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Assessment of Spatial Variably Saturated Flow by Irrigation Moisture Sensors in 2-Dimensions using COMSOL-Multiphysics 4.1

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Abstract: This paper reports on the application of COMSOL Multiphysics' Richard Equation interface in the assessment of using irrigation moisture sensors for detecting the level of water saturation in a variably partially saturated soil. The Richard Equation (in COMSOL) provides the interface which automates the van Genuchten equation(that contains parameters which account for the Soil Water Retention (SWR) and different pore space geometry). A hypothetical soil column 4 m by 4 m was set up with seven irrigation sensors inserted and spaced at 0.5 m. In other words, the primary aim of this study is to determine and visualize the localized effective saturation distribution around the sensors after various time steps. The simulation was done in both vertical and horizontal orientations. The modeling procedure was simplified using a 2-D cross-section. The meshing was user defined. From the result, comparative analysis was done between the two orientations using the effective saturation, velocity field and pressure head plots. Finally, this study could aid in determining the best orientation and optimal location in placing irrigation sensors and also in sustainable irrigation water management.

Keywords: Richards Equation, van Genuchten, Irrigation Sensors, COMSOL Interface, Effective Saturation

1. Introduction

In sustainable irrigation practice, the knowledge of water movement in the soil is very paramount. Increasingly, irrigation water use efficiency is a major factor to be considered because of competition for water among agricultural, municipal and industrial users which is likely to increase in future (Kandelas et al 2011). Over the past years, many computer models have been developed to predict the flow of water in a saturated and unsaturated porous media (Gardner, 1958; Brooks and Corey, 1964; Clapp and Hornberger, 1978; van Genuchten, 1980; Ghanbarian-Alarijeh *et al* 2010). The applications of these models have always been restricted due to lack of knowledge of the hydraulic properties involving the soil water retention curve (SWRC) (Ghanbarian-Alarijeh et al 2010). The Richards Equation (RE) represents the process of the fluid movement through a saturated or unsaturated porous media. RE is the foundation of all hydrologic models that simulate the movement through permeable materials such as rocks, soils and aquifers (Buchan, 2003; Lee and Abriola, 1999; Chen et al 1993; Clapp and Hornberger, 1978). One of the major challenges using RE is the complexity and non-linearity of its coefficients (Ghanbarian-Alarijeh et al 2010; COMSOL Multiphysics 4.1, 2011). When quantifying the fluid flow in a variably saturated porous media, the main challenge has always been how to illustrate the capacity of the media to transmit and store fluid changes as fluid enter and fill the pore spaces (COMSOL Multiphysics 4.1, 2011). It's always a daunting task to obtain an experimental data which illustrate this process. The foundation work of Andrew Hinnel, Alex Furman and Ty Ferre (Department of Hydrology and Water Resources, University of Arizona) helps to characterize the spatial distribution of moisture changes around three impermeable sensors inserted in two soil columns with different soil properties. Their approach solved this problem using Partial Differential Equation (PDE) interface of COMSOL Multiphysics. In the same vein, the Variable Saturated Flow Model of COMSOL approach this problem using the RE interfaces. However, in this paper, we seek to expand this research problem by quantifying the localization of moisture distribution around seven irrigation sensors inserted in both vertical and horizontal orientations in an homogeneous soil column. The work would be very relevant in determining the best orientation and positioning of irrigation sensors for optimal irrigation water use efficiency and management.

2. Governing Equations

The governing equation for this problem uses the RE which illustrates the flow of water through saturated and unsaturated soil. An important assumption in this application is that RE is only applicable to water since the soil is at atmospheric pressure (COMSOL Multiphysics 4.1, 2011). The following gives the governing equation for the model (COMSOL Multiphysics 4.1, 2011)

$$[C + \mathrm{Se}S]\frac{\partial H_p}{\partial t} + \nabla \cdot [-K\nabla(H_p + D)] = 0 \qquad (1)$$

Where:
H_p = Pressure head [m]
C = Specific Capacity [m^{-1}]
S_e = Effective Saturation
S = Storage Coefficient [m^{-1}]
t = time [s]
K = Hydraulic Conductivity [ms^{-1}]
D = Coordinate (x,y,z) for the vertical elevation [m]

The volumetric fraction of water is θ given by van Genuchten Formula (van Genuchten, 1980):

$$\theta = \begin{cases} \theta_r + \mathrm{Se}(\theta_s - \theta_r) & H_p < 0 \\ \theta_s & H_p \geq 0 \end{cases} \qquad (2)$$

The soil is considered being saturated when the fluid pressure is at atmospheric (H_p=0).

