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Modelling Flow through Fractures in Porous Media

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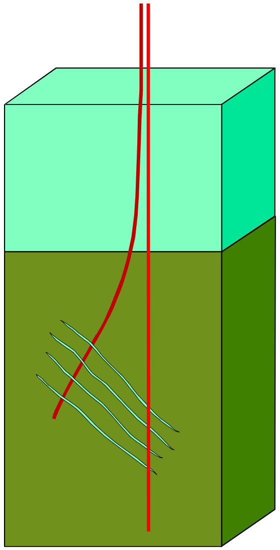
Applications for Fracture Models

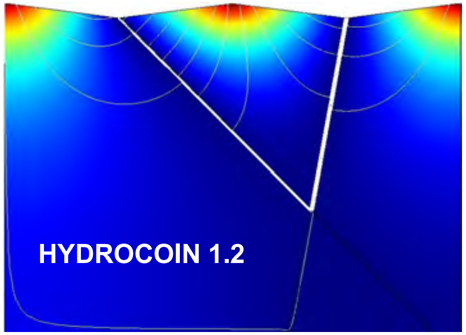
- Material Science
  - Concrete
  - Pavement
- Micro-Technology
  - Low permeable materials
  - Low permeable membranes
- Fractured rocks in geological systems
  - Subsurface waste repositories
  - Geothermics
- Medicine
  - Bones
  - Teeth



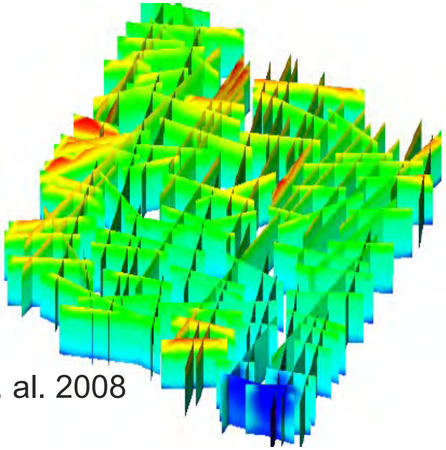
see: Jung, Orzol, Schellschmidt

Classification of Fracture Models

Diodato (1994) suggests a classification into

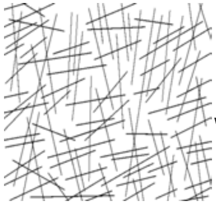


- explicit discrete fracture formulations
- discrete fracture networks
- continuum formulations



conc. fracture dimensionality

- full dimensional
- lower dimensional



Wang et. al. 2008

Pde - Flow Options

- Matrix
  - no-flow
  - Darcy‘s Law

- Fracture
  - Darcy‘s Law
  - Hagen-Poisseuille Laws
    - tubes
    - slices
  - Navier-Stokes equations
  - Brinkman equations
  - Saint-Venant equations
  - Preissman scheme

Example: Brinkman equation (steady state)

$$\frac{\eta}{\mathbf{k}}\mathbf{u} + \nabla p - \frac{\eta}{\theta}\nabla\left(\nabla\mathbf{u} + \left(\nabla\mathbf{u}\right)^T\right) = 0$$

$$\nabla \cdot \mathbf{u} = 0$$

with symbols

$\mathbf{u}$	Darcy velocity
$\mathbf{k}$	permeability tensor
η	dynamic viscosity
p	pressure
θ	porosity

Differential Equations & Non-dimensionalisation

Matrix: $\nabla K_{low} \nabla \varphi = 0$

- low hydraulic conductivity

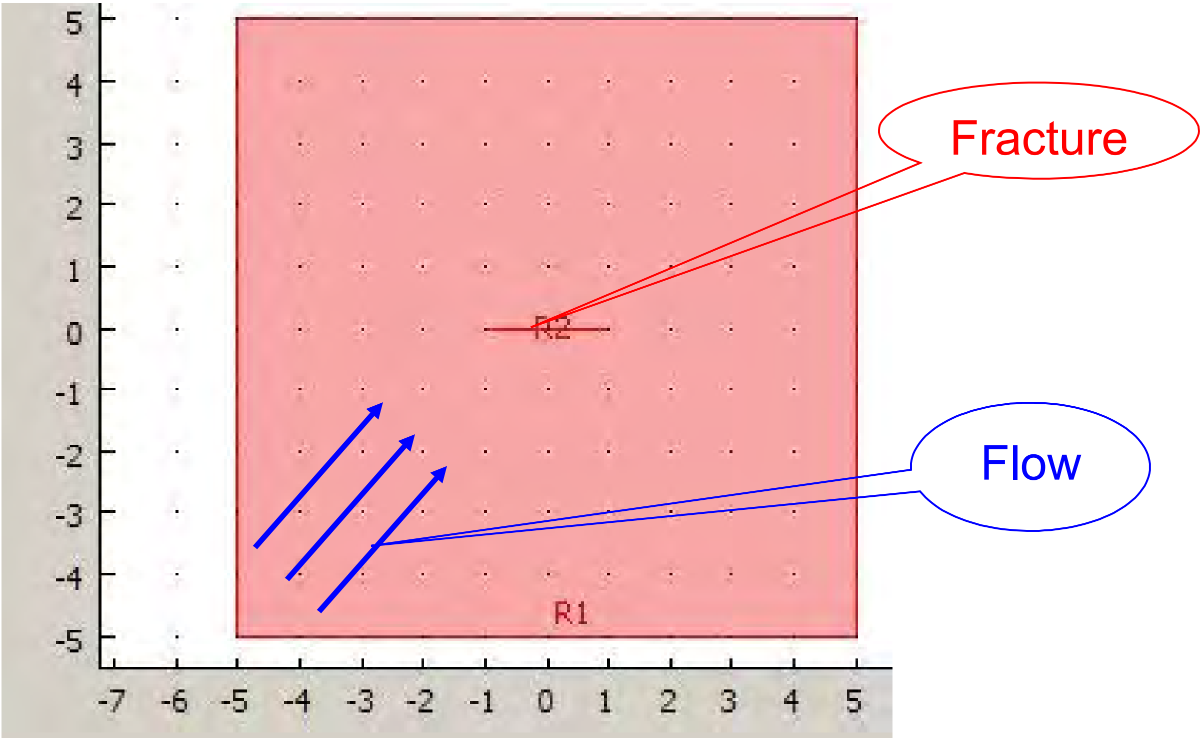
Fracture: $\nabla K_{high} \nabla \varphi = 0$

