Coupling Hydrodynamics and Geophysics with COMSOL Multiphysics: First Approach and Application to Leachate Injection in Municipal Waste Landfills

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Abstract:

Electrical Resistivity Tomography (ERT) is increasingly used to investigate moisture distribution in Municipal Waste Landfills, especially when leachate is injected in waste to enhance its biodegradation. It is proposed here to theoretically assess the quality of ERT mapping, using COMSOL Multiphysics' simulations of both hydrodynamical and geophysical aspects. Leachate injection is simulated using COMSOL's Porous Media and Subsurface Flow module which produces the effective saturation distribution. Virtual current injection following ERT principles is then performed on this variably saturated domain using COMSOL's AC/DC module, to produce the electric potential field and to derive the apparent resistivity distribution. The apparent resistivity data set is then inverted with a specific code to reconstruct a calculated resistivity model, which is finally compared to the initial field of effective saturation. Notable distortions are evidenced, especially in the lower part of the so-called saturation bulb.

Keywords: hydrodynamics, Richard's equation, geophysics, electric resistivity, municipal waste, landfill, leachate, simulation.

1. Introduction

In France, even if the recycling of municipal waste is improving, landfill remains a major tool for waste management. In the last decade, the landfill bioreactor concept has emerged. It aims at enhancing the kinetics of waste biodegradation by leachate recirculation. The efficiency of this technique depends on a homogeneous distribution of leachate in the waste body. Therefore, optimisation of leachate injection systems is a challenging issue for operators. Most studies have shown that surface Electrical Resistivity Tomography (ERT) can be a suitable method to study moisture distribution (2D and 3D)(Moreau, Courant et al. 2007; Clément, Oxarango et al. 2009; Clément, Oxarango et al. 2011). But resistivity inversion models used to date are based on limiting hypotheses which induce distortions in mapping electrical resistivity (Clement, Descloitres et al. 2009).

The main goal of this paper is to estimate the accuracy of the 2D ERT method. Comsol Multiphysics is used to perform coupled numerical simulations of leachate flow and electrical processes occurring during a 2D ERT experiment. The resulting sets of synthetic data are then interpretation (inverted) using Bert software package. Finally the maps of initial and inverted electrical resistivity are compared.

2. Use of COMSOL Multiphysics

COMSOL Multiphysics 4.1 has been used to evaluate distortions in ERT mapping, following two steps:
- Step 1: to simulate leachate injection and unsaturated flow in waste using Richard's model,
- Step 2: to simulate electrical resistivity measurements and inversion (mapping) of the water content field obtained in step 1.

2.1 Leachate injection simulation

Hydrodynamic modelling relies on the resolution of Richard's equation. COMSOL Multiphysics' "Porous Media and Subsurface Flow" module solves a specific form of Richard's equation (Richards, 1931), with water pressure as the dependant variable.

Richard's equation is completed with Mualem-Van Genuchten's retention model which expresses the relationship between water pressure and the effective saturation (retention properties), as well as between relative permeability and the effective saturation.
Model parameters for municipal solid waste are set within literature limits (see table 1).

<table>
<thead>
<tr>
<th>Porous medium</th>
<th>$K_s$ (m.s$^{-1}$)</th>
<th>$\alpha$ (m$^{-1}$)</th>
<th>n</th>
<th>$\theta_s$</th>
<th>$\theta_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical sand</td>
<td>$10^9$</td>
<td>4</td>
<td>2.5</td>
<td>0.4</td>
<td>0.04</td>
</tr>
<tr>
<td>Typical clay</td>
<td>$10^8$</td>
<td>5</td>
<td>1.5</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>MSW (Zardava and Powrie, 2009)</td>
<td>$5.10^9$</td>
<td>1.66</td>
<td>1.5</td>
<td>0.35</td>
<td>0.034</td>
</tr>
<tr>
<td>MSW, present model</td>
<td>horiz.:</td>
<td>2</td>
<td>1.5</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>vert.:</td>
<td>$5.10^6$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 1.** Typical Van Genuchten’s model parameters from literature, for soils and Municipal Solid Waste (MSW)

Where $K_s$ is the hydraulic conductivity (saturated state), $\alpha$ and $n$ are key parameters of Mualem-Van Genuchten’s constitutive law, $\theta_s$ is the saturated liquid volume fraction and $\theta_r$ is the residual liquid volume fraction.

