

# Determination of Fundamental Frequency of A Thin Rectangular Perforated Plate by Using COMSOL Multiphysics

Sodisetty V N B Prasad<sup>1</sup>, B.Raghavendra<sup>2</sup>,

<sup>1</sup>BITS Pilani K K Birla Goa Campus, <sup>2</sup>BITS Pilani Campus

<sup>1</sup>Hostel-CH2-103, BITS Pilani K K Birla Goa Campus,  
Zuari Nagar, Near NH17B, Goa-403726, India.

<sup>2</sup>Hostel-MLB-2108, BITS PILANI CAMPUS,

Vidya vihar campus, Pilani, Rajasthan-333031, India.

<sup>1</sup>bhavaniprasad.478@gmail.com <sup>2</sup>braghavendra05@gmail.com

**Abstract:** A plate is an integral part of any structure. It may be defined as a solid body bounded by two flat surfaces. The distance between the two surfaces defines the thickness of the plate, which is assumed to be small compared to the lateral dimensions, such as the length and width in case of a rectangular plate and the diameter in case of a circular plate. A plate is usually considered to be thin when the ratio of its thickness to its smaller lateral dimension (such as width in case of rectangular plate and diameter in case of circular plate) is less than 1/20.

Study of vibration of plates is performed in engineering disciplines such as civil, mechanical and aeronautical engineering. Study of plates includes members, viz., beams, plates and shells. These found wide applications in real time systems such as bridge decks, pressure vessel covers, hydraulic systems, pavements of highways and airport runways, ship decks, airplanes and machine parts. This project includes calculation of the first few modes of vibration characteristics of plates by using COMSOL, Analytical and Experimental methods. Approximate method such as Rayleigh's Ritz method for plates had been used to find natural frequencies. Rayleigh's Ritz method has an advantage over general discretization methods such as Finite element methods in terms of problems having large degree of freedom systems. To solve differential equations MAPLE software has been used. Experimental vibration analysis is done by using Fast Fourier Transform (FFT) Analyzer.

## Keywords:

Lateral vibrations, Fundamental frequency, FFT Analyzer, Rayleigh's quotient.

## 1. Introduction

### 1.1 Rayleigh's method

In most structural and mechanical systems, the fundamental or lowest natural frequency is the most important. Rayleigh's method can be used to find out the fundamental natural frequency of the system. This method is based on the Rayleigh's Principle which can be stated as follows: The frequency of vibration of a conservative system vibrating about an equilibrium position has a stationary value in the neighbourhood of a natural mode. This stationary value, in fact, is a minimum value in the neighbourhood of the fundamental natural mode.

Kinetic Energy of Plate T is given as,

$$T = \frac{1}{2} \iint_R h \rho \dot{w}^2(x,y,t) dx dy$$

Where h is the height of plate and  $\rho$  is the density of plate material. Assuming that the plate is undergoing harmonic vibrations, then vibrating middle surface of the plate can be approximated by equation,

$$w(x,y,t) = W_1(x,y) \cos \omega_1 t$$

where  $W_1(X,Y)$  is a continuous function that approximately represents the shape of the plate.  $\omega_1$  represents the natural frequency of the plate pertinent to the assumed shape function.

Assume  $\omega = \omega_1$  be its fundamental frequency  
Maximum value of kinetic energy is obtained at

$$\sin^2 \omega_1 t = 1$$

$$T_{\max} = \frac{1}{2} \omega_1^2 \sin^2 \omega_1 t$$

$$\iint_R h\rho W_1^2(x,y) dx dy = \frac{1}{2}\omega_1^2 \iint_R h\rho W_1^2(x,y) dx dy$$

Maximum Strain Energy is obtained at

$$U_{\max} = \frac{1}{2} \iint_R D[(\nabla^2 W_1)^2 + 2(1-\nu) \left\{ \left( \frac{\partial^2 W_1}{\partial x \partial y} \right)^2 - \frac{\partial^2 W_1}{\partial x^2} \frac{\partial^2 W_1}{\partial y^2} \right\}] dx dy$$

For conservative system by Rayleigh's principle,

$$T_{\max} = U_{\max}$$

$$\omega_1^2 = \frac{\iint_R D[(\nabla^2 W_1)^2 + 2(1-\nu) \{W_{1,xy}^2 - W_{1,xx} W_{1,yy}\}] dx dy}{\iint_R h\rho W_1^2 dx dy}$$

where  $W_{1,xy} = \frac{\partial^2 W_1}{\partial x \partial y}$ ,  $W_{1,xx} = \frac{\partial^2 W_1}{\partial x^2}$   
 $W_{1,yy} = \frac{\partial^2 W_1}{\partial y^2}$

This equation is called as Rayleigh's Quotient and gives fundamental natural frequency of plate.

