Modeling Ferrofluid Sloshing Vibration Energy Harvesting using Level-Set method

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Abstract

Ferrofluid sloshing vibration energy harvesters are the recent addition in the domain of vibration energy harvesting systems. These systems are unique in using a liquid state transduction mechanism to harvest ambient vibrations/oscillations to generate electric power. In this paper, a 2-D representation of one such system is numerically simulated. The simulation consists of interface tracking between air and ferrofluid via the level-set method, modeling permanent magnets using the AC/DC module, and the general incompressible Navier-Stokes equations using the CFD Module. To establish the accuracy of the simulation, grid convergence studies are performed. Secondly, an additional validation of numerical algorithms in COMSOL is performed by mimicking a published experimental configuration for Ferrofluid Vibration Energy Harvesting. The extracted RMS voltages from the experimental results and COMSOL numerical simulation agree within 5% of the variance. Finally, the proposed system with four permanent magnets is analyzed.

Introduction

Ferrofluids are colloidal suspensions of ceramic-coated nanosized ferromagnetic particles like magnetite, titanium dioxide, etc., in a base fluid (or carrier fluid), which exhibits a fundamental property known as super-paramagnetism[1]. Super-paramagnetism allows the ferrofluid to orient itself in the direction of an externally applied magnetic field, as the ferromagnetic particles become magnetized. This unique property paves the way for various engineering applications of ferrofluids involving active flow control, like self-pumping, electronic cooling, mixing, navigation, refrigeration, lab on chip applications, microfluidics, etc.[1]–[4]. Sloshing refers to the movement of liquids inside partially filled containers, by virtue of external excitation[5], [6]. It is characterized as an internal free surface flow. Sloshing has typically been an area of interest for the shipping industry, the automobile industry, and intergalactic rocketry[5]. Most of these applications have tried to mitigate the effects of sloshing because, at large scales, a sloshing fluid can cause significant structural damage to the container[7]. Therefore, most research about sloshing has been done to mitigate the effects of sloshing by the use of baffles etc. However, recently a new use for sloshing has been developed for generating power from external oscillations. Performance characterization of one such energy harvesting configuration was conducted, and a plethora of knowledge regarding design rules for such a configuration was proposed[8].

In this work, a unique configuration for a ferrofluid vibration energy harvester (VEH) is proposed, exploiting the two symmetry planes of the harvester. The Level-set method is introduced to capture the movement of the free surface. Although the level-set method has been used to perform numerical studies on ferrofluid droplets[9], [10], it is believed that this is the first endeavor using the level-set method for ferrofluid sloshing. The following sections explain the proposed configuration and validation of COMSOL's computational algorithms for Ferrofluid sloshing[11].

Proposed Configuration & Governing Equations

Figure 1 represents the configuration with four magnets symmetrically placed at the center planes of the harvesting tank. The four magnets have length dimensions in terms of the length of the tank. The thickness of the magnets is a tenth that of the tank, and their width is a fifth that of the tank. The height of the tank is half of the length, and the fill level is 40 %. The length of the tank is set at 10 cm. The four magnets are also labeled from 1 to 4.

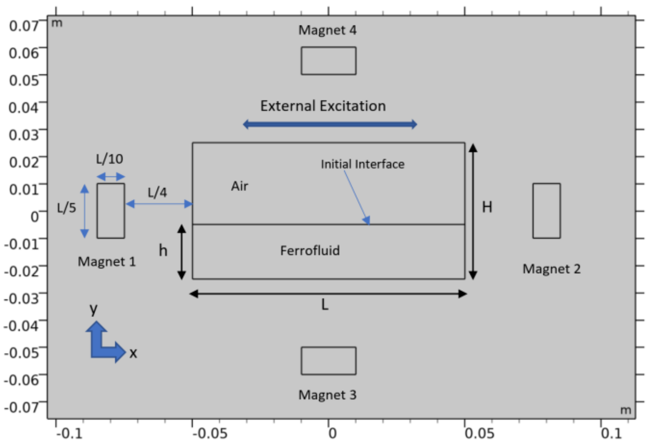

Figure 1 Symmetric Ferrofluid Vibration Energy Harvester configuration[11]

For convergence of the magnetic field, a large domain is created around the harvester. The domain is 10 times larger than the length scale of the harvester. The harvester is assumed to receive an external excitation which is periodic in nature and causes the ferrofluid to slosh inside.