It has to be underlined that we set the saturated hydraulic conductivity to be orthotropic, i.e. the hydraulic conductivity matrix is diagonal, with a horizontal conductivity ten times larger than the vertical one, which is in full accordance with the compacted, stratified nature of landfilled municipal waste.

The studied domain represents a cross sectional portion of a landfill cell, 20 m wide and 10 m deep, and as been spatially discretized with the unstructured triangular mesh presented in figure 1.

**Boundary conditions are:**
- no flow on lateral vertical boundaries,
- Atmospheric pressure ($p = 0$ Pa) on bottom horizontal boundary, to simulate perfect drainage,
- no flow on top horizontal boundary to simulate impervious landfill cover, except on a central 0.5 m wide strip where the following time-dependant hydraulic head function $h(t)$ is applied to simulate progressive leachate injection:

\[
h(t) = 10^{-7} \cdot e^{-t}
\]

2.2 Electrical resistivity simulation

The ERT method is well described in the literature (Chapellier 2000). It consists in measuring the apparent electrical resistivity of the medium using a large set of distributed electrodes acting alternately as current injection (with two electrodes A and B) and electrical potential measurement points (With two electrodes M and N). An interpreted resistivity model is then derived using an
Electric field distribution is built using the AC/DC module (quasi-stationary electromagnetic field with the theory of electromagnetic field) to evaluate the potential differences induced by the injected current. 48 electrode nodes have been designed using COMSOL's FEM tool (Figure 3).

The resistivity model has been created with electrical insulation at the boundaries and a virtual ground node (point) placed at the centre of the model.

The resistivity value has been derived from the 2D hydrodynamic model. We used a relationship between water content and resistivity proposed by Grellier (2005):

$$\rho = \rho_w \times \theta^{-2.5}$$

where:
- $\rho$ is the resistivity of the media express in ohm.m
- $\rho_w$ is the leachate resistivity (1 ohm.m)
- $\theta$ is the value of water content simulated with Comsol

We simulated the apparent resistivity for each quadripole, using the comsol geo-electrical guide.

**Figure 2.** Simulated leachate injection in municipal solid waste: map of effective saturation obtained using COMSOL Multiphysics 4.1 (Richard's model)
3. Results

Figure 2 shows a typical simulated field of effective saturation resulting from ~11 hours (40000 s) of continuous leachate injection in municipal solid waste with the above mentioned parameters. One can observe a 3.5 m deep, quasi circular saturation bulb.

Inversion procedures aims at inverting the apparent resistivity data set to reconstruct calculated resistivity model. This was performed using the BERT software package (Günther and Rücker 2006), using standard parameters. The 2D inversion model domain is 48 m long, and has a thickness of 15 m. We decided to use unstructured triangular finite element meshes for forward calculation.

Figure 3: ERT measurement of apparent resistivity with different quadripoles for different acquisition levels.

The following parameters have been used in this study: a Gauss-Newton minimisation with isotropic smoothness constraints (no preferential direction) and a constant regularisation parameter $\lambda=30$.

These results (fig. 4) allow us to show that electrical resistivity measurement and inversion induce notable distortions, especially in the lower part of the so-called saturation bulb. These distortions could lead to severe misinterpretation.

4. Conclusions

The paper shows that reliable moisture distribution in waste can be simulated using COMSOL's Richard's equation together with representative parameters.

It is also shown that COMSOL's AC/DC module can reliably simulate field resistivity measurements applied to simulated moisture fields.

This study is a first approach aimed at enhancing interpretation of geophysical measurements applied to the characterization of landfill waste and bioreactor landfill operation.

COMSOL appears to be a well adapted tool for this kind of approach combining hydrodynamic and geophysical modelling.

5. References


Figure 4: Comparison of
(a) Bulk electric resistivity distribution derived from effective saturation field simulated with Comsol’s Porous Media and Subsurface Flow module (see fig. 2) and using equation 3.
(b) Distribution of apparent electric resistivity simulated with Comsol’s AC/DC module, from above bulk resistivity distribution.
(c) Reconstructed field of electric resistivity using BERT inversion software, from above apparent electric resistivity field.

One can observe notable distortions between (a) and (b) which should ideally correspond. Distortions are flagrant in the lower part of the so-called saturation bulb.