Ultrasound Piezo-Disk Transducer Model for Material Parameter Optimization

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Ultrasound imaging transducer and FEM

- Ultrasound imaging transducers generate a pressure field into the human body
- Differences in acoustic properties of different types of tissue allow the scanner to generate an image
- Quality of the resulting image is strictly related to:
  - technology level of the materials involved in the transducer manufacturing
  - understanding of their interactions
- Finite Elements Model (FEM) greatly help in the study and optimization of transducer electroacoustical performances and image quality improvement
Our work: simulation and measurements

- Finite Element Model (FEM) for an ultrasound piezoelectric disk transducer has been developed.
- The FEM design followed a "step approach":
  - Development of the model along with the transducer manufacturing stages (starting from the choice of piezoelectric material, up to the complete transducer assembly).
  - Comsol simulation with non-optimized model
  - Measurement comparison
  - Optimization procedure
Transducer Electro-acoustical measurements

- The most important measurements to determine the quality of the transducer and its consequent imaging performances are:
  - **Electrical impedance**
    (performed with Network Analyzer):
    - Determines resonance frequency of principal vibration modes
    - Determines piezoelectric coupling efficiency
  - **Emitted pressure bandwidth**
    (performed with Pulser-water tank-hydrophone-oscilloscope system):
    - Determines transducer spatial resolution
    - Determines transducer sensitivity for different frequencies

**Corresponding Simulation**: a FEM frequency response analysis is to be performed in order to compare simulation results with the measurements above
Main results

Measured and simulated transducer performances comparison

• Electrical impedance
  – error of less than 3% in resonant frequencies and amplitudes

• Far field pressure level measured in water
  – error of less than 1dB in maximum pressure amplitude

Optimization procedure

• Taking advantage of the simplicity of the problem under study, it was possible to establish an optimization procedure that could be followed for future works
Main results

Piezoceramic disk electrical impedance magnitude: measured (solid) and simulated (dotted)

Complete transducer emitted far field pressure level in water: measured (solid) and simulated (dotted)

Frequency (MHz)

- Measured (solid)
- Simulated (dotted)
- 3% shift

1dB
Piezoelectricity equation in COMSOL

The constitutive equations for a piezoelectric material are *(stress-charge form)*:

\[
\begin{align*}
T &= \left[ c^E \right] S - \left[ e^r \right] E \\
D &= \left[ e \right] S + \left[ \varepsilon^S \right] E
\end{align*}
\]

- **T**: stress vector,
- **c**: elasticity matrix,
- **S**: strain vector,
- **e**: piezoelectric matrix,
- **E**: electric field vector,
- **D**: electric displacement vector,
- **\varepsilon**: dielectric permittivity matrix.

- Elasticity, piezoelectric and dielectric permittivity matrices must be specified to build the model in Comsol.
- Unfortunately manufacturer data are often incomplete and should be checked for the particular operating condition of the piezoelectric material.
- Physical insight is the starting point for the model.
- Optimization procedure should be used.
Electric equation in COMSOL

The electrical impedance $Z$ of a piezoelectric disk can be expressed by the general ohm law:

$$Z = \frac{V}{I}$$

$V$: potential difference voltage across the two disk faces,
$I$: current flowing inside.

As regard the electric current flowing in the disk, the following integral holds (axial symmetry):

$$I = \int_{0}^{r} j_z(r) 2\pi r dr$$

where $j_z$ is the current density component along $z$ axis.

This integral has been used in COMSOL as integration variable across the disk surface, in order to use the optimization module with objective function given by the difference of measured and simulated electrical impedance.
Acoustic equation in COMSOL

Pressure waves emitted from the piezoelectric transducer in a biological medium are solution to the wave equation (time domain):

\[ \nabla^2 p(r,t) - \frac{1}{c^2} \frac{\partial^2 p(r,t)}{\partial t^2} = 0 \]

where \( p \) is the pressure, \( c \) is the speed of sound in the medium.

For homogeneous media, axially symmetric geometry, we have (Helmholtz-Kirchhoff) (neglecting the oscillating phase factor):

\[
p_{far}(\mathbf{R}) = -\frac{1}{2} \int_S e^{jk(z^2 - r)} \left[ J_0 \left( \frac{kr}{|\mathbf{R}|} \right) \nabla p(\mathbf{r}) \cdot \mathbf{n} - \frac{jk p(\mathbf{r})}{|\mathbf{R}|} \left( j n_r R J_1 \left( \frac{kr}{|\mathbf{R}|} \right) + n_z Z J_0 \left( \frac{kr}{|\mathbf{R}|} \right) \right) \right] dS
\]

where \( k \) is the wave number, \( J_i \) are Bessel functions, \( r \) and \( z \) are the radial and axial components of the vector distance \( \mathbf{r} \) between observation point and source, \( R \) and \( Z \) are the radial and axial components of the vector distance \( \mathbf{R} \) between observation point and reference system origin, \( \mathbf{n} \) is the normal vector pointing into the domain that \( S \) encloses, \( \mathbf{n}_r \) is the normalized vector in \( r \) direction, \( \mathbf{n}_z \) is the normalized vector in \( z \) direction.
Building the model with a “step approach”

