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Shape Optimization of Electric and Magnetic System using Level Set Technique and Sensitivity Analysis

Y. S. Kim*, A. Weddemann, J. Jadidian, S. R. Khushrushahi, M. Zahn
Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology
*Corresponding author: 77 Massachusetts Ave., Cambridge, MA 02139 USA, youngsun@mit.edu

Abstract: This paper aims to propose a level set method for topology optimization of an electromagnetic system. The classical shape optimization method has a meshing problem for shape changes. The level set method is employed to overcome this difficulty, due to its efficient representation of evolving geometry. The velocity field is required to solve the level set equation of the Hamilton-Jacobi equation. It is obtained using the continuum shape sensitivity in a closed form by the material derivative concept. The optimization problem is modeled as a coupled system of Poisson's equation and the level set equation. They are solved using a standard FEM in the time domain. The design goal is to obtain the maximum torque for an operating electrostatic actuator and synchronous reluctance motor (SynRM) respectively.

Keywords: Optimization, level set method, FEM, sensitivity analysis.

1. Introduction

The classical optimization method has been applied to many design problems for electromagnetic systems. One of its major difficulties is related to meshing problems arising from shape modifications. Several researches have tried to formulate shape optimization with fixed mesh analyses based on fixed grid finite elements to circumvent these kinds of technical difficulties with moving mesh problems. This approach was naturally associated with the level set description of geometry to provide an efficient treatment of problems involving geometry changes and discontinuities. The level set method is a numerical technique first developed to track moving interfaces. It was first devised by Osher and Sethian [1] and has been recently introduced to the field of structure shape optimization [2, 3].

The level set method has several advantages. Its main advantage is that it enables an accurate description of the boundaries on a fixed mesh. Therefore, it provides us with fast and efficient numerical algorithms. It can also handle topological changes, since it allows boundaries to naturally split or merge without using additional techniques, by controlling the level-set function of the Hamilton–Jacobi equation. Such treatment of topological changes can transform a difficult topology optimization problem into a relatively easier shape optimization problem. In this paper, we apply the level set method to topology optimization of an electrostatic actuator and synchronous reluctance motor (SynRM) using continuum sensitivity [4-7] for the velocity field that is inserted into the Hamilton–Jacobi equation for the level set function. The goal of the actuator and rotor of SynRM design is to generate maximum torque operation [8]. The numerical algorithm is implemented with a standard finite element procedure.

2. Concept of LSM

We employ the level set method to define evolving boundaries, since it provides a convenient means to describe closed interfaces of curves and surfaces. The level set function is expressed in the implicit form of a high-dimensional function; then, the boundary changes are traced by the deformation of this function. The design boundaries are changed in shape optimization to minimize or maximize an objective function. Generally, for a given region Ω with an arbitrary boundary, we assume an implicit function $\phi(\mathbf{x})$, as given by

$$\begin{array}{lll} \phi(\mathbf{x}) > 0 & \mathbf{x} \in \Omega^+ & : \text{free space} \\ \phi(\mathbf{x}) = 0 & \mathbf{x} \in \partial\Omega & : \text{design boundary} \\ \phi(\mathbf{x}) < 0 & \mathbf{x} \in \Omega^- & : \text{material region} \end{array} \tag{1}$$

To compute an evolving domain, we can define the function and determine the evolution of domain Ω^- via

$$\Omega^-(t) = \left\{ \phi(\mathbf{x}, t) < 0 \right\} \tag{2}$$

The boundary $\Gamma(t)$ of $\Omega^-(t)$ is given by the zero level set,

$$\Gamma(t) = \left\{ \phi(\mathbf{x}, t) = 0 \right\} \tag{3}$$

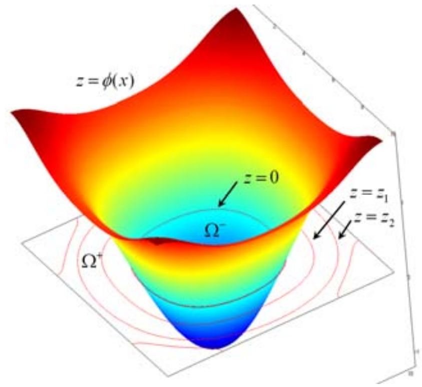


Figure 1. Distribution of level set function and boundary represented by the zero level set($z=0$), where the Ω^- and Ω^+ regions imply the design domain and free space, respectively.

