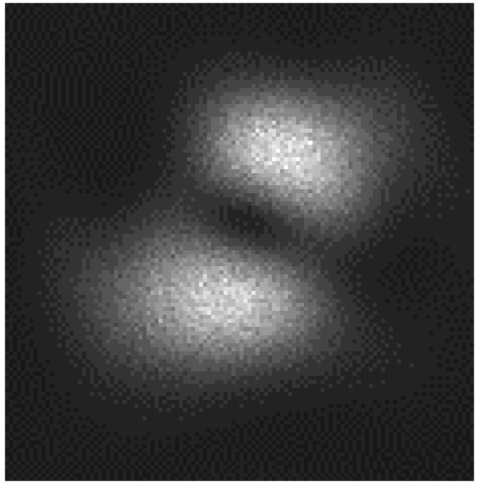

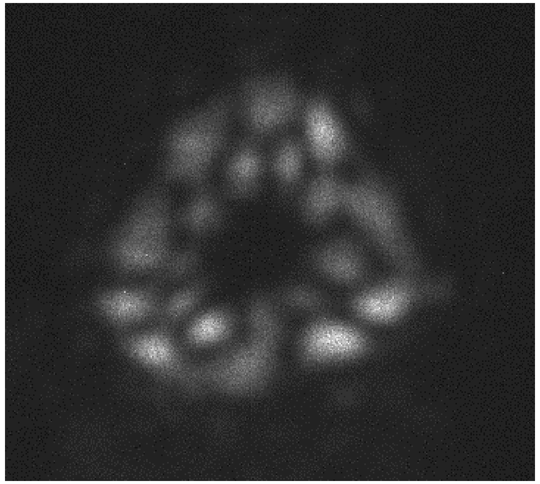

A Numerical Study of the Paraxial Wave Equation

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Motivation:
Laser beam propagation in maritime environment

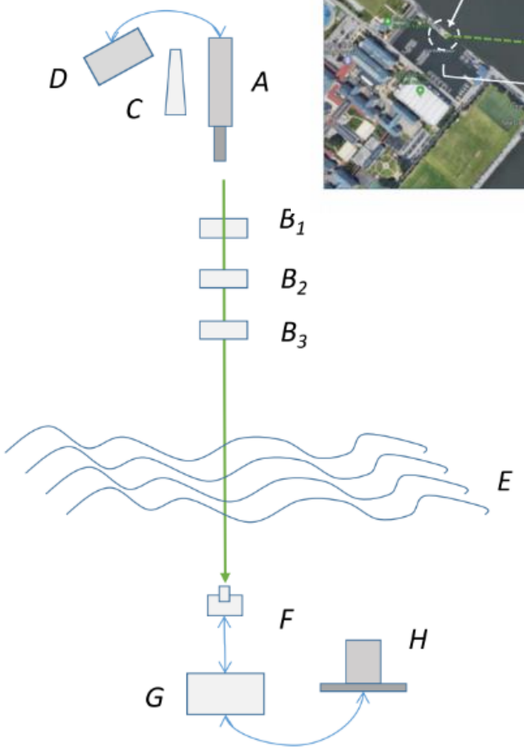
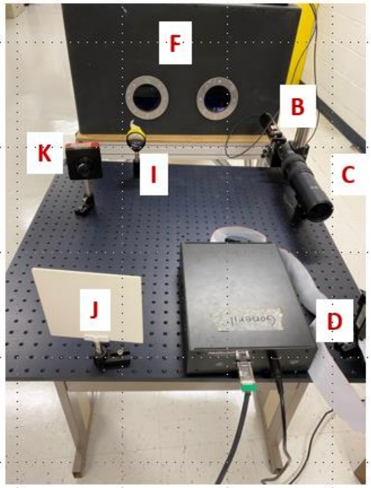
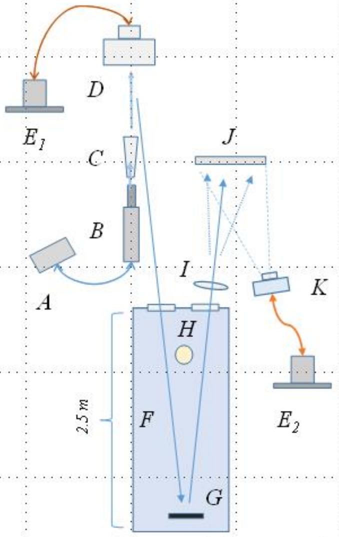


Fig. 2. Experimental equipment schematic, A—532 nm laser diode source, B_1—topological charge 1 plate with flip mount, B_2—topological charge 6 plate with flip mount, B_3—topological charge 8 plate with flip mount, C—optical scope, D—power supply, E—atmospheric channel, F—photodetector, G—DAQ, H—computer.





