Using COMSOL-Multiphysics in an Eddy Current Non-Destructive Testing Context

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Abstract: EDDY current testing (ECT) is widely used to check the integrity of electrically conducting parts and notably to detect flaws. It is based on the interaction between a probe and the part under testing. The finite element method (FEM) is well fitted to the modeling of these kinds of problems because of its large flexibility which allows to deal with complex probe and part configurations. In this context, the benchmark problems TEAM Workshop 15-1 and JSAEM 2-5 have been considered to validate a COMSOL-Multiphysics 3D-resolution using a 3D electromagnetic formulation with Whitney edge elements. A good agreement is obtained between the numerical results and the experimental ones.

Keywords: Eddy current, non-destructive testing, edge elements, harmonic quasi-static fields.

1. Introduction

A typical geometry ECT problem is constituted of a conducting domain in which exists a defect (Figure 1), a probe (coil driven by an excitation current density which may include a magnetic core) and the surrounding air. The benchmark problems TEAM (Testing Electro-magnetic Analysis Methods) Workshop 15–1 and JSAEM (Japan Society of Applied Electromagnetic and Mechanics) 2-5 have been considered in order to validate these approaches. The forward problem typically consists in the determination of the probe impedance variation. The real and imaginary parts of the probe impedance are determined by using the magnetic energy and the power losses, respectively. Both are deduced from the FEM results.

A circular air-cored coil is moved along the length of a rectangular slot in a conductive plate. Both frequency and coil lift-off are fixed. The objective is to compute the change of the impedance of the coil (comparatively to its value over an unflawed portion of the plate) as a function of coil position, and to compare the computed results to the experimental ones.

2. Use of COMSOL Multiphysics in ECT

The release of the software used for the study is the 3.5 release.

2.1 Model definition

The frequency is sufficiently low so that the displacement currents can be neglected (electro-technical hypothesis). Since the exciting coil is driven by a sinusoidal current, the time harmonic approach is used. The presence of the defect implies a 3D FEM resolution:

\[
\left( j \omega \sigma - \omega^2 \varepsilon_0 \varepsilon_r \right) \vec{A} + \nabla \times \left( \mu_0^{-1} \mu_r^{-1} \nabla \times \vec{A} \right) = \vec{J} 0
\]

This Finite Element Formulation results from the Maxwell equations. It uses Whitney edge elements. The scalar unknown is then the circulation of the magnetic vector potential along the element edges of the mesh.
2.1 Geometry definition

The predefined forms with CSG (Constitutive Solid Geometry) approach are used to define the geometry in the software.

<table>
<thead>
<tr>
<th>Geometric parameters</th>
<th>JSAEM</th>
<th>TEAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil inner radius (mm)</td>
<td>0.6</td>
<td>6.15</td>
</tr>
<tr>
<td>Coil outer radius (mm)</td>
<td>1.6</td>
<td>12.4</td>
</tr>
<tr>
<td>Coil length (mm)</td>
<td>0.8</td>
<td>6.15</td>
</tr>
<tr>
<td>Lift off (mm)</td>
<td>0.25</td>
<td>0.88</td>
</tr>
<tr>
<td>Plate length (mm)</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>Plate width (mm)</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>Plate thickness (mm)</td>
<td>1.25</td>
<td>12.22</td>
</tr>
<tr>
<td>Cracks length (mm)</td>
<td>10</td>
<td>12.6</td>
</tr>
<tr>
<td>Cracks depth (mm)</td>
<td>0.75</td>
<td>0.5</td>
</tr>
<tr>
<td>Cracks width (mm)</td>
<td>0.21</td>
<td>0.28</td>
</tr>
</tbody>
</table>

A typical physical parameter of this kind of problem is the skin depth \( \delta \) defined by:

\[
\delta = \frac{1}{\sqrt{\mu_0 \sigma \omega}} = \frac{1}{\sqrt{\mu_0 \sigma f}}
\]

With \( \mu \) the magnetic permeability, \( \sigma \) electrical conductivity, \( \omega \) the pulsation, \( f \) the frequency.

For the JSAEM problem the skin depth (1.30 mm) is greater than the plate thickness. In this case, to avoid numerical problems due to the boundary conditions, it is necessary to add an air domain under the plate.

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2.2 Physical properties

<table>
<thead>
<tr>
<th>Physical parameters</th>
<th>JSAEM</th>
<th>TEAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core relative permeability</td>
<td>1.</td>
<td>1.</td>
</tr>
<tr>
<td>Coil number of turns</td>
<td>140</td>
<td>3790</td>
</tr>
<tr>
<td>Intensity (A)</td>
<td>0.8</td>
<td>6.15</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>1.5e5</td>
<td>900</td>
</tr>
<tr>
<td>Plate conductivity (S/m)</td>
<td>1e6</td>
<td>30.666</td>
</tr>
</tbody>
</table>

Taking into account the high number of turn of a coil, this one is assumed as a medium having an uniform vector current density \( J_0 \) (Figure 3). The resistance of the coil wire is then not taken into account in the modeling. This field is defined in the volume domain corresponding to the coil. Its norm is constant through the coil volume and defined by:

\[
J_0 = \frac{Nb_{\text{coil turn}} * I}{h_{\text{coil}} * (r_{\text{coil out}} - r_{\text{coil in}})}
\]

With \( Nb_{\text{coil turn}} \) the coil number of turns, \( h_{\text{coil}} \), the coil length, \( r_{\text{coil out}} \) the coil outer radius, \( r_{\text{coil in}} \) the coil inner radius.