The evolution of the shape is determined by the velocity $\mathbf{V}$.

$$\frac{d\mathbf{x}(t)}{dt}=\mathbf{V}(\mathbf{x}(t),t) \tag{4}$$

Since the zero level set holds at any time t, its total derivative is expressed using an Eulerian formulation and chain rule, as given by

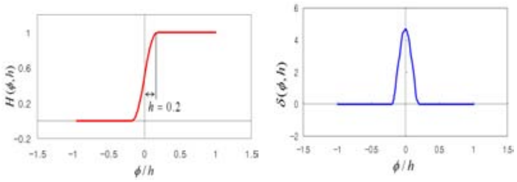
$$\frac{\partial\phi}{\partial t}+V_n\,|\nabla\phi|=0 \tag{5}$$

This is called the Hamilton-Jacobi equation, and the level set function is determined by solving it. The velocity $\mathbf{V}$ must be based on each objective of the design problem.

The boundaries of the design domain can be expressed by the zero level set using the signed distance function and the boolean union function min [1,2].

$$\begin{aligned}&\phi(\mathbf{x})=\min(\mathbf{x}-\mathbf{x}_I)\qquad \text{for all}\quad \mathbf{x}_I\in\Omega\\&\phi(\mathbf{x})=0\quad \text{on the boundary where}\quad \mathbf{x}\in\partial\Omega\end{aligned} \tag{6}$$

It is appropriate to use the Heaviside function to set the material properties of the design region. It is difficult to set the properties strictly for the boundary in numerical processing, because the boundaries can intersect an element. So, we used a numerically smeared Heaviside function, as shown in Figure 2. This provides a continuous material property distribution near the zero level set. Also, it is possible to control the slope of this function using the tuning parameter h.



(a) (b)

Figure 2. Numerically smeared (a) Heaviside and (b) Dirac-Delta functions, for which the function can be controlled by the tunable parameter. This is concerned with element size.

The material property is determined using (7) and is applied to the governing equation of the electrostatic or magnetostatic field, as given by (9).

$$p=p_0H(\phi,h)+(1-H(\phi,h))p_rp_0 \tag{7}$$

$$p(\phi,h)=\begin{cases}p_0 & \phi>0\\ p_rp_0 & \phi\le 0\end{cases} \tag{8}$$

$$\nabla^2V=-\rho_v/p(\phi,h)\quad \text{or}\quad \nabla^2A=-p(\phi,h)J \tag{9}$$

where p denote the material property of electrostatic ε or magnetostatic μ. We can avoid remeshing the model geometry that is changed by moving objects by introducing the level set function to the setting of material properties.

The integration of the surface and boundary integral are used when we calculate the area of a domain or the sensitivity of a boundary. These are expressed using the Heaviside function $H(\phi)$ and the Dirac-Delta function $\delta(\phi)$`.

$$\int_S f(\mathbf{x})\;dS=\int_\Omega f(\mathbf{x})\,H(\phi(\mathbf{x}))\,d\Omega \tag{10}$$

$$\int_\Gamma f(\mathbf{x})\,d\Gamma=\int_\Omega f(\mathbf{x})\,\delta(\phi(\mathbf{x}))\,|\nabla\phi(\mathbf{x})|\,d\Omega \tag{11}$$

3. Velocity field derivation

The total derivative of any objective functions in electromagnetic systems can be derived in a closed form using the material derivative concept of continuum mechanics and an adjoint variable technique. First, the continuum sensitivity formula of switching position A is represented as

$$\frac{dF}{dt}=\int_\gamma G_A(V,\lambda)V_{nA}\,d\Gamma \tag{15}$$

where

$$G_A(V,\lambda) = \varepsilon_0(\varepsilon_r - 1)\left[\varepsilon_r\, E_n(V^*)\, E_n\,(\lambda^*) + E_t(V^*)\, E_t\,(\lambda^*)\right] \text{ or}$$

$$G_A(A,\lambda) = \left(\frac{\mu_r - 1}{\mu_0}\right)\left[\, B_t(\lambda^*)\, B_t\,(A^*) + \frac{1}{\mu_r} B_n(A^*)\, B_n\,(\lambda^*)\right] \tag{16}$$

V_{nA} is the normal component of the velocity field vector, λ is an adjoint variable, and γ is a design boundary. The sensitivity of switching A position represents the relationship between the objective function and the velocity field. If the velocity field is assumed to be

$$V_{nA} = G_A(V,\lambda) \quad \text{: switching A position} \tag{17}$$

the system energy will increase, since it is in a gradient direction. In addition, if the sensitivity of switching B position is chosen as

$$V_{nB} = -G_B(V,\lambda) \quad \text{: switching B position} \tag{18}$$

the system energy will decrease. The velocity field is inserted into the level set equation (5) to obtain the maximum torque of an electromagnetic system.