A - power supply. B - stabilized 2 mW, 632.8 nm He-Ne laser. C - beam expander. D - SLM. F - water tank. G - mirror. H- thermal generator. I - lens. J - display screen. K - camera. $E_{1,2}$ - mobile workstations.

- **Could we design laser beams that give us the advantage in constructing communication links in complex media?**
  - Consider transmitter (beam generation) , propagation environment and receiver (light intensity recoding and decoding of the transmitted message)
- Experiments with structured laser light conducted at the Naval Academy
- Field and lab measurements
  - Spatially partially coherent light
  - Light carrying orbital angular momentum
- OBJECTIVE Simulating structured light propagating through the media where the refractive index changes in space and time

COMSOL Simulations: Paraxial Wave Equation

Bi-harmonic PDE

$$\frac{\partial^2 u}{\partial \bar{x}^2} + \frac{\partial^2 u}{\partial \bar{y}^2} + i\gamma^2 \frac{\partial u}{\partial \bar{z}} = 0$$

PWE normalized the radial distances (x, y) and the propagation distance z, using beam radius at the source, w_0, beam curvature, F_0.

$$\bar{x} = \frac{x}{w_0} \quad \bar{y} = \frac{y}{w_0} \quad \bar{z} = \frac{z}{F_0} \quad \gamma^2 = 2k\frac{{w_0}^2}{F_0}$$

Separation of real and imaginary part of the field u.

$$u(\bar{x}, \bar{y}, \bar{z}) = p(\bar{x}, \bar{y}, \bar{z}) + iq(\bar{x}, \bar{y}, \bar{z}).$$

Gives us bi-harmonic PDE equations

$$\Delta\Delta p + \gamma^4 \frac{\partial^2 p}{\partial \bar{z}^2} = 0 \qquad \Delta\Delta q + \gamma^4 \frac{\partial^2 q}{\partial \bar{z}^2} = 0$$

Note: At this conference we present a poster and submit a paper on solving non-dimensional in cylindrical cavity using COMSOL and Garlekin method.

COMSOL set up

$$e_a \frac{\partial^2 s}{\partial t^2} + \nabla(-\nabla c)s + as = 0$$

Find the beam propagation along the z axis, assume that $\bar{z} = t$

Set the auxiliary variables $m_{real} = p$ $\quad n_{real} = \nabla\nabla m_{real}$

$$m_{imaginary} = q \qquad n_{imaginary} = \nabla\nabla m_{imaginary}$$

$$\nabla(-\nabla c)m_{real} - an_{real} = 0$$

$$\gamma^4 \frac{\partial^2 m_{real}}{\partial t^2} + \nabla(-\nabla c)n_{real} = 0$$

$$\nabla(-\nabla c)m_{imaginary} - an_{imaginary} = 0$$

$$\gamma^4 \frac{\partial^2 m_{imaginary}}{\partial t^2} + \nabla(-\nabla c)n_{imaginary} = 0$$

COMSOL MODEL

$$\left\{\begin{bmatrix} 0 & 0 & 0 & 0 \\ \gamma^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma^4 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \begin{bmatrix} m_{real} \\ n_{real} \\ m_{imaginary} \\ n_{imaginary} \end{bmatrix} = 0$$

COMSOL simulation : Boundary and Initial conditions

We chose Dirichlet boundary conditions for our COMSOL model.