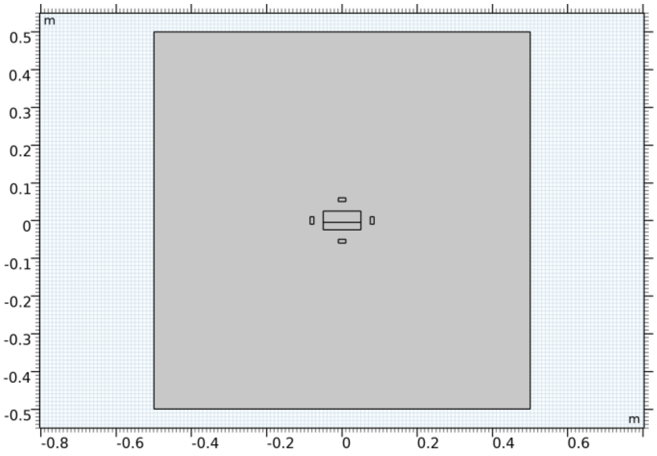

Figure 2 Harvester and the large computational domain surrounding it [11]

The flow inside the domain is governed by Navier Stokes equations and Maxwell's equations. The Navier-Stokes equations are as follows
Continuity:

$$\rho\nabla.\vec{V} = 0 \tag{1}$$

Momentum:

$$\rho\frac{\partial\vec{V}}{\partial t} + \rho(\vec{V}.\nabla)\vec{V} = -\nabla p + \mu\nabla^2\vec{V} + \overrightarrow{F_m} + \rho\vec{g} + \overrightarrow{F_{st}} \tag{2}$$

where, $\vec{V}$ is the velocity vector, $\overrightarrow{F_m}$ is the magnetic force, $\vec{g}$ is the external acceleration on the system, which consists of the external excitation and the acceleration due to gravity, and $\overrightarrow{F_{st}}$ Is the force of the surface tension. Here the external excitation is given as:

$$x = X_0 Sin(\omega t) \tag{3}$$

where X_0 is the amplitude of external excitation and ω the angular frequency, which is related to the excitation frequency f as $2\pi f$. In this study, f is varied such that the set of frequencies contain the first modal frequency. This makes the acceleration to be:

$$a_0 = -X_0\omega^2 Sin(\omega t) \tag{4}$$

Level-set equations for interface tracking:

$$\frac{\partial\phi}{\partial t} + \vec{V}.\nabla\phi = \gamma\,\nabla.\left(\epsilon_{ls}\nabla\phi - \phi(1-\phi)\frac{\nabla\phi}{|\nabla\phi|}\right) \tag{5}$$

where ϕ is the level-set variable, γ is the re-initialization parameter and ϵ_{ls} is the parameter which controls the thickness of the fluid-air interface.

For the fluid domain inside the tank, the properties must be expressed in terms of the level set variable. Density is given as:

$$\rho = \rho_1 + (\rho_2 - \rho_1)\phi \tag{6}$$

Viscosity is given as:

$$\mu = \mu_1 + (\mu_2 - \mu_1)\phi \tag{7}$$

where the indices 1 and 2 denote fluid 1 and fluid 2, which in our case is the ferrofluid and air, respectively. Similarly, to calculate the Kelvin body force, the magnetic properties of the fluids must be expressed in terms of the level set variable:

$$\overrightarrow{F_m} = \vec{M}.\nabla\vec{H} \tag{8}$$

where, $\vec{H}$ is the magnetic field, and the magnetization $\vec{M}$ is given by the constitutive relation:

$$\vec{M} = \chi_m\vec{H} \tag{9}$$

where, χ_m is the magnetic susceptibility and is obtained from the level set function for the whole domain as:

$$\chi_m = \chi_{m1} + (\chi_{m2} - \chi_{m1})\phi \tag{10}$$

Maxwell's conservation laws for the system are[1]:

$$\nabla.\vec{B} = 0 \tag{11}$$

$$\nabla\times\vec{H} = 0 \tag{12}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \tag{13}$$

also, the magnetic field can be written as:

$$\vec{H} = -\nabla V_m \tag{14}$$

where, V_m is the magnetic scalar potential.
The boundary conditions for the domain are the no flux boundary condition for the ends of the domain surrounding the harvester. The magnets are such that they have a uniform magnetization of 400 kA/m. The following are the constitutive relations for different regions:

$$\vec{\mathrm{B}} = \mu_0\vec{\mathrm{H}}\ \text{(for air)}\ ;\ \vec{\mathrm{B}} = \mu_0\left(\vec{\mathrm{H}} + \overrightarrow{\mathrm{M_s}}\right)\ \text{(for the magnets)}\ \&$$