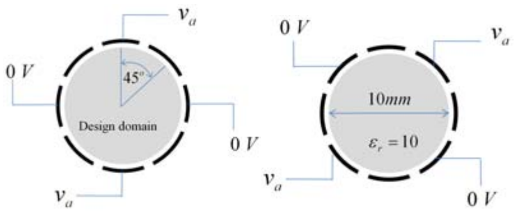
$$\frac{\partial\phi}{\partial t} + [G_A(V,\lambda) - G_B(V,\lambda)]\,|\nabla\phi| = 0 \tag{19}$$

Here, we can see that the optimization procedure is transformed into a Hamilton-Jacobi equation of a time-domain first-order partial differential equation (PDE). The time of the above equation does not mean the real physical time, but it implies a "pseudo time" in the iterative optimization procedure.

4. Optimization problems

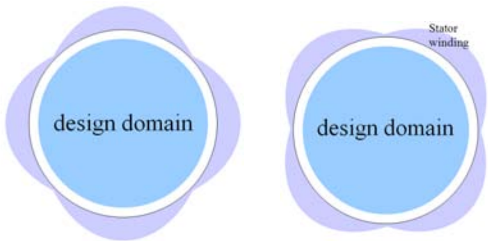
4.1 Description of electromagnetic system

Figure 3 shows a design domain for an electrostatic actuator structure with 8 electrodes around dielectric material represented by the gray-colored central region. This is driven by switching the voltages in the clockwise direction and adjacent electrodes are composed of separated segments. The dielectric material for the design domain will be redistributed to maximize the objective function in the optimization process. The maximum torque is generated by the maximum difference of the system energy between the two switching patterns shown in Figure 3(a) and (b). That is, the shape of the dielectric material is optimized so that the system energy represented by Figure 3(a) is maximized and that represented by Figure 3(b) is minimized. Figure 4 shows a design domain for the rotor shape and a distribution of stator windings for a sinusoidal magnetomotive force (MMF) at two positions. The material on the design domain is to be redistributed to maximize the objective function in the optimization process. A maximum torque is generated by a maximum difference of the system energy between the two positions of Figure 4(a) and (b) [8].



(a) (b)

Figure 3. Actuator information and voltage switching pattern of clockwise rotation. (a) Switching position A is used to maximize the system energy and (b) switching position B is used to minimize the system energy.



(a) (b)

Figure 4. Sinusoidal distributions of stator winding and a design domain for a rotor with 4 salient poles. (a) Reference position and (b) 45 [deg.] rotated position.

4.2 Optimization scheme

The system energy of switching position A (or reference position) must be maximized and it must be minimized at switching position B (or rotated position) to generate the maximum torque. That is, the energy difference between the two positions is maximized. Therefore, the optimization problem in this case can be defined as

$$\text{Maximize : } F = (W_A - W_B) \tag{20}$$

Subject to $\nabla^2 V = -\rho/\varepsilon(\phi)$ or $\nabla^2 A = -\mu(\phi)\,J$

$\int_\Omega H(\phi)d\Omega = S^*$: constraint of constant area where the design domain is represented by Ω. The Poisson equations for electrostatics and magnetostatic are used to calculate the electric scalar potential V and the magnetic vector potential A.

The design variables represent the movable boundary between the dielectric and free space. The design boundary $\partial\Omega$ is represented using the level set function such that

$$\phi(\mathbf{x},t)=0 \quad \text{: zero level set} \tag{21}$$

The velocity field determines the shape variation. It results in variation of the objective function via Poisson's equation. The velocity field is modified from V_n to $\hat{V}_n$ using a Lagrange multiplier technique to impose a constraint condition of constant volume , such as

$$\hat{V}_n = V_n - V_0 \tag{22}$$

where $V_0 = \int_\gamma V_n \, d\Gamma / \int_\gamma d\Gamma$.

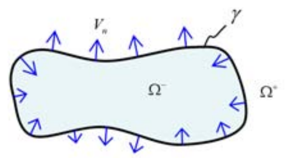


Figure 5. Normal velocity field on the boundary. Its value determines boundary shape.