Initial conditions

A solution for the complex field $u(\bar{x},\bar{y},\bar{z})$in the unbounded space using non-dimensional formulation is given:

$$u(\bar{x},\bar{y},\bar{z}) = I_0 \frac{1}{\left(i\frac{4}{\gamma^2}-1\right)\bar{z}+1} e^{-\frac{\left(1+i\frac{\gamma^2}{4}\right)(\bar{x}^2+\bar{y}^2)}{\left(i\frac{4}{\gamma^2}-1\right)\bar{z}+1}}$$

$$m_{real}(\bar{x},\bar{y},0) = I_0 e^{-(\bar{x}^2+\bar{y}^2)}\left(\cos\left(\frac{\gamma^2}{4}(\bar{x}^2+\bar{y}^2)\right)\right)$$

$$m_{imaginary}(\bar{x},\bar{y},0) = -I_0 e^{-(\bar{x}^2+\bar{y}^2)}\left(\sin\left(\frac{\gamma^2}{4}(\bar{x}^2+\bar{y}^2)\right)\right)$$

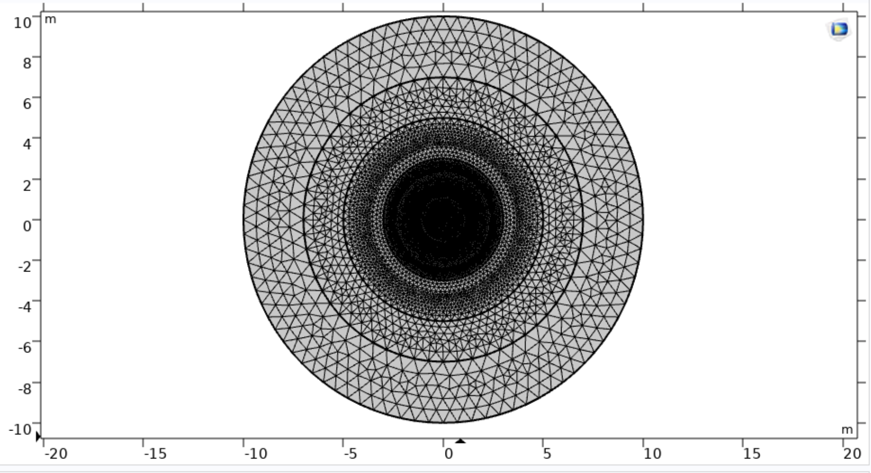
$$\frac{\partial}{\partial\bar{z}} m(\bar{x},\bar{y},0)_{real} = I_0 e^{-(\bar{x}^2+\bar{y}^2)}$$

$$\left(\begin{array}{c}\cos\left(\frac{\gamma^2}{4}(\bar{x}^2+\bar{y}^2)\right)(-2(\bar{x}^2+\bar{y}^2)+1)+\\ \sin\left(\frac{\gamma^2}{4}(\bar{x}^2+\bar{y}^2)\right)\left(\left(\frac{4}{\gamma^2}-\frac{\gamma^2}{4}\right)(\bar{x}^2+\bar{y}^2)-\frac{4}{\gamma^2}\right)\end{array}\right)$$

$$\frac{\partial}{\partial\bar{z}} m(\bar{x},\bar{y},0)_{imaginary} = I_0 e^{-(\bar{x}^2+\bar{y}^2)}$$

$$\left(\begin{array}{c}-\sin\left(\frac{\gamma^2}{4}(\bar{x}^2+\bar{y}^2)\right)(-2(\bar{x}^2+\bar{y}^2)+1)+\\ \cos\left(\frac{\gamma^2}{4}(\bar{x}^2+\bar{y}^2)\right)\left(\left(\frac{4}{\gamma^2}-\frac{\gamma^2}{4}\right)(\bar{x}^2+\bar{y}^2)-\frac{4}{\gamma^2}\right)\end{array}\right)$$