Simulation and measurements comparison for:
- piezoceramic disk alone
- piezoceramic disk bonded on backing substrate
- complete transducer piezoceramic disk, backing substrate and front matching layer

For each step, the optimization is based on minimization of the Root Mean Square deviation of an objective function given by the difference between the measured and simulated electrical impedance of the disk.
Piezoelectric disk alone (1)

Electrical impedance analysis was performed, between 1 MHz and 7 MHz

Agreement between measurements and simulation results is good over the whole frequency range. In particular, resonance and antiresonance frequency fit error is about 3%.

Moreover, both first thickness vibration mode (4 MHz) and radial vibration modes (from 150 kHz up to 2.5 MHz) can be clearly recognized.

**Optimization:** some material parameters are more critical than other in the optimization process. Here we report the most important parameters, along with the effect of each one on the electrical impedance response.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relevant changes on electrical impedance magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{33}$ (Elasticity matrix)</td>
<td>Resonance and antiresonance freq. and amplitude of first thickness vibration mode</td>
</tr>
<tr>
<td>$e_{33}$ (Piezoelectric matrix)</td>
<td>Antiresonance freq. and amplitude of first thickness vibration mode</td>
</tr>
<tr>
<td>$e_{33}$ (Dielectric matrix)</td>
<td>Antiresonance freq. and amplitude of first thickness vibration mode</td>
</tr>
</tbody>
</table>
Piezoelectric disk alone (2)

First radial vibration mode (150 kHz), $z$-$r$ plane

First thickness vibration mode (4.0 MHz), $z$-$r$ plane
Piezoelectric disk with backing substrate

Electrical impedance analysis was performed, between 1 MHz and 7 MHz

Again, fit to measurements is good, with resonance (~3.8 MHz) and antiresonance frequency (~4.7 MHz) error below 3%.

Moreover, as expected, the quality factor at the resonance frequency is reduced and radial modes are almost cancelled by bonding on backing substrate material.

Optimization: The backing substrate parameters greatly influence the electrical impedance response:

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<tr>
<td>$E_{\text{Backing}}$ (Backing substrate Young Module)</td>
<td>Thickness vibration mode amplitude</td>
</tr>
<tr>
<td>$\nu_{\text{Backing}}$ (Backing substrate Poisson coeff.)</td>
<td>Thickness vibration mode amplitude</td>
</tr>
</tbody>
</table>
Complete transducer

Electrical impedance analysis was performed, between 1 MHz and 7 MHz

Again, fit to measurements is quite good, with resonance and antiresonance frequency error below 5%.

Moreover, both resonance due to piezoceramic (∼4.1 MHz) and resonance due to matching layer (∼2.4 MHz) are clearly visible.

**Optimization**: The matching layer parameters greatly influence the electrical impedance response:

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<tbody>
<tr>
<td>( E_{\text{Matching Layer}} ) (Matching Layer Young Module)</td>
<td>Resonance freq. and amplitude due to the matching layer</td>
</tr>
<tr>
<td>( \nu_{\text{Matching Layer}} ) (Matching Layer Poisson coeff.)</td>
<td>Resonance freq. due to the matching layer</td>
</tr>
</tbody>
</table>
Final pressure field simulation

After all optimization stages were performed, the transducer model was completed with the front acoustic domain, as shown below:

Far field simulated pressure on $z$ axis was compared to the measurements performed with a membrane hydrophone field placed at about 30 cm depth, in a specialized water tank. As a final check of the model efficiency, the directivity performance of the transducer was both measured and simulated, at 3 MHz operating frequency. This frequency corresponds to the first maximum of the emitted pressure field level.
Field pressure level results

Frequency response analysis was performed, between 1 MHz and 7 MHz

Discrepancies between simulation and measurements are less than 5% for maximum amplitude frequencies and less than 1dB for corresponding amplitude values
Also in this case the agreement between measurement and simulation is good, with less than 5% amplitude error at -6dB from maximum.
Conclusion

A Finite Element Model (FEM) for an ultrasound piezoelectric transducer has been developed.

The FEM design follows a “step approach” which consists in the development of the model along with the transducer manufacturing stages.

Final results for the far field pressure level show a good agreement between measured and simulated transducer performances, even better results are obtained in terms of electric impedance.
Future work

The optimization procedure that could be established for the present study will be followed for future works. These will be the development of models for imaging probe arrays, with much more complicated geometries and operating conditions.

Indeed, Esaote complete probes for ultrasound diagnostic application consist in arrays of sub-mm piezoelectric elements which must be studied with particular care to electromechanical coupling phenomena. This will be our major task in the simulation and optimization process of such high technology devices.
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