## 2. Use of COMSOL Multiphysics

### 2.1 Modeling in COMSOL Multiphysics

A rectangular plate with four circular cut outs is modeled and material properties are assigned to it. The dimensions of rectangular plates are 216mm x 138mm x 2mm and the following material properties are given  $E = 2.1 \times 10^{11} \text{ N/m}^2$ ,  $\nu = 0.3$ ,  $\rho = 7850 \text{ kg/m}^3$ . Diameter of circular cut outs is 25 mm. The model is as shown in the figure 1.

### 2.2 Model Navigator

In this analysis Eigen frequency module from structure mechanics is used. This module is used for finding out natural frequencies and their corresponding mode shapes.

### 2.3 Eigen frequency

This module is used for finding out natural frequencies and their corresponding mode

shapes. The first five frequencies of plate are extracted and their mode shapes are studied. The first natural frequency obtained was 673.43 Hertz.

### 2.4 Subdomain settings.

The linear elastic material is selected and value of Young's modulus, poisons ratio, density are given as  $E = 2.1 \times 10^{11} \text{ N/m}^2$ ,  $\nu = 0.3$ ,  $\rho = 7850 \text{ kg/m}^3$ .

### 2.5 Boundary settings.

Rectangular plate's four sides are clamped, Degrees of freedom of all four edges are made zero. The desired number of Eigen frequencies is set to 5 and they are searched around 100.

### 2.6 Meshing solving and post processing

In this analysis Free tetrahedral element is selected for meshing, mesh size is selected as extremely small. The number of elements after meshing are found to be 28,111, the total degrees of freedom are 1,70,949. The plate after imposing boundary conditions and meshing is shown in figure 2. The mode shape for Eigen frequency 1574.62 is as shown in figure 3.

## 2.7 Results and discussion

In this Structural analysis first 5 Eigen Frequency values are found from COMSOL Multiphysics simulation and they are listed out in Figure 4.

## 3. Figures

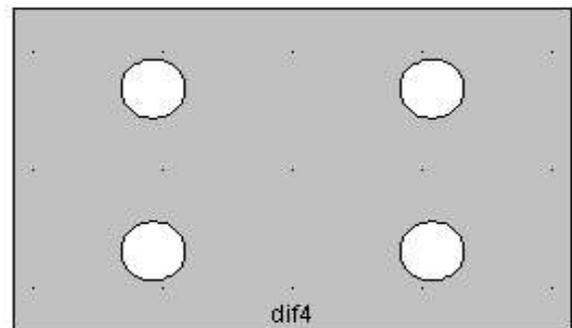


Figure 1. Rectangular plate with circular cutouts for simulation.

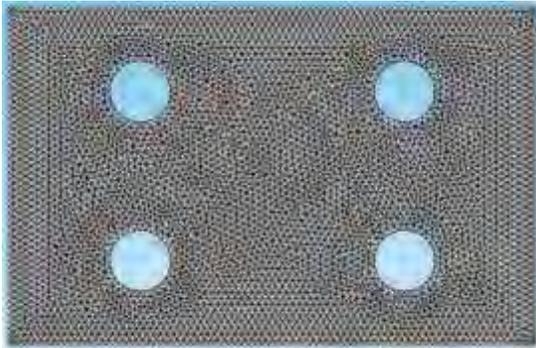


Figure 2. Rectangular plate after meshing.

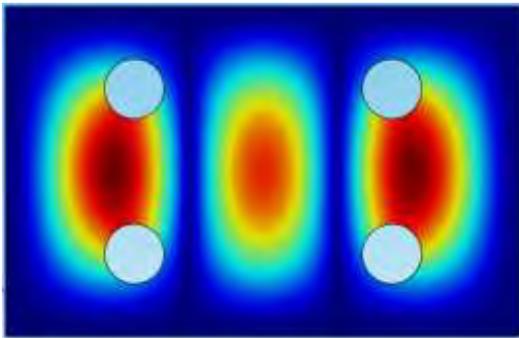


Figure 3. Mode shape corresponding to 1574.62 Hz

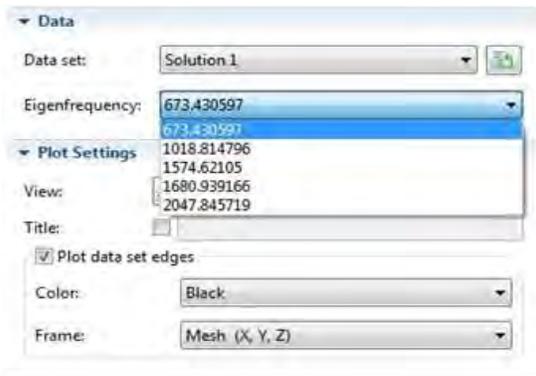


Figure 4. Simulated result showing the first five Fundamental natural frequencies of the perforated plate.