- high hydraulic conductivity

Normalization: $K_{low} = 1 / K_{high}$

- normalized velocity $v_0 = \sqrt{K_{matrix} K_{fracture}}$
- normalized length H (height)

Set-up 1



Thin fracture in a constant flow field

Mathematical approach: Darcy‘s Law in Fracture and Matrix

Analytical Solution

Complex potential for an impermeable line obstacle according to Churchill & Brown (1984):

$$\Phi(z) = \Phi_o(z\cos(\alpha) - i\sqrt{z^2 - a^2}\sin(\alpha))$$

with	α	angle fracture – baseflow direction
	a	half length of fracture
	Φ_0	baseflow potential

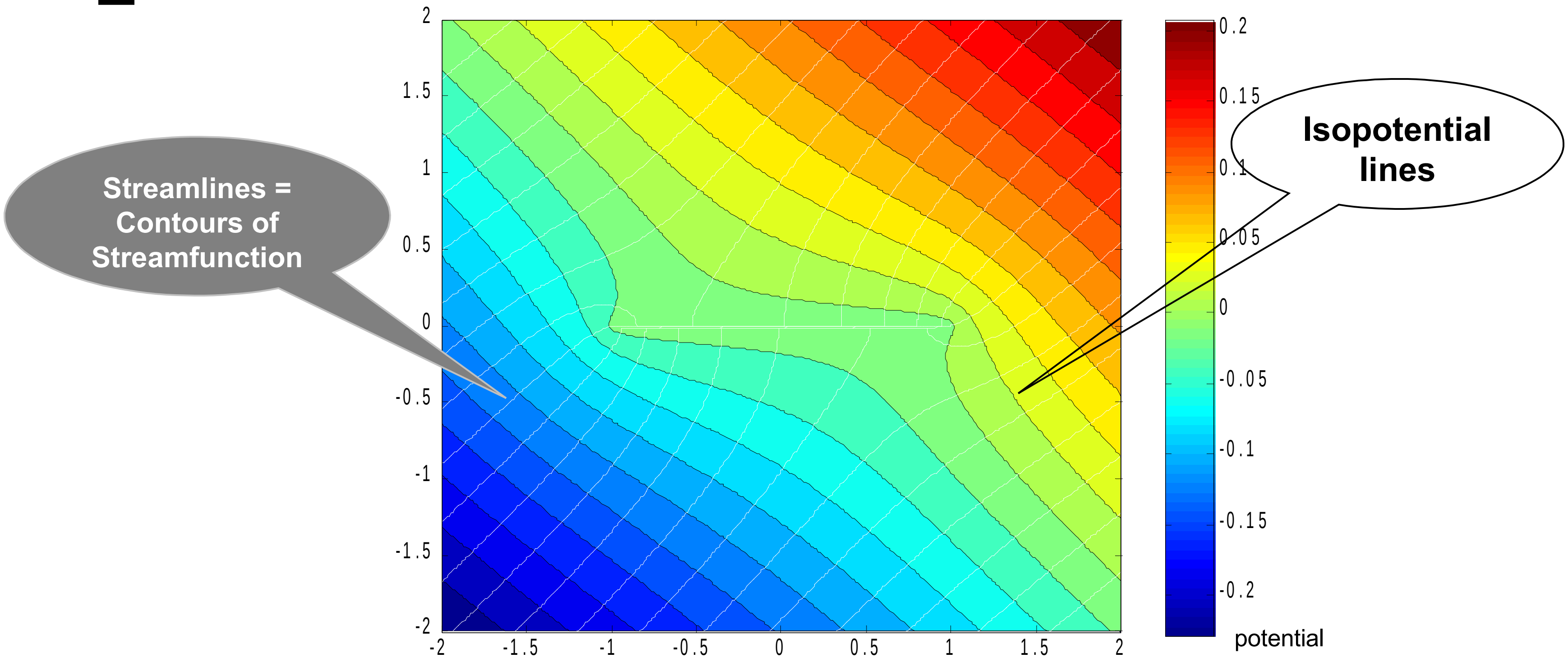
Complex potential contains real potential φ in real part and streamfuntion Ψ in imaginary part