The algorithm of the level set method for optimization problem is numerically implemented using the finite element method. Generally, the finite difference method has been used for the time-dependent solution for the level set equation. However, we add an artificial diffusion term to the level set equation for numerical stability and usage of the standard finite element method for 2nd order PDE. Thus, the level set equation of first order PDE is transformed into second order PDE as

$$\frac{\partial\phi}{\partial t} + V_n |\nabla\phi| = \alpha\nabla^2\phi \tag{23}$$

where the coefficient α of the artificial diffusion term must be sufficiently small.

5. Numerical examples

5.1 Electrostatic actuator

The difference of the electrostatic system energy between the two voltage switching positions has to be maximized to generate the maximum torque. The numerical results showed that the proposed algorithm produced the optimal shape and topology of the salient pole actuator without prior information of the dielectric layout, as shown in Figure 6. The initial shape of the actuator is given by the 19 dielectric regions that will be topologically modified to generate the maximum torque.

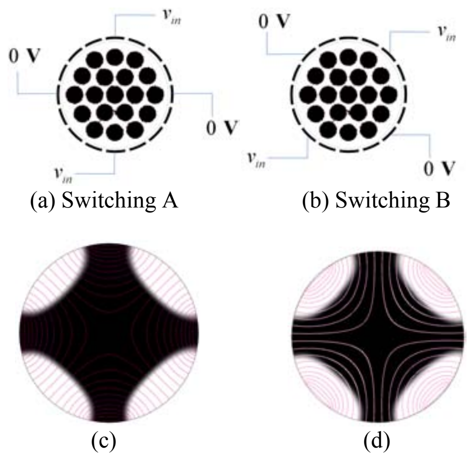


Figure 6. Initial shape of electrostatic actuator containing 19 dielectric regions. Voltage switching position of (a) maximizing and (b) minimizing electrostatic energy. Final shapes and equi-potential line made by (c) switching A and (d) switching B.

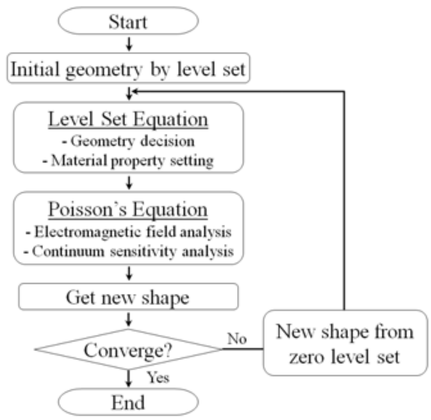


Figure 7. Flowchart of proposed algorithm for optimization of electrostatic actuator.

Figure 7 summarizes the overall flowchart of the proposed algorithm. Figure 8 show the shape variations of the dielectric material in the time domain where we can see that the design shape changes by freely producing various intermediate topologies and shapes. Figure 9 shows the variation of the electrostatic system energy in the iterative optimization procedure. The values converge to a maximum after about 2.7 seconds of pseudo-time. Figure 10 show the distributions of the level set surfaces in the initial and final steps, where the level set value is negative in the dielectric material and positive in air. In addition, computational time and number of iterations are about 450 seconds and 370 steps, until the shape converged.

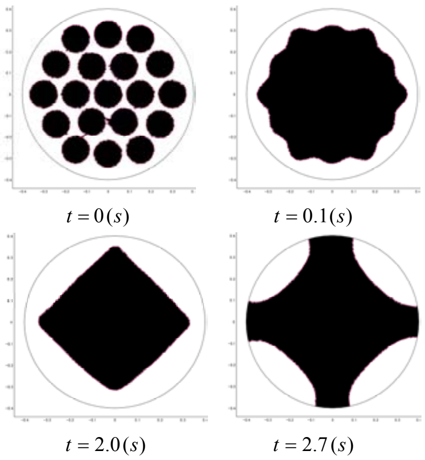


Figure 8. Optimization procedure of electrostatic actuator with material density and zero level set, which imply the material boundary.

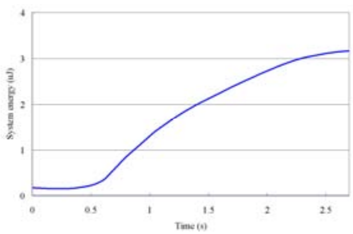


Figure 9. Changes of electrostatic system energy in time domain.

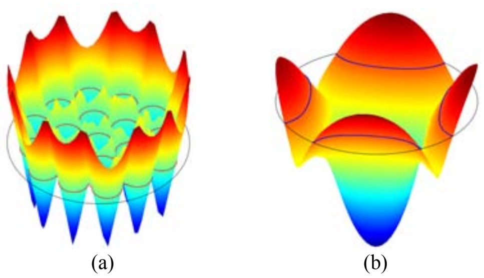


Figure 10. Distributions of level set function and zero level set at (a) initial shape and (b) converged final step.

