Electromagnetic and Coupled Field Computations: A Perspective

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Outline

- Introduction
- Classification of Electromagnetic Field Problems
- Coupled Field Computations
  - Circuit-Field
  - Magnetic-Thermal
  - Magnetic-Structural
- Case Studies
- Concluding Remarks
Introduction
An Overview of EM Applications

EM principles form the core of electricity generation, transmission and distribution

EM, and its computational version, are used to design, test and validate devices across a wide range of sizes – from the smallest micro-electro-mechanical (MEMS) devices to the very large transformers and generators

Low Frequency Devices:
- Transformers
- Electric motors
- Power generators
- EM forming and welding
- EM interference/coupling
- Induction heating devices

High Frequency Devices:
- Antennas
- Waveguides and Resonant cavities
- Magnetic storage and imaging systems
- Optoelectronics and photonics
- Microwave circuits and devices
- Plasma devices
Need for EM Field Computation

- Computation of magnetic fields is required in all low frequency and high frequency devices for:
  - Evaluation and improvement of performance parameters at the design stage
  - Reliability enhancement
  - Investigative analysis

- Field computation provides a non-destructive technique for testing and evaluation

- In order to optimize material costs, in the present-day highly global market, an accurate understanding and analysis of the field distribution is necessary
Low Frequency Devices: Performance Parameters

- Inductances and capacitances
- Insulation design for high voltage applications
- Eddy currents
- Forces in windings and current carrying bars
- Torques in rotating machines
- Mechanical stresses / deformations
- Temperature profiles and hot-spots
- Noise level
High Frequency Devices: Performance Parameters

- S parameters
- Power flow/Poynting vector
- Propagation constants
- Characteristic impedance
- Radiation patterns
- Modal field distributions
Computational Methods

- **Difference methods:**
  - Finite difference method (FDM)
  - Finite-difference time-domain method (FDTD)

- **Variational / Weighted residual approach:**
  - Finite element method (FEM)

- **Integral methods:**
  - Method of moments (MoM)
  - Boundary element method (BEM)
  - Charge simulation method (CSM)
Finite Element Method

- The method has emerged as the forerunner among all the numerical techniques
  - Geometrical complexities can be handled in better ways using FEM
  - Anisotropic, non-uniform and non-linear media can be incorporated
  - Availability of several commercial softwares makes the applicability to real-life problems easier
  - Finite element method can also be used in solving problems involving coupling of electromagnetic fields with circuits and/or other physical fields
Classification of Electromagnetic Field Problems
Basic Governing Equations

- Maxwell’s equations:
  \[ \nabla \cdot D = \rho_v \]
  \[ \nabla \times E = -\frac{\partial B}{\partial t} \]
  \[ \nabla \cdot B = 0 \]
  \[ \nabla \times H = J + \frac{\partial D}{\partial t} \]

- Constitutive relations:
  \[ D = \varepsilon E \]
  \[ B = \mu H \]
  \[ J = \sigma E \]

- Continuity equation:
  \[ \nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \]

- Lorentz force equation:
  \[ F = Q (E + v \times B) \]
Typical entities and problem types: Low frequency

- Current fed massive conductors
- Voltage fed stranded conductors
- Moving conductors
- Eddy currents, non-magnetic region
- Eddy currents, magnetic region
- Air/oil/insulation
- Laminated Iron
- Neumann boundary
- Dirichlet boundary
Classification of Electromagnetic Field Problems

- **Electrostatics:**
  - Analysis of the electric field in capacitive or dielectric systems
    \[
    \varepsilon_x \frac{d^2V}{dx^2} + \varepsilon_y \frac{d^2V}{dy^2} = -\rho_v
    \]
  - Computation of electric field, capacitance, electrostatic forces and torques, etc.
Magnetostatics:

- This analysis type is used to analyze magnetic field produced by direct electric current, permanent magnet or applied magnetic field

\[ \frac{1}{\mu_x} \frac{d^2 A}{dx^2} + \frac{1}{\mu_y} \frac{d^2 A}{dy^2} = -J_o \]

