Simulation of the Spread of Epidemic Disease Using Persistent Surveillance Data

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Agenda

• Significance of Mathematical Modeling in Epidemic Disease.
• SIR (Susceptible-Infected-Recovered) --family models and their shortcomings.
• Principle of SIR-HT (Heat Transfer) model, which is proposed as an innovative method in this field.
• Mathematical Description of SIR-HT Model
• Simulation Using COMSOL 3.5a
• Conclusion and Future Work
Significance of Mathematical Modeling in Epidemic Disease

• Explore the transmission mechanism of epidemic diseases;

• Obtain insight into potential cost and outcomes of the breakout of the disease;

• Evaluate the effectiveness of prevention / control strategies such as immunization and segregation.
Existing Epidemic Models (Deterministic)

- **S**: susceptible
- **I**: infected
- **R**: recovered
- **μ**: death
- **B**: birth
- **M**: immunity from mother
- **E**: exposed rate in latent period

**Recovered People return to susceptible**

Birth/death

Exposed rate

Immunity from Mom
SIR Model – Mathematical Description

Define $s(t)$, $i(t)$ and $r(t)$ be the proportion of the number of susceptible, infected and recovered individuals at time $t$

\[
\begin{align*}
\frac{ds(t)}{dt} &= -\beta s(t) i(t) \\
\frac{di(t)}{dt} &= \beta s(t) i(t) - \gamma i(t) \\
\frac{dr(t)}{dt} &= \gamma i(t)
\end{align*}
\]

$s(t) + i(t) + r(t) = 1$

Scalar $\beta$ is contact rate; scalar $\gamma$ is the mean recovery rate
Short-Comings of Existing Epidemic Model

• In an isolated community: no interaction between neighboring communities.

• No spatial variable such as distance, location, route of transmission, etc.

• No transmission media.
Derivation of SIR-HT Model

SIR-HT, which couples SIR, GIS and PS based on heat-transfer platform.
Principle of SIR-HT: Similarity in Diffusion Mechanism of Disease and Heat

**Spread of Epidemic Disease**
- Contact infection
- Personnel's movement

**Diffusion of Heat Energy**
- Vibration of atom influences neighboring atoms
- Free electrons carry energy
## Principle of SIR-HT: SIR-HT vs. Standard Heat-transfer

<table>
<thead>
<tr>
<th>SIR-HT model</th>
<th>Counterpart in Heat transfer model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of infective population</td>
<td>temperature ((T))</td>
</tr>
<tr>
<td>Change rate of infective population</td>
<td>Heat-flux((Q))</td>
</tr>
<tr>
<td>Personnel exchange between neighboring community</td>
<td>Conductivity ((k))</td>
</tr>
<tr>
<td>Road (including local, high-way, free-way)</td>
<td>Thin but highly conductive layer</td>
</tr>
<tr>
<td>Terrain conditions (lake, mountain, etc)</td>
<td>Boundary conditions</td>
</tr>
<tr>
<td>Persistent surveillance data</td>
<td>Initial conditions</td>
</tr>
<tr>
<td>Conservation of infective</td>
<td>Law of conservation of energy</td>
</tr>
<tr>
<td>The transmission of infectious between neighboring communities is proportional to the difference of their infective rate</td>
<td>Fourier law</td>
</tr>
</tbody>
</table>
**Mathematical Description of SIR-HT: Governing Equations**

\[
\rho C_p \frac{\partial i(X,t)}{\partial t} + \nabla \cdot ( -K(X) \nabla i(X,t)) = Q_{\text{inf}} - Q_{\text{rec}}
\]

\[
Q_{\text{rec}} = \gamma(X)i(X,t)
\]

\[
Q_{\text{inf}} = \beta(X)s(X,t)i(X,t)
\]

- \(\beta(X)\) is location-related **contact infection tensor**;
- \(\gamma(X)\) is **recovery rate**;
- \(\rho\) is population density;
- \(C_p\) is a time-scaling coefficient (dimensionless);
- \(Q_{\text{inf}}\) is the incremental infective caused by contact infection;
- \(Q_{\text{rec}}\) is the decremented infective caused by recovery;
- \(\nabla \cdot ( -K(X) \nabla i(X,t))\) indicates the infective change caused by inter-community personnel exchanging.
Mathematical Description of SIR-HT: BCs Introduced by Road

\[
\begin{align*}
    d_{rd} \rho_{rd} C_{rd} + \nabla \cdot \left( -d_{rd} \kappa_{rd} \nabla i(X,t) \right) &= -n \cdot q \\
    q &= -K(X) \nabla i(X,t)
\end{align*}
\]

- \( d_{rd} \) is transportation bandwidth;
- \( \rho_{rd} \) is the passenger density;
- \( C_{rd} \) is a coefficient;
- the transportation network is translated into boundary condition, denoted as \( \partial \Omega_{rd} \);
- Counter-part term in COMSOL: thin but highly conductive layer
Mathematical Description of SIR-HT: Initial Condition and BCs

• **Initial conditions** are derived from persistent surveillance data;

• The disease **transmission media** is defined according to geographic information.
Flowchart of SIR-HT Framework

1) Formulate the heat-transfer medium according to geographic information

2) Instill the SIR model into SIR-HT model

3) Obtain the initial/boundary conditions of the heat transfer problem according to persistent surveillance data

4) Simulate the spread of epidemic disease by solving the transient heat-transfer problem
Simulation Experiment: Spread of Flu at a Sample Site Near Minneapolis

Map of a sample site near Minneapolis

Heat-transfer model derived from sample site
Experiment: Animation of the Spread of Epidemic Flu
Experiment: Spread of Epidemic Flu with Time

Day 1

Day 3

Day 5

Day 10
Conclusions

• A novel deterministic epidemic model is developed and implemented using COMSOL 3.5a;
• The simulation result shows infectious disease spread within residential area or along transportation network, which is basically consistent with our expectation;
• A more critical validation about the SIR-HT model is needed with the support and collaborations of experts in multidisciplinary areas such as medical science, sociology, statistic, optimization, geology science, and public health, etc.
Future Work

• **Validation** of proposed mathematical model;
• Effect of public **prevention strategy** and **medical treatment** over the SIR-HT;
• Introduction probability into SIR-HT model to achieve **stochastic** description about the spread of epidemic disease;
• Effects of **air-line** transportation over SIR-HT;
• **Global** tracking/analysis platform for epidemic disease;
• Promote SIR-HT framework into other applications such as immigration of locust, spread of cancer cells, etc.
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Question and Answer

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