Modification for a highly permeable fracture:

$$\bar{\Phi}(z) = -i\Phi_o(z\cos(\alpha) - i\sqrt{z^2 - a^2}\sin(\alpha))$$

See also: Sato (2003)

MATLAB Visualization

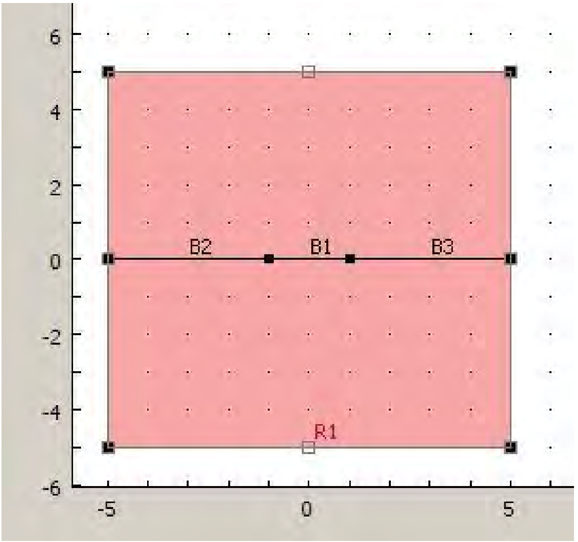


Jump in streamfunction: $\Psi^{+} - \Psi^{-} = \dfrac{K_{high} d}{K_{low}} \dfrac{\partial \varphi}{\partial s}$

Numerical Solution

2D Geometry

❖ (di) total domain: diffusion equation for real potential φ
❖ (di1) upper part: diffusion equation for streamfunction Ψ
❖ (di2) lower part: diffusion equation for streamfunction Ψ



1D Geometry (for lower dimensional case)

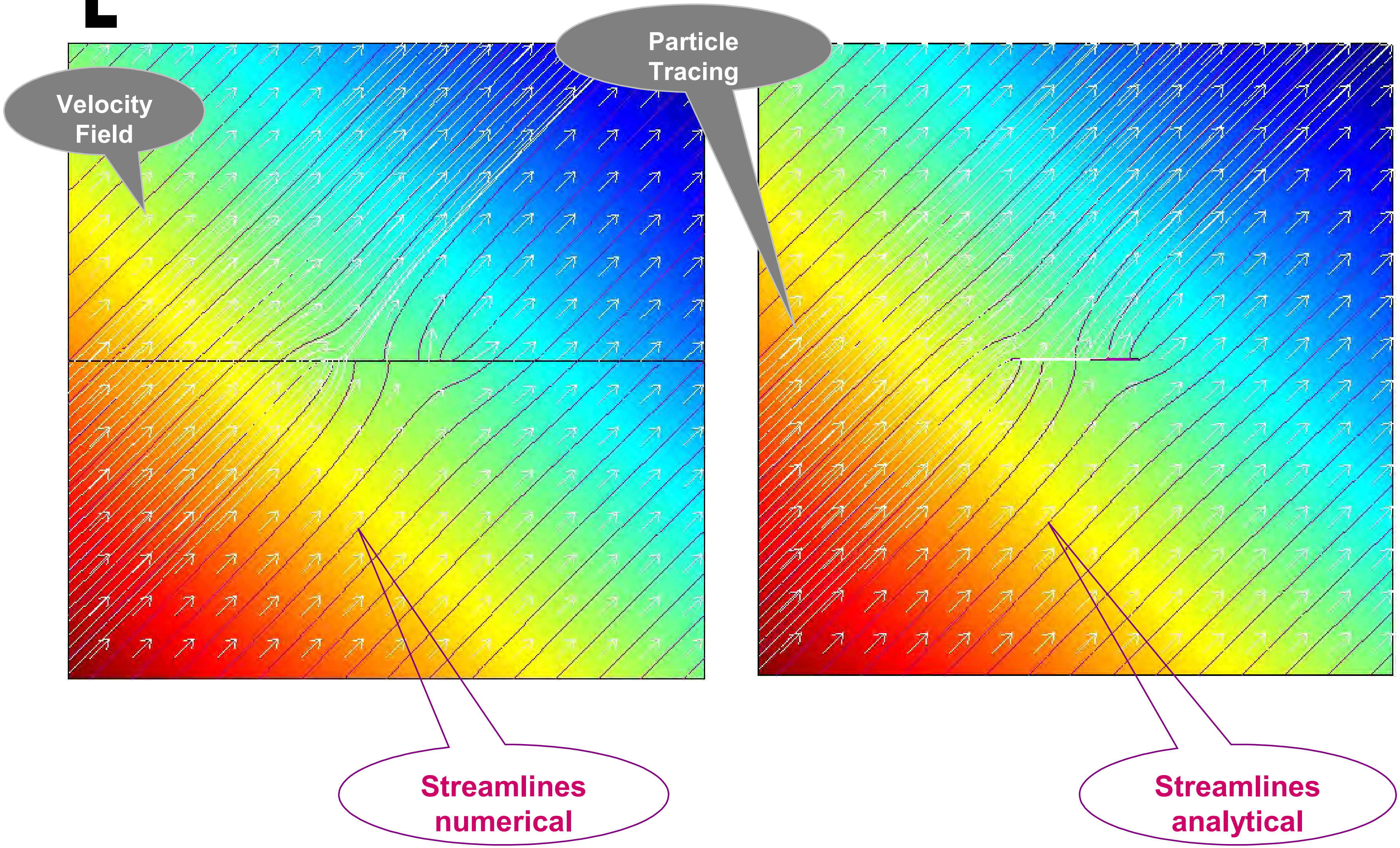
❖ (di0) diffusion equation for real potential φ