- The static analysis is used to compute parameters such as magnetic flux, self and mutual inductances, forces, torques, etc.
Time-harmonic: Diffusion Equation

- Harmonic analysis is used for sinusoidal excitations and linear materials
- The analysis can be carried out for a single frequency or a range of frequencies to compute eddy currents, stray losses, skin effect and proximity effect

\[
\frac{1}{\mu_x} \frac{d^2 A}{dx^2} + \frac{1}{\mu_y} \frac{d^2 A}{dy^2} + \frac{1}{\mu_z} \frac{d^2 A}{dy^2} = \sigma (\nabla V + j\omega A)
\]
- **Time-harmonic: Wave Equation**
  - The wave equations for time harmonic electric and magnetic fields with angular frequency $\omega$ are:
    \[
    \nabla^2 \mathbf{E} + \omega^2 \mu \varepsilon \mathbf{E} = 0 \\
    \nabla^2 \mathbf{H} + \omega^2 \mu \varepsilon \mathbf{H} = 0
    \]
  - \[\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\]

**Rectangular Waveguides**

- TE20
- TM11
- TE22
- TM21
Eigenvalue FEM analysis of a plasmonic waveguide

- Photonic circuits of nanoscale dimensions: A potential research area
- These could form harmonizing links between nano-scaled electron devices and micro-scaled optical devices

- FEM analysis*: Metal-dielectric-slotted metal structure
- A good confinement and a propagation length of 10 µm is observed

* Courtesy: Ms. Padmaja, Research Scholar, EE Dept, IIT Bombay
Transient:

- Transient magnetic analysis is a technique for calculating magnetic fields that vary over time, such as those caused by surges in voltage or current or pulsed external fields.

\[
\frac{1}{\mu} \frac{d^2 A}{dx^2} + \frac{1}{\mu} \frac{d^2 A}{dy^2} = \sigma \left( \nabla V + \frac{\partial A}{\partial t} \right)
\]

- Performance parameters such as inrush current, eddy currents and forces can be computed when electrical machines are subjected to transient stresses.
Potentials Used in Computational Electromagnetics

- Electric scalar potential ($V$):
  - Used in electrostatic formulations: Insulation design in high voltage equipment
  - Analysis of frequency-dependent performance of dielectrics

\[
\nabla \cdot \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) = 0
\]

If the effects of magnetic field can be neglected,

\[
\nabla \times \mathbf{E} = 0 \implies \mathbf{E} = -\nabla V
\]

\[
-\nabla \cdot (\sigma \nabla V) - \nabla \cdot \left( \frac{\partial}{\partial t} (\varepsilon \nabla V) \right) = 0
\]

\[
-\nabla \cdot \left( (\sigma + j \omega \varepsilon) \nabla V \right) = 0
\]

\[
-\nabla \cdot \left( (\sigma_{\text{eff}} + j \omega \varepsilon') \nabla V \right) = 0 \quad \sigma_{\text{eff}} = \sigma + \omega \varepsilon'' = \sigma + \omega \varepsilon' \tan \delta
\]
Magnetic vector potential (A):

- In the case of 2-D models, the formulation based on magnetic vector potential (MVP) is generally used.
  - The number of unknowns at any point reduce from 2 (Bx, By) to one (Az), with the current in z-direction.

\[ \phi = \oint_S \mathbf{B} \cdot d\mathbf{S} = \oint_L \mathbf{A} \cdot d\mathbf{l} \]  

  Flux passing through any two points 1 and 2 \( \rightarrow \phi = A_1 - A_2 \) (2-D case)

- However in 3-D models, the MVP formulation has three degrees of freedom per node, Ax, Ay and Az, making it computationally unattractive.

- MVP formulation can be used for static, time-harmonic or transient magnetic analyses.