COMSOL Simulation



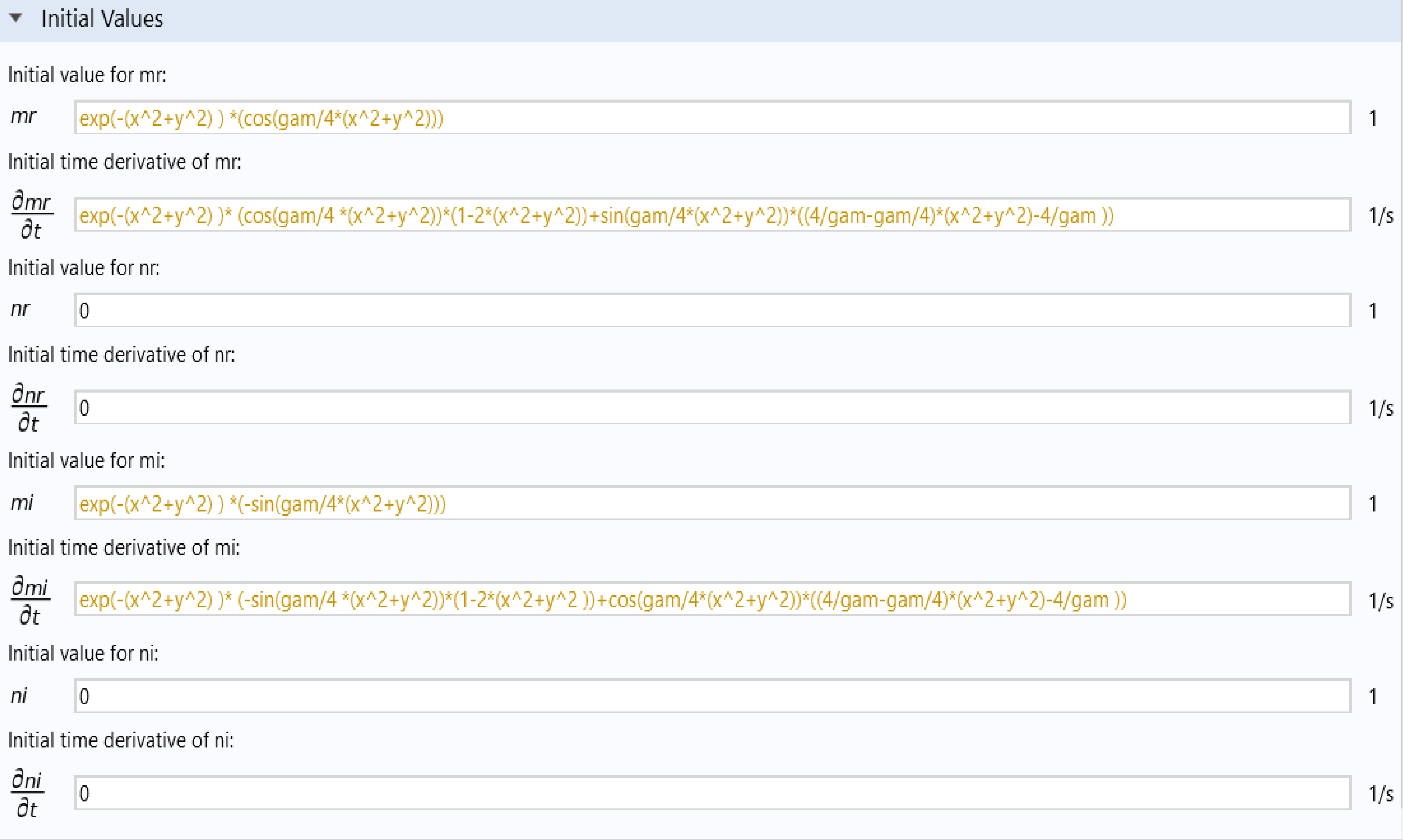
Mash (radius 10 m)

- Large enough so that the boundary conditions do not interfere wuth the beam propagation on axis
- using concentric circles to focus on the area of the beam intensity concentration

Initial conditions required by COMSOL

Note $\boldsymbol{n_r}, \boldsymbol{n_i}, \frac{\partial n_r}{\partial z}(\bar{x}, \bar{y}, 0)$ $\boldsymbol{and}$ $\frac{\partial n_i}{\partial z}(\bar{x}, \bar{y}, 0)$ are not necessary to solve the bi-harmonic equation, so we set them to zero. We run another COMSOL simulation in which we provided exact formulas for these IC.

Both COMSOL simulations produced identical results for the beam intensity on axis.