Couplings:

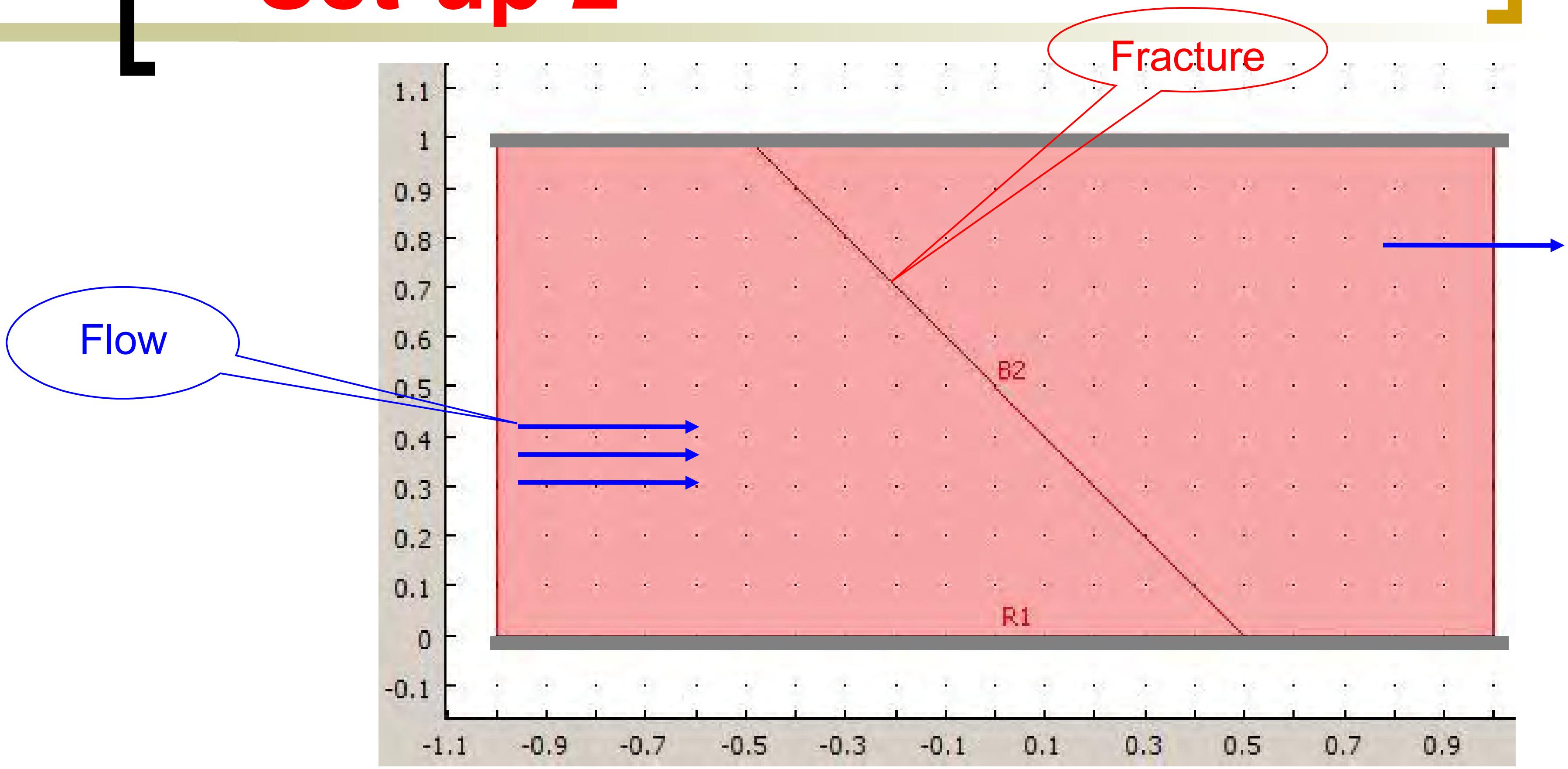
❖ di-di0: solutions identical at fracture (B1)
❖ di1-di2: jump condition at fracture boundary, based on solution of di (B1)
❖ di1-di, di2-di: total flux as boundary condition for Ψ taken from solution of di (boundary integration)