- If there are eddy current regions, additionally the electric scalar potential needs to be considered (A-V, A formulation).
Magnetic scalar potential ($\Omega$):

- In 3-D problems, $A$ has three unknowns at every point
- The scalar potential formulation has only one degree of freedom per node
- The magnetic scalar potential based formulation is therefore suitable for 3-D magnetostatic problems
- The scalar potential cannot be used for current carrying regions and/or in any part which surrounds such regions
- Reduced scalar potential is used to circumvent above restrictions
- Reduced Scalar Potential ($\Omega_r$) or Total Scalar Potential ($\Omega$) is selected as variable depending on presence and absence of current carrying domains, respectively: Hybrid formulation
- **Electric Vector Potential (T)**
  - Used for solving eddy current problems
    \[ \nabla \times T = J \]
    \[ H - T = -\nabla \Omega \quad \Rightarrow \quad H = T - \nabla \Omega \]
  - Compare with: \[ E = -\frac{\partial A}{\partial t} - \nabla V. \]
  - \( T \) represents induced eddy currents like the term \( -\frac{\partial A}{\partial t} \) does in the \( A \)-based formulation
  - The formulation is advantageous for the analysis of eddy currents in laminated structures
- **Nodal Vs Vector (Edge) Elements**
  - Nodal formulation: Popularly used for low frequency computations
    - Simpler and easy to implement
    - ‘Scalar’ FEM
  - Edge elements: Better suited for high frequency computations
    - Degrees of freedom are associated with edges
    - Continuity of tangential components of field vectors is ensured
    - Spurious modes/solutions are avoided as the divergence condition is satisfied
    - They are better in handling singularities
Coupled Field Computations
About Coupled Fields

- **Classification:**
  - Weakly coupled
  - Strongly coupled

- **Weak or indirect coupling:**
  - Solution of one field acts as load to another field
  - It is flexible, modular and easy approach

- **Strong or direct coupling:**
  - Coupled field equations are solved simultaneously
  - The approach is used when field interactions are highly nonlinear and the coupled fields have comparable time constants.
Coupled Systems: Real-Life Design Problems

Coupled interactions in Transformers

Coupled Field Computations

Circuit – Field
Field-Circuit Coupling

Electromagnetic model:

\[ \frac{\partial}{\partial x} \left( \nu \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \nu \frac{\partial A_z}{\partial y} \right) = -J_z \]

Circuit coupling:
Conductor Models

Stranded conductor:

\[ J_{str} = \frac{N_{str} I_{str}}{A_{str}} \]

Solid (massive) conductor:

\[ J_{sol} = \sigma \frac{V_{sol}}{l_{sol}} - \sigma \frac{\partial A_z}{\partial t} \]

Figure 3.2: (a) Stranded (b) solid conductor models
Field-Circuit Coupling

FEM formulation for stranded conductor

\[ \frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial y} \right) = -J_z \]

\[ J_z = \frac{N_{str} I_{str}}{A_{str}} \]

Finite element discretization leads to:

\[ [K] \{A\} + [P] \{I\} = 0 \]

where,

\[ [K] = \sum_{\Omega} \int_{\Delta_e} \left( \frac{\partial N_e^T}{\partial x} \frac{\partial N_e}{\partial x} + \frac{\partial N_e^T}{\partial y} \frac{\partial N_e}{\partial y} \right) dx dy \]

\[ [P] = \sum_{\Omega} \frac{N_{str}}{A_{str}} \int_{\Delta_e} N_e^T \ dx dy \]
Field-Circuit Coupling

Circuit equations can be written as:

$$\{U\} = \left\{ \frac{d\Phi}{dt} \right\} + [R] \{I\} + [L] \left\{ \frac{dI}{dt} \right\}$$

$$\frac{d\Phi}{dt} = \frac{L_{str}}{A_{str}} \int_{\Omega} \frac{\partial A_z}{\partial t} d\Omega$$

$$\{U\} = [G] \left\{ \frac{dA}{dt} \right\} + [R] \{I\} + [L] \left\{ \frac{dI}{dt} \right\}$$

