Super-resolution Properties of the Maxwell Fish-Eye

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Outline

1. Introduction
2. Spherical Geodesic Waveguide
3. Simulations in COMSOL Multiphysics
4. Conclusions
Maxwell fish-eye

\[ n(\rho) = \frac{2}{1 + (\rho/a)^2} \quad (\rho^2 = x^2 + y^2) \]

All Geometrical Optics rays from point P are perfectly imaged on point Q in Geometrical Optics. J.C. Maxwell 1854
Maxwell fish-eye

\[ n(\rho) = \frac{2}{1 + (\rho/a)^2} \]

\[ \rho > a, \quad n(\rho) < 1 \]
Maxwell fish-eye

What happens in Wave Optics?


\[ E = \frac{P_v(-\cos \theta) - e^{i\nu \pi} P_v(\cos \theta)}{4 \sin(\nu \pi)} \]

\[ \nu(\nu + 1) = (ak_0)^2 \]
Experimental demonstration of $\lambda/5$ super-resolution

Super-resolution stands for the capacity of an optical system to resolve below by the diffraction limit.

[Yungui Ma, Singapore]
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Spherical Geodesic Waveguide

Cylindrical 3D MFE lens

\[ n(\rho) = \frac{2}{1 + (\rho/a)^2} \]

Each \( z = \text{constant} \) plane is mapped on a different sphere via stereographic projections.

Material with spherical symmetry

\[ n(r) = \frac{a}{r} \]

\[ r = \sqrt{x'^2 + y'^2 + z'^2} \]

[J.C. Miñano, P. Benítez, J.C. González, NJP, 12 (2010)]
Spherical Geodesic Waveguide

\[ D_e = 10 \text{ mm} \quad D_i = 5 \text{ mm} \quad L = 20 \text{ mm} \]

\[ R_M = 1005 \text{ mm} \quad R_m = 1000 \text{ mm} \]

\[ n(r)=\frac{R_M}{r} \approx 1 \]
Microwave circuit made up of the two ports and the spherical waveguide

When \( V_d^+ = 0, I_d^+ = 0 \)

\[
\begin{bmatrix}
V_s^- \\
V_d^-
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
V_s^+ \\
V_d^+_s
\end{bmatrix}
\]

\[
V_d^- = S_{21} V_s^+ \\
V_s^- = S_{11} V_s^+
\]

\[
P_I = \frac{1}{2} \frac{|V_s^+|^2}{Z_o} \\
P_T = P_I |S_{21}|^2 \\
P_R = P_I (1 - |S_{21}|^2)
\]
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Meshing in Comsol
Transmitted power for different frequencies

Source

Drain (passive)

$Z_0$

Graph showing transmitted power for different frequencies.
Transmitted power for different frequencies

$\theta = 2^\circ, \ l = \lambda/33$
$\lambda = 1.15\ m$
Simulation of $\lambda/500$ super-resolution
Simulation of $\lambda/500$ super-resolution
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Conclusions

- Super-resolution properties of the Maxwell Fish-eye are analyzed using Spherical Geodesic Waveguide (SGW).
- Simulations of the SGW show super-resolution up to $\lambda /500$ at microwave frequencies.
- The super-resolution is achieved using an approximate model of the SGW (the model having $n=1$ inside the waveguide) convenient for manufacturing.