Couplings are introduced using integration and extrusion variables

Set-up 1, Numerical Solution

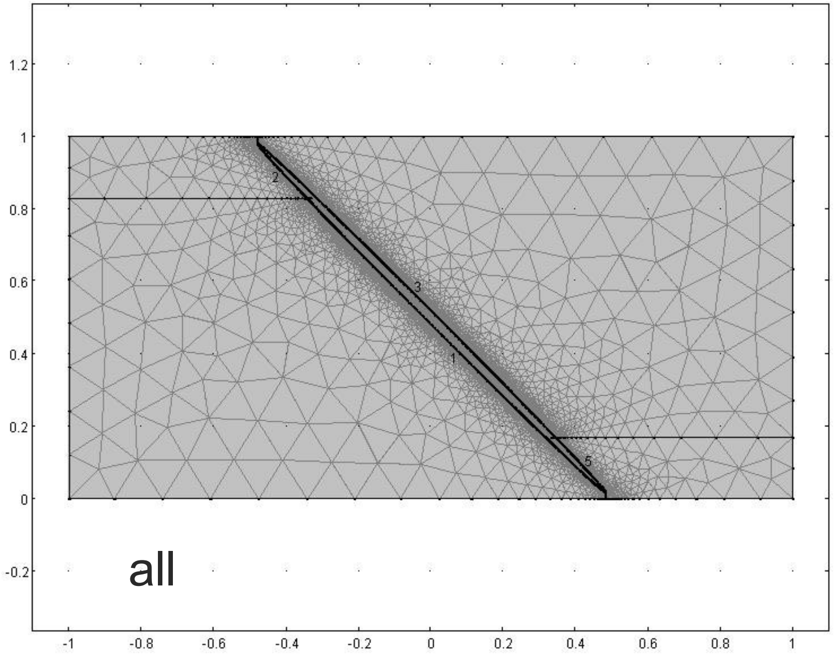


Set-up 2

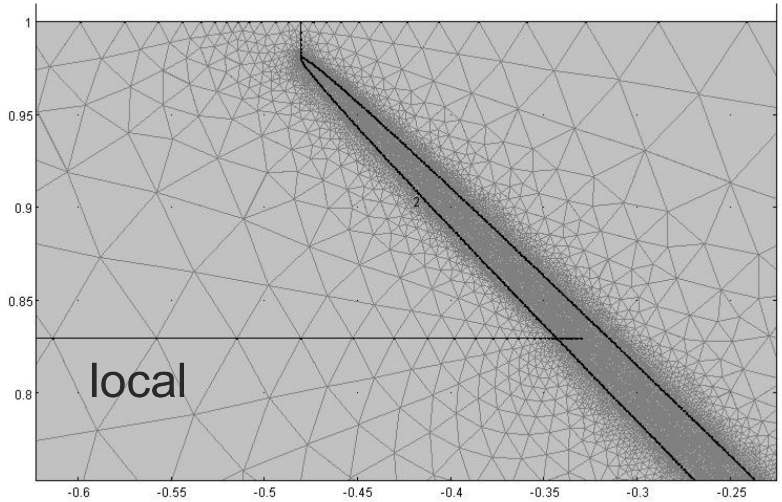


Coupled potential equations for (real) potential and streamfunction

Meshing

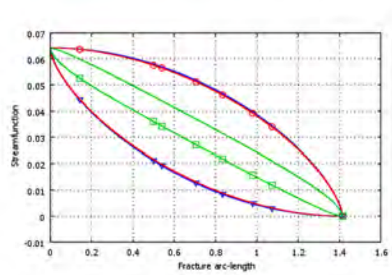


for 2D full-dimensional elliptic fracture