$$\vec{\mathrm{B}} = (1 + \chi_m)\vec{\mathrm{H}}\ \text{for sloshing}$$

The electromotive force from sloshing is picked up by an external coil with N turns (which are varied to optimize power harvested). The electromotive force is given as:

$$\epsilon = -N\frac{d\Phi}{dt} \tag{15}$$

where Φ is the magnetic flux obtained by integrating the magnetic flux density over the area.
To model the above set of governing equations, the following modules were used in COMSOL Multiphysics:

a.) The CFD Module

b.) The AC-DC Module
c.) COMSOL Multiphysics

In the CFD Module, the inbuilt laminar flow with the level-set interface is used for simulating the separated flow through interface tracking, and the flow is assumed to be laminar. The magnetic fields and no current (mfnc) interface is used from the AC-DC module to obtain the magnetic scalar potential. The solver solves for the level-set variable, the pressure, and velocity and then the magnetic scalar potential in three segregated steps inside one segregated iteration. The BDF method is used for the time-dependent solver, and a time step of 2 milliseconds and a grid size of 0.001 m was found to be stable and accurate[11]. Also, the discretization was performed by first-order elements for all the four dependent variables.

Validation Study

To validate COMSOL and the level set method used in the computations, a comparative study was run replicating the published experimental and computational results by Liu et al. [12]. The combination 1H, as shown in Figure 3, is simulated, and the results are compared in Figure 4. The predictions from the numerical scheme adopted in the present work match very closely the reported computational and experimental data in [12], as shown in Figure 4 b) and a), respectively.

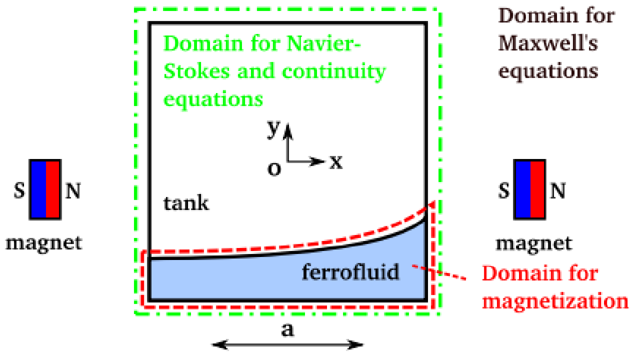

Figure 3 Ferrofluid Sloshing VEH configuration used for validating COMSOL sloshing simulations[12]

The peak voltage near the resonance frequency almost matches perfectly with the experimental data. However, since the spectrum of frequencies in the frequency sweep performed in the present work is not as closely spaced as reported by Liu et al., the internal energy pump, which occurs at around 1.5 Hz, is not as prominent in our simulation of their configuration as shown in Figure 4 b). Based on this validation, it is inferred that the level-set formulation and the solvers chosen can simulate the complexities involved in this flow.

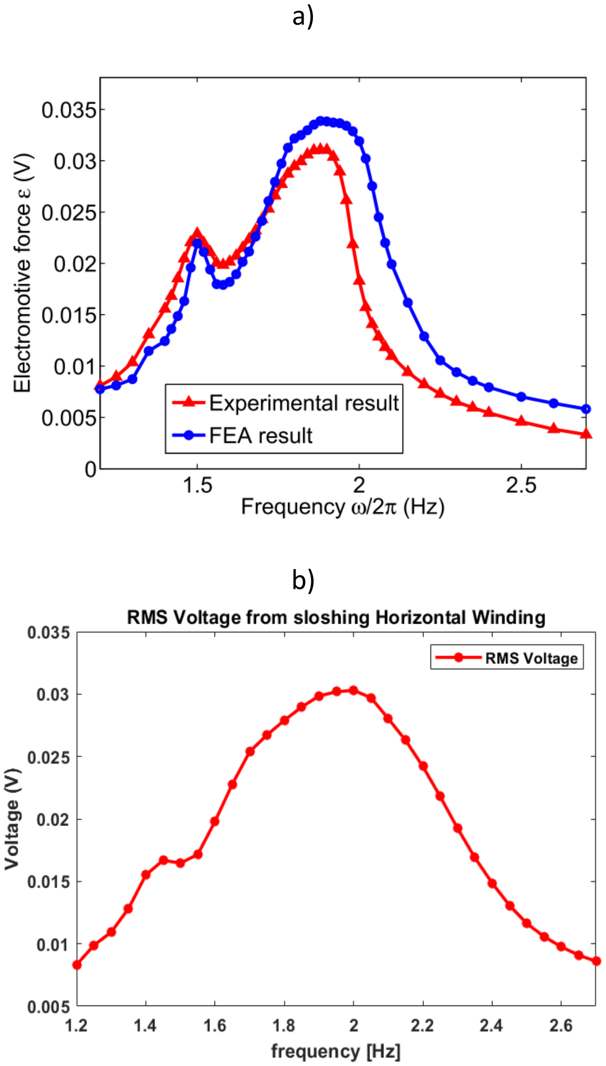

Figure 4 Comparison of Results from a) Ferrofluid sloshing configuration by Liu et al. [12] and b) COMSOL simulation of same configuration[11]

