A Flow Induced Vertical Thermoelectric Generator and Its Simulation Using COMSOL Multiphysics

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Abstract: In this study, a new thermoelectric harvester with fluid flow for increased performance. The thermoelectric generator is 3-D vertical configuration with p- and n-doped Silicon thermolegs. There is water flow between channels integrated through the thermoelectric columns, providing forced convection on the heat flow path. By this method, the temperature difference between the hot and cold junctions of the generator is highly increased, resulting in an increased voltage and power output. The Thermoelectric and Navier-Stokes equations are coupled in this work to obtain a whole performance profile of the thermoelectric energy generator.

Keywords: vibration energy harvesting, electromagnetics, MEMS, energy scavenging, coupled physics

1. Introduction

Energy harvesting from heat has been used in a wide range of applications including industry, aerospace and automotive for many decades. However, thermoelectric energy harvesting at miniaturized scales has been popular in recent years. There is a need for scaled down generators for specific applications like notebooks, smart phones, watches or other hand held devices. One of the most outstanding examples of thermoelectric energy harvesting in small scale is the thermoelectric generator for Seiko thermic watch [1]. It that specific study, BiTe thermoelectric material to supply all the power that the Seiko thermic watch needed.

We present a new vertical thermoelectric generator which can be used for energy harnessing in hot spots of PC’s. Especially in water-cooled systems, the flow can be introduced through the columns of the generator, dramatically increasing the voltage and power output.

2. The Vertical TE Generator Design with Flow Channels and Theory

The thermoelectric generators are used to convert heat into electricity by the use of Seebeck effect. The Seebeck effect is shown in Figure 1. In thermoelectric materials, a Seebeck voltage occurs between the ends of a thermocouple when a temperature difference is induced between the junctions.

![Figure 1. Seebeck effect on a thermocouple.](image)

In Figure 1, $Q_h$ is the input heat, $\Delta T$ is the temperature difference between junctions due to heat supplied, and $V_{oc}$ is the open-circuit voltage generated by the Seebeck effect.

The Seebeck voltage generated by a thermoelectric generator is calculated by (1).

$$V_{oc} = n \cdot \alpha \cdot \Delta T$$  

(1)

Where $n$ is the number of thermocouples, $\alpha$ is the combined seebeck coefficient and $\Delta T$ is the temperature difference between the hot and cold junction.

When the thermoelectric generator is made to drive an electrical load, the electrical power generated is calculated as in (2).

$$P = \frac{V_{oc}^2}{2(R_{te} + R_{load})}$$  

(2)

Here $R_{te}$ is the internal thermoelectric resistance, and $R_{load}$ is the electrical resistance of the load. The thermoelectric generators are mostly designed to achieve maximum efficiency or maximum electrical power. For maximum electrical power, the load resistance should be...
equal to thermocouple resistance \( R_{te} = R_{load} \). On the other hand, for maximum efficiency
\[
R_{load} = R_{te}\sqrt{1 + ZT_H} \quad [2].
\]
Here \( Z \) is the thermoelectric figure of merit and \( T_H \) is the absolute temperature of the hot junction.

3. The Vertical TE Generator Design and Calculations

The thermoelectric generators proposed consist of p- and n-doped silicon vertical legs as the p- and n-type thermoelectric material. In systems where the dimensions are limited in horizontal and vertical dimensions, these miniaturized thermoelectric energy converters are used to convert waste heat into electricity. There are fundamentally two designs introduced in this work. In the first design only natural convection of air occurs between the thermocouples, whereas in the second design there is water cooling between the thermocouples. The two designs have exactly the same geometric properties. The thermoelectric generator with intermediate water channels is shown in Figure 2.

![Water Flow](Image)

**Figure 2.** Vertical TE generator design with flow channels.

The thermoelectric generator is 1 mm x 1 mm with varying thermoelectric material thickness between 10 µm and 100 µm. The cross section area of the thermoelectric columns is 50 µm x 50 µm. The thermoelectric dimensions are chosen considering fabrication availability. The thickness of the heat source and sink can be adjusted to fit system limitations. In this design, one side is the heat source (top-in this configuration) and one side is the heat sink (bottom). The bottom can be attached to a component (a larger heat sink).

The thermal conductivity of the whole system is the main consideration calculating the temperature between hot and cold junctions. Although the top of the heat source and bottom of the heat sink are tried to be kept at constant \( T \) in most configurations, the temperature difference in the generator section (through thermolegs) varies with the geometry and materials used.

For the first design, the heat source is kept at 310 K and the bottom of the heat sink is kept at 300 K. Apart from conduction through the thermocouples, there is natural air convection from the thermocouple surfaces. The rate of heat transferred to the surrounding by convection is calculated by (3).

\[
\dot{Q}_{conv} = h \cdot A \cdot (T_s - T_\infty) \quad (3)
\]

Where \( h \) is the convection coefficient, \( A \) is the area where convective heat transfer occurs, \( T_s \) is the surface temperature and \( T_\infty \) is the ambient temperature.