with half-axes ratio 1/400

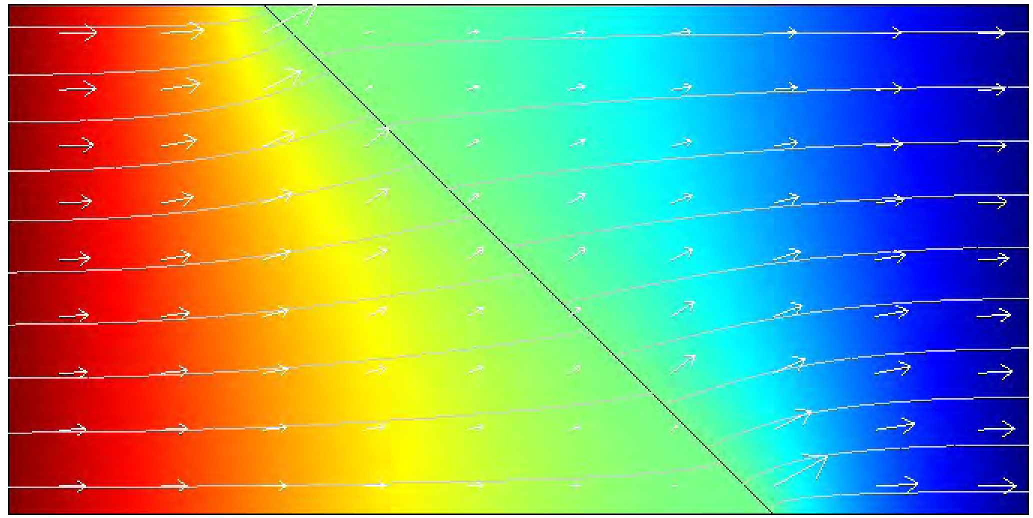


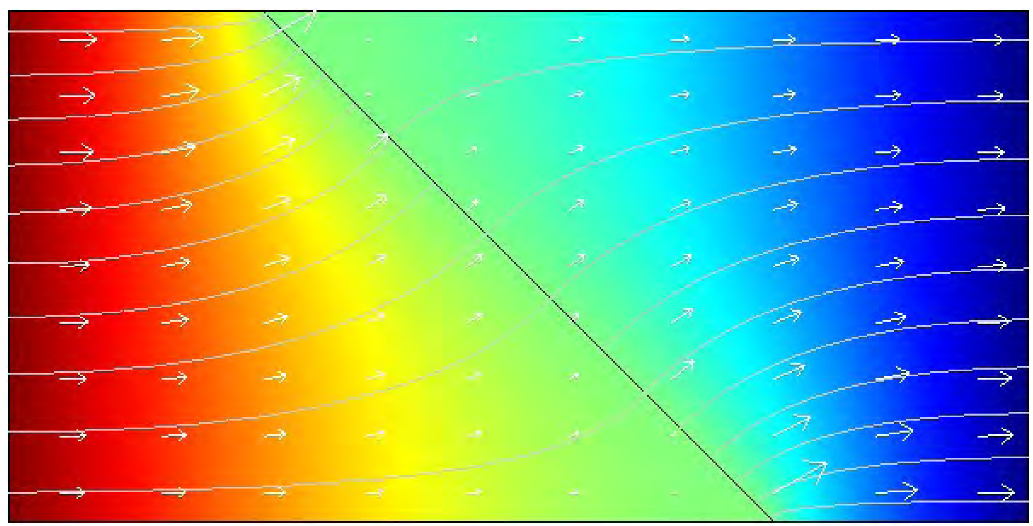
Results; Variation of K_{ratio}

Angle: 45°
Width: 0.01
K_{ratio} : 100 (top)
and 10000 (bottom)



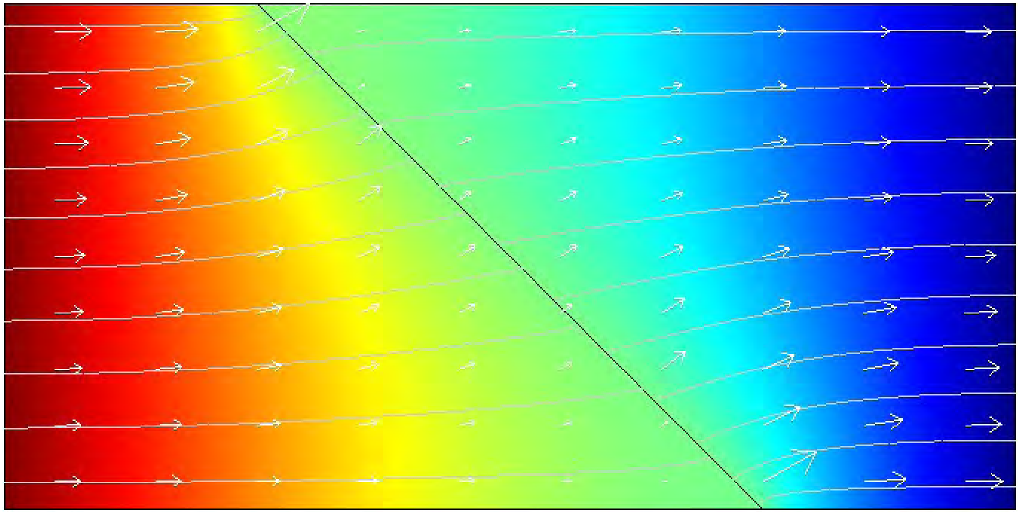
1D lower-dimensional fracture





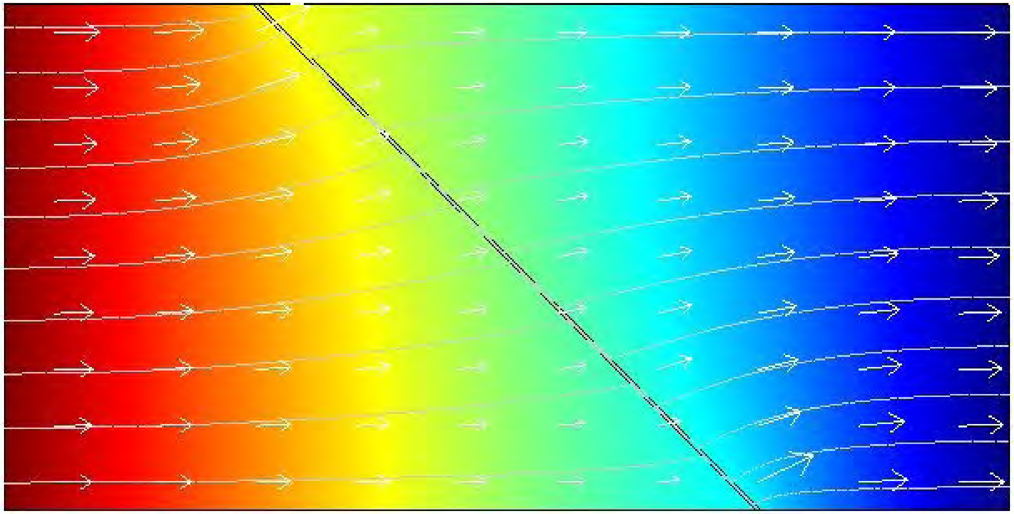
Comparison: 1D and 2D model approach for fracture