The challenge of non-linear relationships occur when C, S_e and K varies with H_p and θ.

The specific moisture capacity is defined as:

$$C=\delta\theta/\delta H_p \qquad (3)$$

In modeling the storage coefficient, the specific storage option would be used which sets:

$$S=\rho_f g(\chi_p+ \theta\chi_f) \qquad (4)$$

Where:

ρ_f = fluid density [kgm^{-3}]

g = acceleration due to gravity [ms^{-2}],

χ_p = compressibility of solid particle [ms^2kg^{-1}]

χ_f = compressibility for fluid [ms^2kg^{-1}]

In water retention process, van Genuchten (van Genuchten, 1980) and the Brooks and Corey (Brooks and Corey, 1964) formulas both substantiate the variation in C, Se, K and θ with H_p. These are defined as (COMSOL Multiphysics 4.1, 2011):

$$\mathrm{Se} = \begin{cases} \dfrac{1}{[1+|\alpha H_p|^n]^m} & H_p < 0 \\ 1 & H_p \geq 0 \end{cases} \qquad (5)$$

$$C = \begin{cases} \dfrac{\alpha m}{1-m}(\theta_s - \theta_r)\mathrm{Se}^{\frac{1}{m}}\left(1-\mathrm{Se}^{\frac{1}{m}}\right)^m & H_p < 0 \\ 0 & H_p \geq 0 \end{cases} \qquad (6)$$

$$k_r = \begin{cases} \mathrm{Se}^l\left[1-\left(1-\mathrm{Se}^{\frac{1}{m}}\right)^m\right]^2 & H_p < 0 \\ 1 & H_p \geq 0 \end{cases} \qquad (7)$$

Where θ_s and θ_r are saturated and liquid volume factors respectively while n, m, l and α are constants

3. Numerical Model

This paper approach the problem by considering a hypothetical homogeneous soil column with 4 m by 4 m dimension partially saturated with water. The hydraulic and soil properties of this hypothetical soil were adapted from the Variably Saturated Model of the COMSOL Multiphysics 4.1. Within this column seven irrigation sensors were inserted (inform of impermeable rods) which are 0.5 m wide. The pressure head is assumed uniform and as soon as the sensors were inserted, water begin to flow downward inform of a steady drainage (COMSOL Multiphysics 4.1, 2011). For simplicity, a 2-Dimensional geometry was applied and the meshing procedure was also user defined. Vertical and

Horizontal orientation of sensors placement were considered which can be seen in Figures 1 and 2 below.

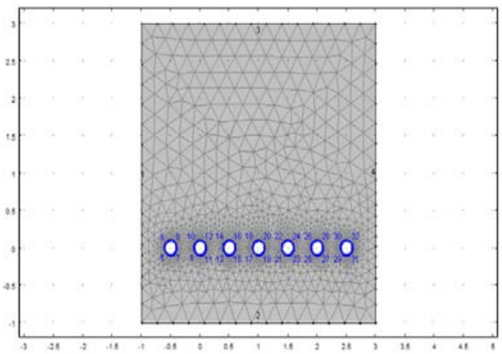

Figure 1: The Meshed Geometry in Horizontal Direction.