5.2 SynRM rotor

The difference of the magnetic system energy between two winding positions has to be maximized to generate a maximum torque of a synchronous reluctance motor. The initial shape on the design domain is taken as in Figure 11(a) and (b) for the optimal topology design of the rotor shape. The initial design consists of 25 ferromagnetic cylinder bars, which could produce any free and arbitrary topology and shape and occupies 70% of the area of the design domain. The stator winding is set to be distributed to make a sinusoidal MMF along the air gap as shown in Figure 11(a) and (b). The final design of the rotor shape is obtained as in Figure 11(c) after the optimization process of the coupled system of the level set equation and the magnetic field equation is executed. The final shape of the rotor is unique and matches our predictions of a very symmetric rotor design.

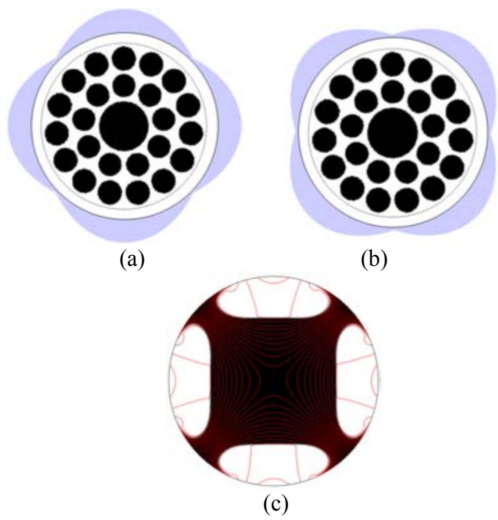


Figure 11. 25 ferromagnetic bars on initial rotor design domain and two winding models in (a) the reference model and (b) the rotated model and (c) the final optimized rotor shape with equi-potential line.

Figure 12 shows the shape changing process in the time domain, where we can see that the design shape changes freely producing various intermediate topologies and shapes. Figures 13 show the distributions of the level set surfaces of the initial and the final steps, where its value is positive in the ferromagnetic material and negative in air.

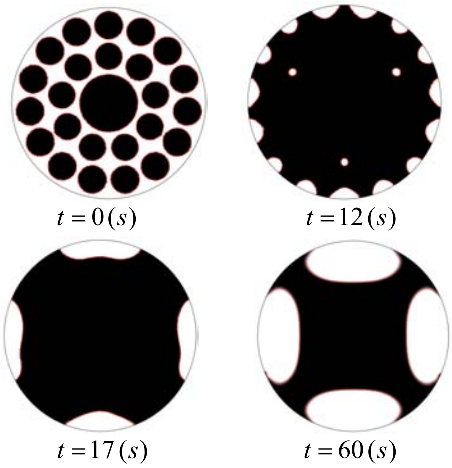
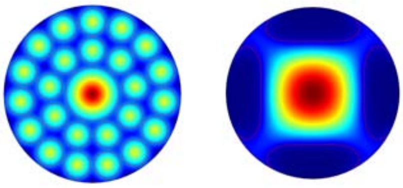


Figure 12. Optimization procedure of 4 pole rotor for synchronous reluctance motor with material density and zero level set.



(a) (b)

Figure 13. Distributions of level set functions at (a) initial shape and (b) converged final step. Here material regions are set as positive value of the function

6. Conclusion

We presented the theoretical algorithm and numerical techniques for topology optimization of an electromagnetic system to generate the maximum torque using the level set method and design sensitivity analysis. The objective function for the maximum torque is defined as the difference of the system energy with respect to the two switching and winding positions. Numerical results showed that the level set method coupled with the finite element method of the electric and magnetic field is a feasible and effective method to design the shape and topology of an electrostatic and magnetostatic system. From numerical analysis of view, this work has some advantages as follow: (1) Because the level set and field equations are solved using finite element code without remesh step for changing shapes, its computational efficiency is not only better than existing technique and it is but also more convenient to apply to the standard finite element analysis. (2) Since the shape design sensitivity of analytical form is employed for the normal velocity term of the level set equation, the optimization process is numerically accurate. (3) In the level set method the design shape is easily and efficiently calculated using the level set function of high order implicit form, whereas the conventional design techniques require definition of design parameters that are complicated and depends on the geometry of a given problem. (4) Because the shape and topology can be freely changed during the optimization procedure, the design domain is larger. Also, we think that optimization with the level set method better tends to converge to the global minimum.

7. References

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