The results obtained from COMSOL Multiphysics simulation are to be compared with both Analytical and Experimental methods.

The Analytical analysis is carried out using Rayleigh's Ritz method which is discussed in the section 4 and the first Fundamental frequency is found out from this method.

The Experimental analysis is carried out using Fast Fourier Transform (FFT) Analyzer which is discussed in the section 5.

## 4. Equations by Analytical method

### 4.1 Rayleigh's Ritz Method

The deflection function due to the free vibration of a rectangular plate whose dimensions with a and b is

$$W(x,y) = \sum_m \sum_n A_{mn} \Phi_m(x,y) \psi_n(x,y)$$

Maximum kinetic energy of plate is given by

$$T_{\max} = \frac{1}{2} \omega^2 \int_R ab h \rho W_1^2(x,y) dx dy$$

The maximum strain energy is given by

$$U_{\max} = \frac{1}{2} Dab \int_0^1 \int_0^1 [W_{xx}^2 + \beta W_{yy}^2 + 2\nu \beta W_{xx} W_{yy} + 2(1-\nu)\beta W_{xy}^2]$$

$\beta = \frac{a}{b}$  is the side ratio.

Putting the deflection function into the above maximum kinetic and strain energies and minimising the rayleigh quotient with respect to the constants  $A_{mn}$  the eigen value problem is

$$\sum_m \sum_n [C_{mnij} - \lambda E_{mi} F_{nj}] A_{mn} = 0$$

where

$$C_{mnij} = E_{mi} F_{nj} + \beta E_{mi} F_{nj} + \nu \beta [$$

$$E_{mi}^{(0,2)} F_{nj}^{(2,0)} + E_{mi}^{(2,0)} F_{nj}^{(0,2)} ] + 2(1-\nu)\beta^2 E_{mi}^{(1,1)} F_{nj}^{(1,1)}$$

$$m, n, i, j = 1, 2, 3, \dots, p, q = 0, 1, 2$$

$$E_{mi}^{(p,q)} = \int_0^a (d^p \phi_m / dx^p)(d^q \phi_i / dx^q) dx$$

$$F_{nj}^{(p,q)} = \int_0^b (d^p \phi_n / dy^p)(d^q \phi_j / dy^q) dy$$

$$\lambda = \frac{(h \times \rho)}{D} \omega_1^2$$

Where  $\lambda$  = Rayleigh's quotient

The deflection functions considered are

$$\Phi(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

$$\psi(y) = b_0 + b_1 y + b_2 y^2 + b_3 y^3 + b_4 y^4$$

Consider a plate with four edges clamped, the boundary conditions are

$$\Phi_1(0) = 0, \Phi_1(1) = 0, \Phi_1'(0) = 0, \Phi_1'(1) = 0$$

$$\psi_1(0) = 0, \psi_1(1) = 0, \psi_1'(0) = 0, \psi_1'(1) = 0$$

By substituting the boundary conditions in the deflection functions we get

$$\Phi(x) = (x^2 + x^4 - 2x^3) * a_4$$

$$\psi(y) = (y^2 + y^4 - 2y^3) * b_4$$

#### 4.2 Orthogonal functions

Orthogonal functions should satisfy the equation

$$\int_0^a \Phi(x) * \Phi(x) dx = 1$$

$$\rightarrow (a_4^2) \int_0^a x^4 (1 + x^2 - 2x)^2 dx = 1$$

$$\rightarrow a_4 = \frac{1}{3\sqrt{70} \sqrt{a^5 (70a^4 - 315a^3 + 540a^2 - 420a + 126)}}$$

The expression for  $\Phi(x)$  is given by

$$\Phi(x) = 3(x^2 + x^4 - 2x^3)$$

$$\sqrt{70} \sqrt{\frac{1}{a^5 (70a^4 - 315a^3 + 540a^2 - 420a + 126)}}$$

Similarly the expression for  $\psi(y)$  is given by

$$\psi(y) = 3(y^2 + y^4 - 2y^3)$$

$$\sqrt{70} \sqrt{\frac{1}{b^5 (70b^4 - 315b^3 + 540b^2 - 420b + 126)}}$$

here we are considering for the first fundamental frequency so we consider only each one function in both x and y directions.