The \( J_0 \) vector is then expressed in the \((x, y, z)\) coordinate system by the following formula:

\[
J_0 = \left[ 0, \frac{\|J_0\|_z}{\sqrt{y^2 + z^2}}, \frac{\|J_0\|_y}{\sqrt{y^2 + z^2}} \right]
\]

With \( x \) the vertical component of the coordinate system and with a coordinate system reference located at the center of the coil (Figure 4).
The defect is supposed to be an insulating crack and this domain has the same physical properties than an air domain.

2.3 Boundary conditions setting

In all the boundaries of the study domain, the magnetic induction B is supposed tangent and then the magnetic insulation condition (nxA=0) is imposed (default condition in COMSOL-Multiphysics).

2.4 Mesh generation

The quality of the mesh is crucial for a finite element simulation. It allows to obtain an accurate solution with a reduced number of elements and so a reasonable computational time. The mesh is generated with tetrahedral elements. A fine mesh is required in the region where the variation of the fields is important. As an example we have to refine the mesh around the coil or in the skin depth in the plate since the magnetic induction field varies significantly in these regions. In fact to take into account the field variation in the skin depth, three layers of elements are necessary if first order elements are used.

\[ \Delta Z = \frac{\text{Re}(\Delta Z)}{I^2} \text{Re}(Z) \]

\[ \Delta Z = \left[ \int_{\text{domain}} \frac{1}{\sigma} \left( |J|^2 - |I|^2 \right) d\Omega \right] \]

\[ \Delta Z = \left[ \int_{\text{domain}} \frac{1}{\mu} \left( |B|^2 - |B|^2 \right) d\Omega \right] \]

2.5 Impedance variation computation

The impedance variation \( \Delta Z \) is a complex number. The real part is computed with the Joule Losses (JL) in the conductive media and the imaginary part is computed with the magnetic energy (\( W_m \)) in the whole meshed domain:

\[ \text{Re}(Z) = \frac{\text{JL}}{I^2} \rightarrow I^2 \text{Re}(\Delta Z) \]

\[ \text{Im}(Z) = \frac{\text{W}_m}{I^2} \rightarrow I^2 \text{Im}(\Delta Z) \]
\( \mathbf{J}, \mathbf{J} \) and \( \mathbf{B}, \mathbf{B} \) are the current density field and the magnetic induction field respectively calculated with and without the defect and are both software post-treatment quantities.

### 3. Scripting consideration

For the computing process, a script program is used. A loop allows calling a function which computes the impedance at each displacement step.

In this function as an example, the magnetic energy and the Joules losses are directly evaluated using the function ‘postint’ as it is described in the script bellow:

```matlab
% Computation of the Energy
% \( \rightarrow \text{CALCUL DE } b^2/\mu \)
WM=postint(xfem,'(norm\_emqa*norm\_emqa)/(4*pi*1e-7)', 'unit','W*m')

% Computation of the Joule losses
% \( \rightarrow \text{CALCUL DE } j^2/\sigma \)
if (okdefect==0) % no defect
    Joule_Losses=postint(xfem,'(norm\_emqa*norm\_emqa)/(1e6)', 'unit','A*A/m','dl',2) % Defect+ plate domain
elseif (okdefect==1) % with defect
    Joule_Losses=postint(xfem,'(norm\_emqa*norm\_emqa)/(1e6)', 'unit','A*A/m','dl',2) % Only in the plate domain
end

Zr = PJ
Zi = 2*pi*Fq*WM
deltaZr = Zr-Zr0
deltaZi = Zi-Zi0
```

### 4. Results

The circular coil is displaced along the y-axis (Figure 1). The impedance variation is computed as a function of the position of the coil center. Quantitatively, Figures 8 and 9 show a good behavior of the current density. Quantitatively the Figures 10 and 11 show a good agreement of the impedance variation results in comparison with the results obtain in references [1] and [2].

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**Figure 8.** Map of the eddy currents in the plate without defect

**Figure 9.** Map of the eddy currents in the plate with a defect

**Figure 10.** JSAEM 2.5 – Variation of the probe resistance (bottom) and reactance (top) versus the probe position along the crack length
Figure 11. Team-Workshop 15 - Variation of the probe resistance (bottom) and reactance (top) versus the probe position along the crack length.

5. Conclusions

This paper presents a validation of an ECT 3D problem with the software COMSOL Multiphysics 3.5. The use of the software to solve this kind of problem is detailed and explained. Benchmark numerical results are presented.

6. References


2. Y Choua and all, Specific development on a Finite Element Tool for thin crack modeling in EC testing, *ECNDT conference 2006*, Poster 145

7. Acknowledgements

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