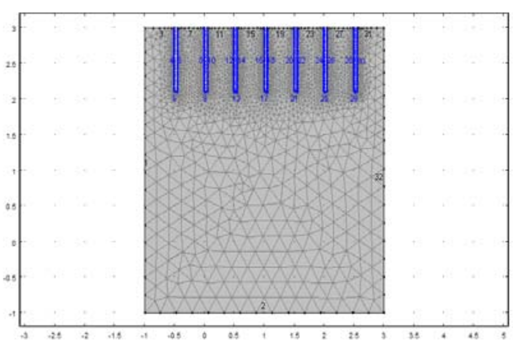

Figure 2: The Meshed Geometry in Vertical Direction.

4. Boundary Condition

The initial boundary conditions need to be defined and specified to solve this problem. This is given as (COMSOL Multiphysics 4.1, 2011):

$$\mathbf{n} \cdot [-\underline{K}\nabla(H_p + D)] \quad \partial\Omega \ \ \text{Sides}$$
$$\mathbf{n} \cdot [-\underline{K}\nabla(H_p + D)] \quad \partial\Omega \ \ \text{Rings}$$
$$H_p = H_{p0} \quad \partial\Omega \ \ \text{Base}$$
$$H_p = H_{p0} \quad \partial\Omega \ \ \text{Surface}$$

Where **n** can be defined as the unit vector normal to the boundary (COMSOL Multiphysics 4.1, 2011)

5. Simulation Results

Figures 3 and 4 show the solutions to the RE problem using COMSOL Multiphysics interface in the two orientations. Each Figure show the effective saturate (as surface plot), fluid velocities (as arrows) and pressure head (as contours) after 7200 seconds. It can be seen that the saturation around the rods (sensors) varies but the saturation through the remaining part of the column is uniform. In Figure 3, the sides of the rods are wetter compared to the lower part. It is observed also in Figure 4 that the upper circumference of the rods is wetter than the lower part. An observation for the two orientations can be seen in Figures 5 and 6.