$\Phi_1(x)$ ,  $\psi_1(y)$  and only one constant  $A_{11}$ .

The value of the expression  $E_{mi}^{(0,0)} * F_{nj}^{(0,0)}$  is given by unity since the functions are orthogonal. So the equation  $C_{m1n1} - \lambda E_{mi}^{(0,0)} F_{nj}^{(0,0)}$

$$\text{becomes } C_{1111} - \lambda = 0$$

$$\rightarrow C_{1111} = \lambda$$

By substituting the values of a, b values for perforated plate in the above expression for  $\lambda$  and by solving it using MAPLE software the value of Rayleigh's quotient is given by

$$\lambda = 57797.65266.$$

The fundamental frequency is calculated by

$$\begin{aligned} \omega_1^2 &= \frac{(\lambda * D)}{(h * \rho)} \\ &= (57797.65266 * 153.85) / (0.002 * 7850) \\ &= 566380.1823 \text{ (Hertz)} \\ \omega_1 &= 752.58 \text{ Hertz} \end{aligned}$$

## 5. Experimental method

The experimental analysis is carried out by using Fast Fourier Transform (FFT) analyzer. The specimen used is shown in the Figure 5



**Figure 5.** The specimen used in experimental setup is shown.

The experimental setup consisting of FFT Analyzer, accelerometer, hammer used is shown in Figure 6.

The plate is clamped on all the four edges such that all the boundaries are fixed. By using an impact hammer small lateral vibrations are induced in the plate.

The experiment is carried out for 8 different points on the plate and mean value of the frequency values are measured from FFT Analyzer. The values are tabulated in table 1.



**Fig 6.** Experimental setup used is shown in the figure.

## 6. Results and discussion

A thin perforated plate of dimensions 216 x 138 x 2 mm is considered for this analysis. Material properties of Mild steel are selected for this structural analysis. The first fundamental frequency for lateral vibrations occurs at 673.43 Hertz. Similarly for next four frequencies values are found out as shown in Figure 7. The values of first five fundamental frequencies are calculated from Experimental methods. According to Experimental method the first fundamental frequency for lateral vibration occurs at 649.37 Hertz which is in close proximity to COMSOL simulation result.

The frequency of vibration depends upon shape, size, material properties of the plate and also the conditions with which the system is vibrating.

## 7. Conclusion

The vibration analysis of perforated plate is carried out in detail. The first few modes of vibrations of perforated plate are found out.

The first five fundamental frequencies of the perforated plate are found out from COMSOL simulation, Experimental methods. The first fundamental frequency of perforated plate is calculated from Rayleigh's Ritz method. The results obtained by simulation in COMSOL are in close proximity to that found by Fourier Frequency Transform analyser and Analytical method.

## 8. References

1. COMSOL Multiphysics 4.0a user manual.
2. D.V. Bambill, et al, Natural frequencies of thin, rectangular plates with holes or orthotropic "patches" carrying an elastically mounted mass. *International Journal of Solids and Structures* 43, 4116–4135(2006).
3. B. J. E. van Rens, et al, Homogenization of the elastoplastic behavior of perforated Plates. *Computers and Structures* 69, 537-545(1998)
4. J. Rezaeepazhand, M. Jafari, Stress analysis of perforated composite plates. *Composite Structures* 71, 463–468(2005).

5.Chakraverty S., 2009. Vibration of plates, First ed. CRC Press Taylor & Francis Group, Boca Raton, 75-163(2009).  
 properties. Journal of the Korean Nuclear Society. Volume 30, Number 5, 416-423(1998).

## 9. Acknowledgements

Thanks to COMSOL Multiphysics for giving an opportunity to solve this multiphysics problem and an extreme support and encouragement from our faculty members especially MR. Kiran D Mali, Lecturer of Mechanical Department, BITS Pilani.

## 10. Appendix

Natural mode	Natural Frequencies from FFT analyzer (Hz)	Natural Frequencies from COMSOL results (Hz)
1	<b>649.37</b>	<b>673.43</b>
2	985.27	1018.81
3	1538.23	1574.62
4	1617.93	1680.93
5	1980.8	2047.84

**Table1.**The Eigen Frequencies obtained from the analysis are tabulated