cavity geometries such as discs and toroids