1D



Angle: 45°
K_{ratio} : 100
Width: 0.01

2D



- Colour for (real) potential
- Streamlines from streamfunction
- Arrows from potential gradient

Comparison: Performance 3.4

Fracture Dimension	$K_{ratio} = K_{high}/K_{low}$	# DOF	# elements	# it.	Exec. Time (s)
1D	100	155486	38048	5	69-112-120-172
2D	100	321176	80229	3	222-257-260-341
1D	10000	155486	38048	13	129-156-218-411
2D	10000	321176	80229	3	121-177-197-329

Free mesh: normal
Maximum meshsize in fracture: 0.001
Starting from initial
Solvers: direct Spooles (linear), damped Newton (nonlinear)
Required accuracy: 10^{-6}

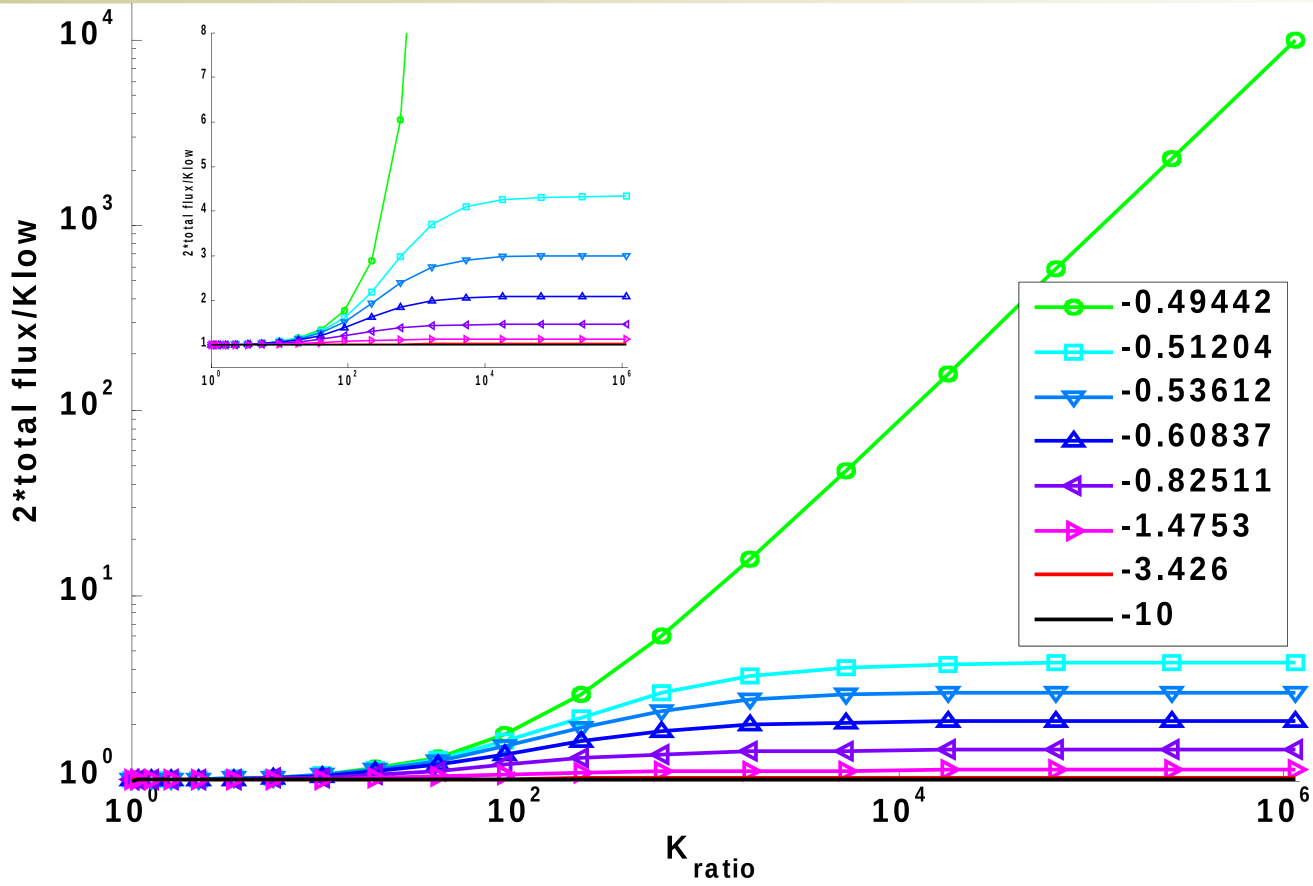
Comparison: Performance 3.5a

Fracture Dimension	K_{ratio} $=K_{high}/K_{low}$	# DOF	# elements	# it.	Exec. Time (s)*
1D	100	155486	38048	5	17.8
2D	100	321176	80229	3	27.8
1D	10000	155486	38048	32	104
2D	10000	321176	80229	3	26.9