It must be noted that the RMS voltage obtained from COMSOL is by running the simulation for only 4 time periods and taking the RMS value of the instantaneous voltages obtained in the last three time periods of oscillation, leaving the first time period for stabilization. In the reference work, the simulation and experiment are performed for around 12 time periods, and the last 9 time periods are used for calculation of RMS voltage and power. This explains the slight difference between the Experimental data from Liu et al. and the validation study in COMSOL performed for the present work.

Results from the proposed configuration

The proposed configuration discussed before has the following orientation of magnets:

a.) Magnet 1 South pole facing the tank wall

b.) Magnet 2 North pole facing the tank wall
c.) Magnet 3 South pole facing the tank wall
d.) Magnet 4 North pole facing the tank wall

To analyze the characteristics of this magnet tank arrangement for ferrofluid vibration energy harvesting, an external acceleration magnitude of 1 m/s^2 is set for the external excitation. Then the system is swept from frequencies from 1.2 Hz to 2.5 Hz. As the frequency changes, the amplitude of the external excitation changes according to the constraint of the acceleration. An external pick up coil is assumed to be wound in either vertical (y-z) or horizontal direction (x-z). The coil wound in the vertical direction will pick up the electromotive force generated by the change of magnetic flux density in the x-direction (with time), and the coil wound horizontally will pick up the electromotive force generated by the change in magnetic flux density in the y-direction (with time). The energy harvester performance is then analyzed by assuming the properties of the pick-up coil. The number of turns of the coil are fixed to be 1000 turns and is made out of copper. The coil is also assumed to have a diameter such that when it is tightly wound around the tank, it will cover the whole pitch length in 1000 turns. Finally, to render the third dimension to the coil, it is assumed that the depth of the tank (dimension in z) is 10 cm. This gives the coil properties are given in Table 1.

Table 1 Electrical properties of the Horizontal and vertical coils

# of turns	Resistance of the horizontal coil (ohm)	Resistance of the vertical coil (ohm)	Inductance of the horizontal coil (H)	Inductance of the vertical coil (H)
1000	3422.47	641.71	1.20	0.77

Figure 5 shows the evolution of the surface of the ferrofluid sloshing inside the tank at 1 second.

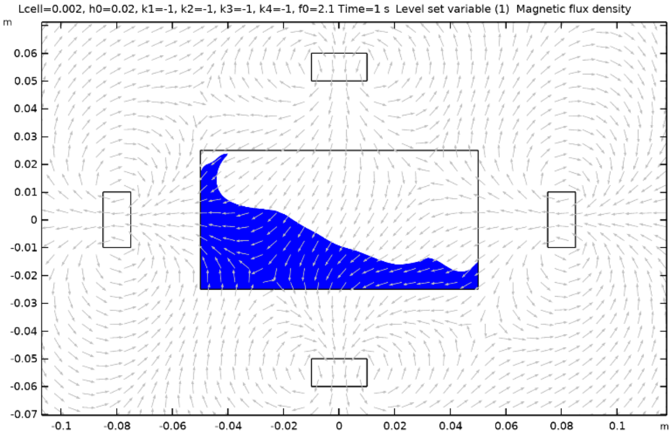

Figure 5 Surface evolution and magnetic flux density arrow lines for near resonance frequency

The magnetic flux density lines are also shown, intersecting the ferrofluid free surface and bending inside the ferrofluid, which is due to the discontinuity in magnetic properties of air and the ferrofluid. Also, the poles of the magnets can be discerned from the figure, where the flux lines emanate from the North Pole and end into the South Pole.