Global system of equations is given as:

$$\begin{bmatrix} [0 & 0] & \{A\} \\ [G & L] & \{I\} \end{bmatrix} + \begin{bmatrix} [K & P] \\ [0 & R] \end{bmatrix} \{I\} = \{0\}$$

$$\begin{bmatrix} [0 & 0] & \{A\} \\ [G & L] & \{I\} \end{bmatrix} + \begin{bmatrix} [K & P] \\ [0 & R] \end{bmatrix} \{I\} = \{0\}$$
Coupled Field Formulations: Solid Conductor

- **Field Equation:**
  \[
  \frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial y} \right) = -\sigma \frac{V}{l_{sol}} + \sigma \frac{\partial A_z}{\partial t}
  \]

- **Internal Current Equation:**
  \[
  I = G_{sol} V - \int_{\Omega} \sigma \frac{\partial A_z}{\partial t} \, d\Omega
  \]

- **Circuit Equation:**
  \[
  \{U\} = \{V\} + \begin{bmatrix} [R] & [I] & [L] \end{bmatrix} \begin{bmatrix} dI \end{bmatrix}
  \]

\[
\begin{bmatrix}
[Q] & [0] & [0] \\
[B]^T & [0] & [0] \\
[0] & [0] & -[M]
\end{bmatrix}
\frac{d}{dt} \begin{bmatrix}
\{A\} \\
\{V\} \\
\{I\}
\end{bmatrix}
+
\begin{bmatrix}
[C] & [B] & [0] \\
[0] & [S] & [P] \\
[0] & [P]^T & -[R]
\end{bmatrix}
\begin{bmatrix}
\{A\} \\
\{V\} \\
\{I\}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\{0\} \\
\{0\} \\
\{U\}
\end{bmatrix}
\]
# 1. Half-Turn Effect

## Single-phase three-limb transformer

<table>
<thead>
<tr>
<th></th>
<th>Measured</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux density in end limbs (T)</td>
<td>1.04</td>
<td>0.93</td>
</tr>
<tr>
<td>Extra core loss due to the half-turn effect (kW)</td>
<td>4.2</td>
<td>3.9</td>
</tr>
</tbody>
</table>

(a) Flux lines (b) flux density plots with half-turn

## Three-phase five-limb transformer

<table>
<thead>
<tr>
<th></th>
<th>Flux density (T) for unbalanced currents in windings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Balanced</td>
</tr>
<tr>
<td>Without half-turn</td>
<td>0.02</td>
</tr>
<tr>
<td>With half-turn</td>
<td>0.04</td>
</tr>
</tbody>
</table>

2. Forces in Split-Winding Transformer

Contour lines of magnetic vector potential
(a) One winding short circuited (b) Both windings short circuited

3. Coupled Circuit – Field Analysis: Interphase Transformer (IPT)

Coupled Field Computations

Magnetic – Thermal
The 2-D transient magnetic and thermal equations are

\[
\nabla \cdot \left( \frac{1}{\mu} \nabla (A_z) \right) = -\sigma(T) \frac{V}{l} + \sigma(T) \frac{\partial A_z}{\partial t}
\]

\[
\nabla \cdot (k \nabla (T)) = -q(A_z, T) + mc \frac{\partial T}{\partial t}
\]

These are coupled by the following relations

\[
\sigma(T) = \frac{\sigma_{ref}}{(1 + \alpha(T - T_{ref}))}
\]

\[
q(A_z, T) = \frac{1}{\Omega_e} \int_{\Omega_e} \sigma(T) \left( -\frac{V}{L} + \frac{\partial A_z}{\partial t} \right)^2 d\Omega
\]

Convection boundary conditions

\[
k \nabla (T) \cdot n + h_c (T - T_a) = 0
\]
Magnetic-Thermal Coupling

Start

Temp, $\sigma$, $\mu$

Electromagnetic (Time Harmonic)

Heat Generated, $k$, $c$, $m$

Thermal (Transient)

$T = t + \Delta t$

Update Electromagnetic Solution?