The temperature profile on the thermoelectric generator for the 1st design is shown in Figure 3. The temperature difference between the hot and cold junctions is 3.04 K. The Seebeck voltage generated in 1mm² area with this temperature profile is 6.56 mV, as shown in Figure 4. The details of the voltage calculation are given in the following section.

![Temperature Profile](Image)

**Figure 3.** Temperature profile of the vertical TE generator with natural convection of air.

![Voltage Generated](Image)

**Figure 4.** The voltage generated from the air-cooled vertical TE generator.

In the second design, flow channels pass between the thermoelectric columns to provide...
forced convection cooling along the legs. Therefore, the temperature difference along the generator area will be increased. The temperature difference is calculated as 5.26 K as shown in Figure 5. The corresponding Seebeck voltage in this case is 14 mV.

![Figure 5. Temperature profile of the water cooled TE generator.](image-url)

In forced convection, it can be said that increasing the overall flow rate $u$ will increase the Nusselt number, thus increasing $h$ value considering (4) [3].

$$\bar{h} = \frac{Nu_k k}{L} \tag{4}$$

Thus, we can conclude that increasing the $h$ will result in larger heat loss along the thermoelectric column and a larger temperature difference between junctions will be created. At steady state, the overall heat transfer equation along the subdomains is given in (5).

$$\nabla \cdot (-k \nabla T) = Q + q_s T - \rho C_p u \cdot \nabla T \tag{5}$$

Where the term on the left hand side is the conduction term, $Q$ is the heat source, $q_s$ is the production/absorption coefficient, $\rho$ is the density of the medium, $C_p$ is the heat capacity and $u$ is the flow field.

### 4. Coupled physics simulation of the MEMS Thermoelectric Energy Harvester using COMSOL Multiphysics

The voltage simulations of the flow induced thermoelectric energy harvester are done by coupling Coefficient Form PDE mode and Navier-Stokes modes together. The adaptation of Coefficient Form PDE mode to Thermolectric is done by using the algorithm reported in [4]. The heat source is heated to 310 K and the heat sink bottom is kept at 300 K. Additionally, there is water flow with an inlet velocity of 2 m/s, which requires the addition of Navier-Stokes mode.

The PDE formulation for Navier-Stokes flow can be expressed as in (6). The equations governing Weakly Compressible Navier-Stokes Flow can be found in [5].

$$\rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u = \nabla \cdot \left(-p I + \eta (\nabla u + (\nabla u)^T) - \left(\frac{2\eta}{3} - \kappa_{dv}\right)(\nabla \cdot u)\right) + F \tag{6}$$

Here, $\rho$: fluid density, kg/m$^3$

$u$: velocity field, m/s

$p$: pressure, Pa

$I$: identity matrix

$\eta$: dynamic viscosity, Pa.s

$\kappa_{dv}$: dilatational viscosity, Pa.s

$F$: volume force field, kg/m$^3$

The equations expressing the physics involved in thermoelectric generation are the heat flow equation (5) and continuity of electric charge (7) [6].

$$\partial \cdot \left( j + \frac{\partial D}{\partial t} \right) = 0 \tag{7}$$

Here, $q$: heat flux vector, W/m$^2$

$T$: absolute temperature, K

$[\Pi]$: Peltier coefficient matrix, V

$[J]$: electric current density vector, A/m$^2$

$[\Lambda]$: thermal conductivity matrix,

$[\sigma]$: electrical conductivity matrix, S/m

$E$: electric field density vector, V/m

$[\alpha]$: Seebeck coefficient matrix, V/K

$[\epsilon]$: dielectric permittivity matrix, F/m

The constitutive equations involving Seebeck, Peltier and Thomson effects are expressed as (8)-(10). These equations are used to couple (5) and (7).

$$q = [\Pi] \cdot J - [\Lambda] \cdot \nabla T \tag{8}$$

$$J = [\sigma] \cdot (E - [\alpha] \cdot \nabla T) \tag{9}$$

$$D = [\epsilon] \cdot E \tag{10}$$
Once the temperature profile with the water flow is simulated to obtain the result shown in Figure 5, all of the modules are solved together with parametric segregated solver. The voltage profile of the thermoelectric energy harvester is shown in Figure 6.

![Figure 6](image)

Figure 6. The voltage generated from water cooled TE generator in 1mm² area.

For benchmarking, the designs with and without water flow are compared. With flow induced design, an electrical power of 4.2 mW can be obtained from 1mm x 1mm device area. The flow induced model proved to generate 350% more electrical power than the conventional design. The power obtained with varying thermocouple thickness is shown in Figure 7.

![Figure 7](image)

Figure 7. The generated power levels of flow induced TE generator, and TE generator without fluid flow.

5. Conclusions

A new flow induced vertical thermoelectric energy harvester and finite element analysis for performance was introduced. The thermoelectric performance was analyzed coupling the Heat Transfer, Navier-Stokes and Coefficient Form PDE modes in COMSOL Multiphysics together. The physics involved in thermoelectric energy conversion was integrated to PDE form as matrices. An electrical power up to 4.2 mW can be obtained from 1cm² thermoelectric device area. Compared to the same design without fluid flow, this novel flow induced TE energy harvester can generate up to 3.5 times the electrical power.

6. References