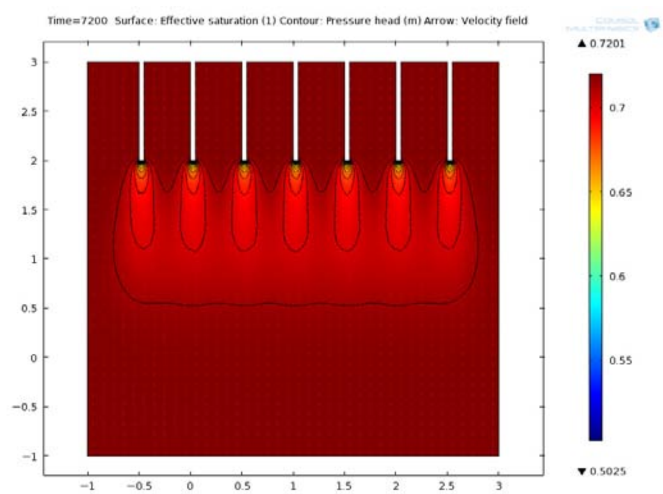

Figure 3: Vertical Sensor Simulation Result after 7200 seconds

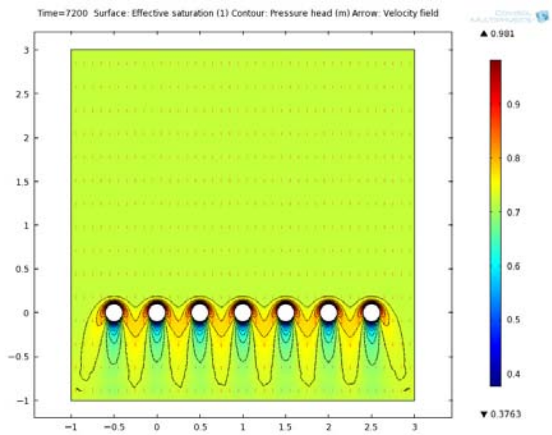

Figure 4: Horizontal Orientation Simulation after 7200 seconds

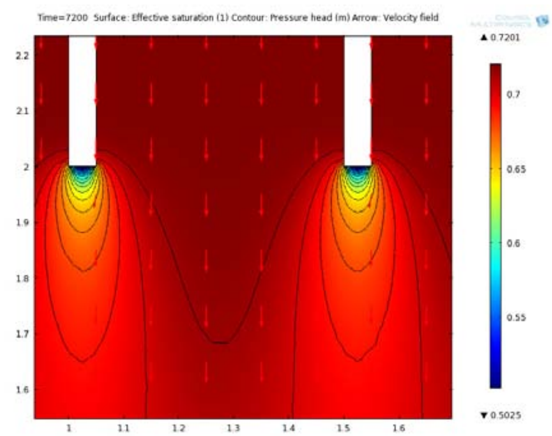

Figure 5: Localized Moisture Distribution around a Vertically Placed Sensor

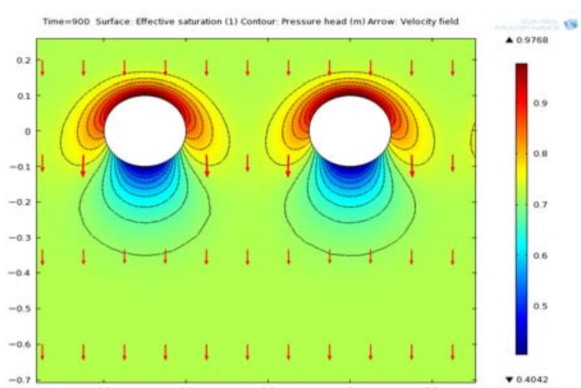

Figure 6: Localized Moisture Distribution around a Horizontally Placed Sensor

Figures 7 and 8 give the effective saturation over time at the soil-rod(sensor) boundary. In Figure 7, it can be seen that there is low effetive saturation under the rods(sensors) (Figure 5) while in Figure 8, it shows that the soil is wetter above the rods.

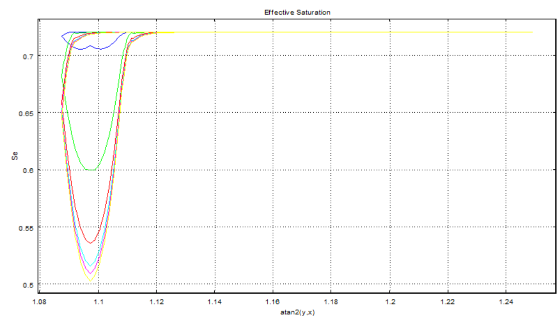

Figure 7: The Effective Saturation around the Sensor in Vertical orientation.

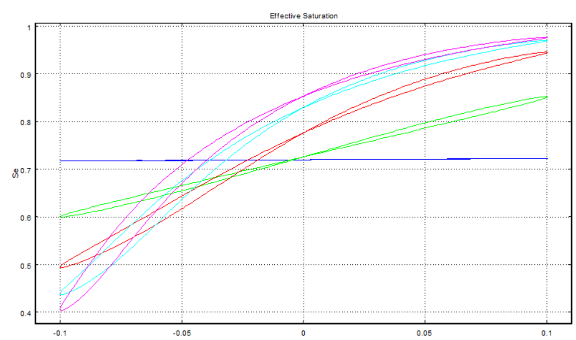

Figure 8: The Effective Saturation around the Sensor in Horizontal orientation.

6. Conclusions

This paper has been able to demonstrate the application of RE in COMSOL Multiphysics 4.1 interface in describing localization of spatial distribution of water around seven irrigation sensors in two different orientations. It's always a daunting task to get an experimental data for this type of process for model validation. Knowing where to locate irrigation sensors and what orientation to use in irrigation water management experiments are very important for sustainable agriculture. Therefore, this paper could help in this regard. COMSOL Multiphysics have been demonstrated as a capable tool to solve this problem. Another area of research that would be interesting and helpful for irrigation engineers, soil scientists and agronomists is the application of COMSOL Multiphysics to predict water and nutrient uptake by plants. This should be considered in future release version of COMSOL Multiphysics. Finally, the applications of this paper could be applied for heterogeneous soils in order to mimic a more realistic situation.

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