Initial Values

Initial value for mr:

mr: exp(-(x^2+y^2)) *(cos(gam/4*(x^2+y^2))) 1

Initial time derivative of mr:

$\frac{\partial mr}{\partial t}$: exp(-(x^2+y^2))* (cos(gam/4 *(x^2+y^2))*(1-2*(x^2+y^2))+sin(gam/4*(x^2+y^2))*((4/gam-gam/4)*(x^2+y^2)-4/gam)) 1/s

Initial value for nr:

nr: 0 1

Initial time derivative of nr:

$\frac{\partial nr}{\partial t}$: 0 1/s

Initial value for mi:

mi: exp(-(x^2+y^2)) *(-sin(gam/4*(x^2+y^2))) 1

Initial time derivative of mi:

$\frac{\partial mi}{\partial t}$: exp(-(x^2+y^2))* (-sin(gam/4 *(x^2+y^2))*(1-2*(x^2+y^2))+cos(gam/4*(x^2+y^2))*((4/gam-gam/4)*(x^2+y^2)-4/gam)) 1/s

Initial value for ni:

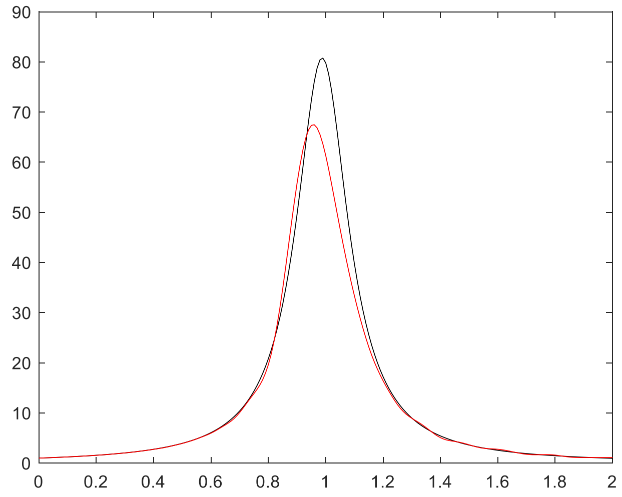
ni: 0 1

Initial time derivative of ni:

$\frac{\partial ni}{\partial t}$: 0 1/s

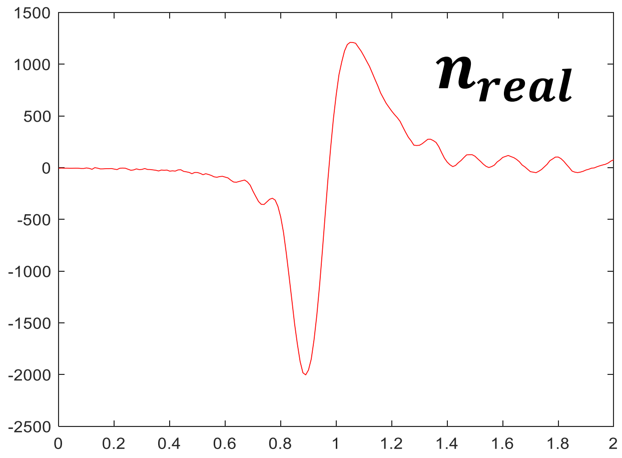
Results

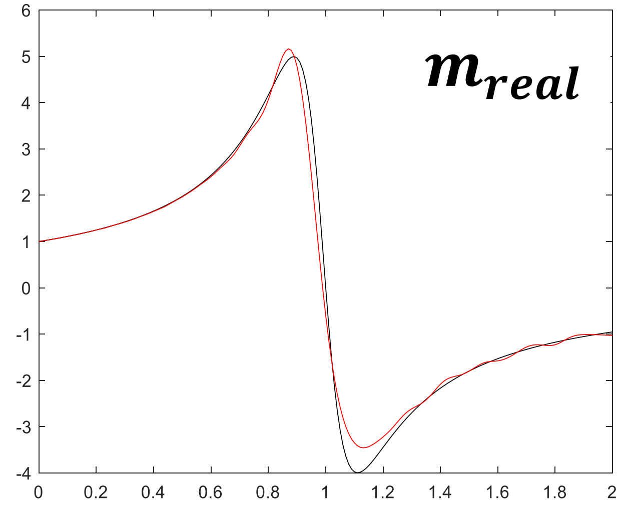
Beam Intensity

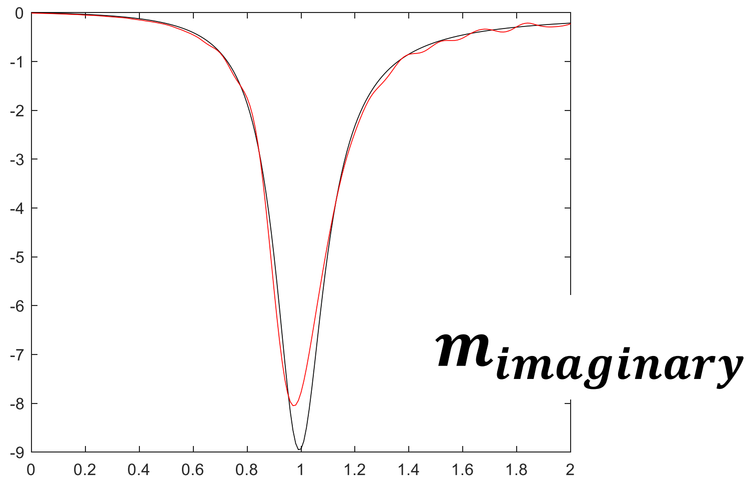


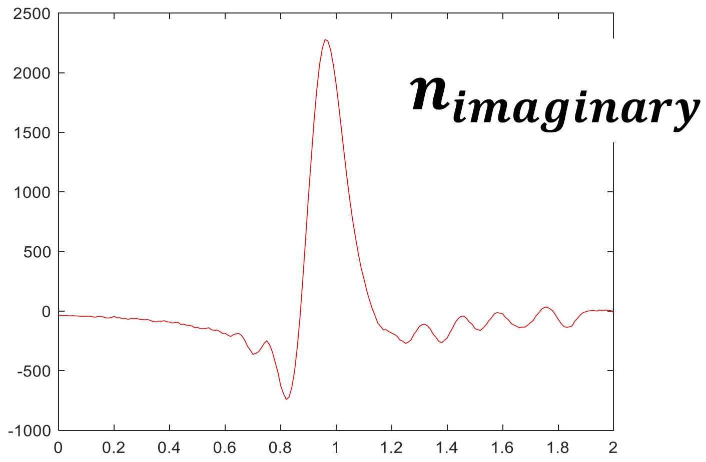
COMSOL simulation solutions (red) were compared with the solutions in the unbound space (black) for the on axis beam propagation.

- Clear discrepancies we observed in peak amplitude and location at the beam waist ($z = 1\ equavalent\ to\ z = 500\ m = F_0$)
- Pronounced oscillations of derivatives at longer propagation distances.

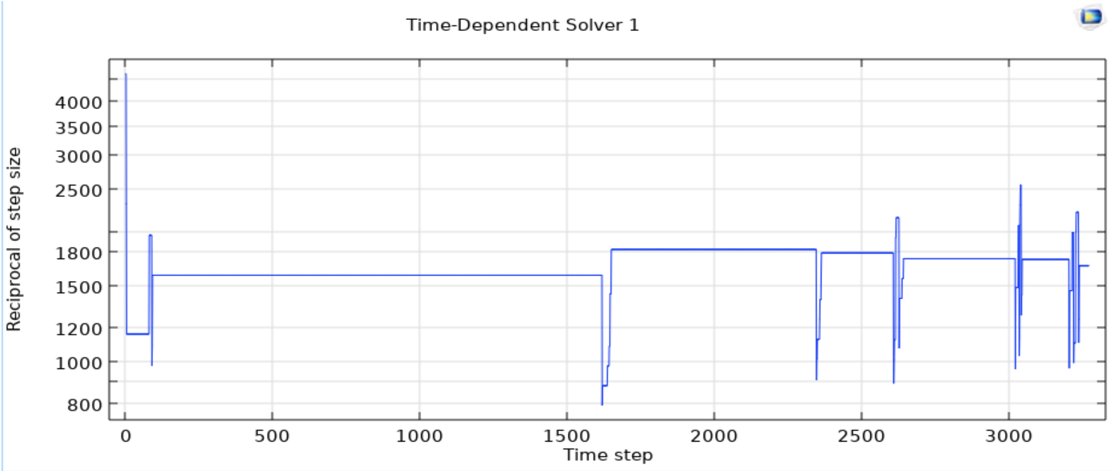








Next steps



➢ Verify the discrepancies between the exact solution in unbound space and COMSOL solution in cylindrical cavity.

➢ Evaluate convergence of COMSOL solution.

➢ If we consider propagation of structured light through complex environment we need to systematically study COMSOL solutions off axis, since the light intensity pattern across the beam carries the information.

➢ Complex medium introduced refractive index fluctuations in space and time and that needs to be taken in consideration.

γ^2*(function of space and time)*