Free mesh: normal
Maximum meshsize in fracture: 0.001
Starting from initial
Solvers: direct Spooles (linear), damped Newton (nonlinear)
Required accuracy: 10^{-6}

* mean from 4 runs

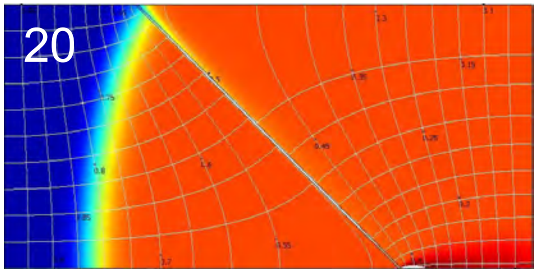
Evaluation Set-up 2

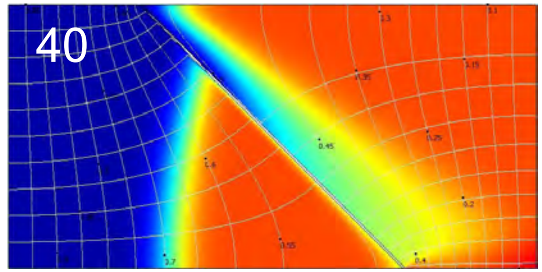


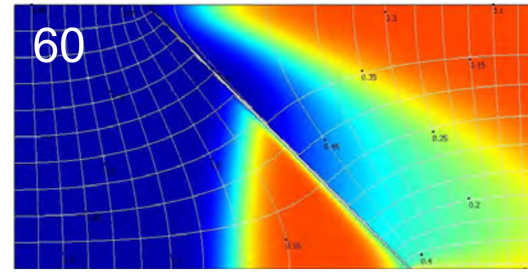
Increased Flux – compared to the no-fracture situation

in dependence of K_{ratio} and fracture angle (in legend given as slope)

Animation of Heat Transfer

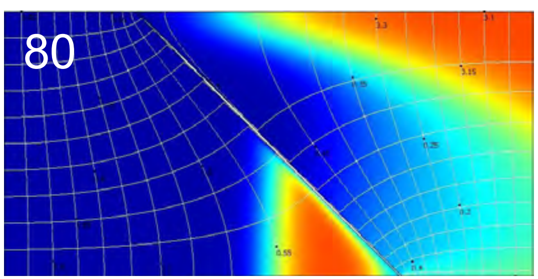


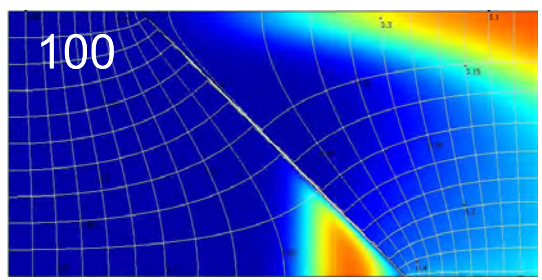


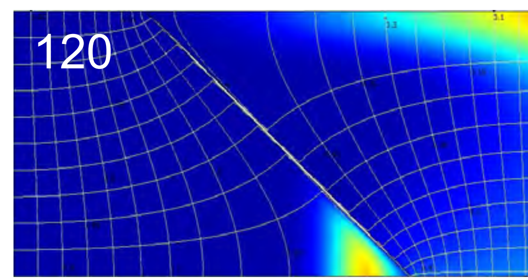


Cold water replacing hot water from the left side

Times: 20, 40, 60 (top); 80, 100, 120 (bottom)







obtained with 2D fracture representation with constant width

Conclusions

- For lower-dimensional fracture representations streamlines through fractures can not be obtained by particle tracking from the (real) potential solution
- Streamlines can be obtained by either using a full-dimensional approach or using the lower-dimensional streamfunction with jump condition at the fractures

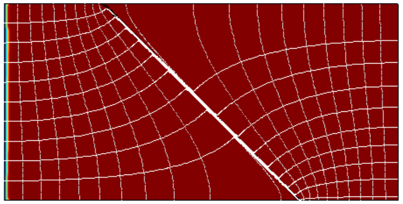
⇨ Execution time of 2D approach, despite of higher DOF, is smaller and this advantage is more pronounced for finer meshes

Conclusions conc. fracture networks from lower dimensional fractures

- Numerical solutions for lower and full-dimensional solutions coincide.
- For single lower-dimensional fracture numerical solutions converge against analytical solution for $K_{ratio} \rightarrow \infty$

⇨ but: analytical solutions can not be combined for fracture networks (even not non-intersecting);

⇨ for numerical solutions, including streamfunction, the entire model region has to be sub-divided in simply connected sub-regions.



Merci beaucoup