Yes

No

End
Formulation

- Magnetic equations for solid conductors (g=-V/L)
  \[
  \frac{\partial}{\partial x} \left( \frac{1}{\mu_0} \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu_0} \frac{\partial A_z}{\partial y} \right) = \sigma \left( g + j\omega A_z \right)
  \]
  \[
  I = \int_{\Omega_c} \sigma \left( g + j\omega A_z \right) d\Omega_c
  \]

- In matrix form
  \[
  \begin{bmatrix}
  K + j\omega V & W \\
  j\omega W^T & G'
  \end{bmatrix}
  \begin{bmatrix}
  A \\
  g
  \end{bmatrix} = \begin{bmatrix}
  0 \\
  I
  \end{bmatrix}
  \]

- Elemental loss
  \[
  Q_e = \int_{\Omega_e} \sigma \left( g + j\omega A_z \right) \left( g + j\omega A_z \right)^* d\Omega_e
  \]

\[\text{[K]} = \sum_{\Omega} \iint_{\Delta_e} \left( \frac{\partial N_e^T}{\partial x} \frac{\partial N_e}{\partial x} + \frac{\partial N_e^T}{\partial y} \frac{\partial N_e}{\partial y} \right) dxdy\]

\[\text{[V]} = \sum_{\Omega} \sigma \iint_{\Delta_e} N_e^T N_e dxdy\]

\[\text{[W]} = \sum_{\Omega} \sigma \iint_{\Delta_e} N_e^T dxdy\]

\(G'\) is the diagonal matrix of the conductance of the bars
1. High Current Terminations

Unequal Current Distribution

Non-uniform Temperature distribution

2. Induction Hardening

Auto-tempering process

Coupled Field Computations

Magnetic – Structural
Coupled Field Formulations: Magnetic-Structural

- Coupled Equations:

\[
\begin{bmatrix}
K & C \\
N & M
\end{bmatrix}
\begin{bmatrix}
\{A\} \\
\{X\}
\end{bmatrix}
= 
\begin{bmatrix}
\{J\} \\
\{F_{ext}\}
\end{bmatrix}
\]

K and M are magnetic and mechanical stiffness matrices respectively. A and X are nodal values of magnetic vector potential and displacements.

- The formulation with suitable modifications can be used for:

- Analysis of core noise: Magnetostriction phenomenon
- Computation of noise due to winding vibrations (J x B Force)
- Analysis of winding deformations due to short circuit forces
- Design of high current carrying bars in large rectifier and furnace duty applications
- $[N]$ and $[C]$ are the coupling matrices, and $\{J\}$ and $\{F_{ext}\}$ are the column vectors representing the magnetic and mechanical source terms, respectively.

- The term $[N]$ represents the effect of magnetic parameters on mechanical displacements, whereas $[C]$ represents the effect of mechanical displacements on magnetic parameters.

- It can be proved that the total magnetic force ($F_{mag}$) can be represented by $-\{N\} \{A\}$

- If the effects of displacements on magnetic fields are not appreciable, $[C]$ can be neglected and the magnetic forces affecting displacements can be added to the mechanical (external) forces:

$$
\begin{bmatrix}
K & 0 \\
0 & M
\end{bmatrix}
\begin{bmatrix}
\{A\} \\
\{X\}
\end{bmatrix} =
\begin{bmatrix}
\{J\} \\
\{F_{ext} + F_{mag}\}
\end{bmatrix}
$$

- The magnetostriction phenomenon can also be considered in a weakly coupled scheme by adding the corresponding vector $\{F_{ms}\}$ to the force terms.
Electromagnetic Forming

Diagram of the Electromagnetic Forming process:
- High Voltage Charging Power Supply
- Spark gap
- Forming Coil
- Switch

Graph showing the current over time:
- X-axis: Time
- Y-axis: Current

Components:
- Coil
- Pipe

Simulation and Verification images.
Concluding Remarks
Current and Emerging Trends

- Competence in 3-D analysis is essential
- Coupled field computations (circuit-field, magnetic-thermal, magnetic-structural) will be increasingly used
- Coupled EM-thermal-structural analysis is not uncommon these days
- Hybrid numerical techniques are being used for complex problems
- Other trends
  - Parallel computing
  - Real time FEM
  - Meshless methods
  - Wavelet based FEM
Thank You!