To obtain the maximum power from the system at different frequencies, an external load resistor is assumed to be connected to the pick-up coil. This load resistor was imparted 25 different resistances from 10^0 ohms to 10^5 ohms in equal exponential increments to find the most suitable resistor. Also, the maximum RMS voltages are found and are shown in Figure 6, for both horizontal and vertical coils. The highest voltage picked up by the vertical coil is around 0.015 V for a frequency of 2.1 Hz. The natural frequency of this sloshing tank without the magnetic fields should be around 2.0 Hz. The increase in the frequency can be attributed to the additional magnetic force on the ferrofluid in the downward direction, enhancing the gravitational field. The voltages in the horizontal coil are slightly higher at lower frequencies, and then for most other frequencies are much lower than the voltages induced in vertically wound coils. This is again dues to the fact that the motion of the ferrofluid in the vertical direction is mostly arrested by gravity and more importantly, the excitation is purely in the horizontal direction, which shows up as the variance in $B_{x,}$ which is responsible for the voltage induced in the vertically wound coil.

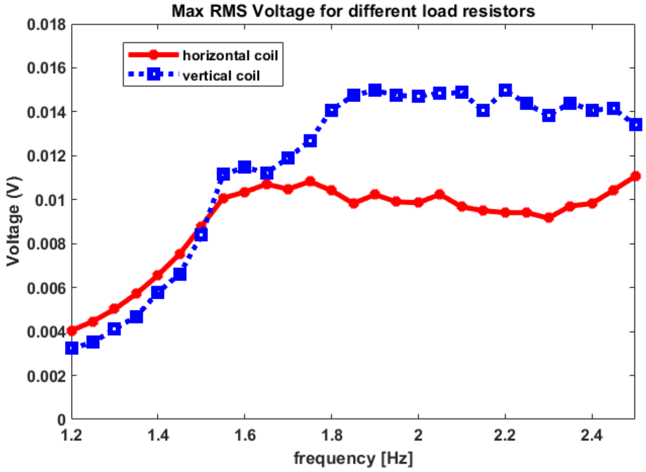

Figure 6 Maximum voltage obtained from impedance matching using 25 load resistors in the R-L circuit

From impedance matching for each frequency, the power output from the load resistor is calculated using the simple R-L circuit[8], [11], [13]. This yields the highest RMS power from the coils to be around 0.14 μW, from the Vertical coil, for a load resistor, the closest to the resistance of the coil. The highest power from the horizontal coils is one to two magnitudes smaller than the power output from the vertical coils. This is because the coil resistance of the horizontal coil is almost five times larger than that for the vertical coil, resulting in much higher dissipation in the pick-up coil. The second biggest reason is because of the lower voltages, which were recorded due to change in magnetic flux density components.

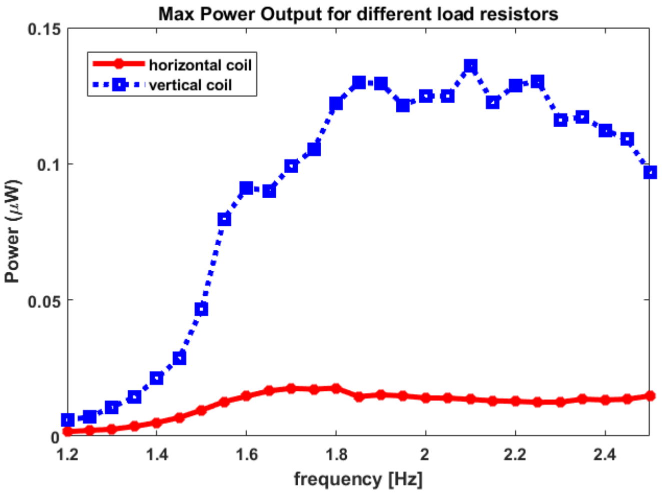

Figure 7 Maximum power output from the load resistor for both horizontal and vertical coils

Conclusions

In this work, a Ferrofluid Sloshing Vibration Harvesting System is simulated using COMSOL multiphysics. This work also serves the purpose of demonstrating the use of COMSOL Multiphysics and the level-set method to simulate such a system. The validation study for COMSOL's capability is conducted by mimicking an experimental study previously published in the literature. Finally, the proposed configuration with four magnets is analyzed, and its performance characteristics are determined. The highest power output from impedance matching is around 0.14 μW from the vertically wound coil.

References

[1] R. E. Rosensweig, *Ferrohydrodynamics*. Dover Publications (February 20, 2014), 1970.

[2] N. Anand and R. Gould, “Study of Enhanced Self Mixing in Ferrofluid Flow in an Elbow Channel Under the Influence of Non-Uniform Magnetic Field.” Jul. 28, 2019, doi: 10.1115/AJKFluids2019-5546.

[3] T. Völker, E. Blums, and S. Odenbach, “Heat and mass transfer phenomena in magnetic fluids,” *GAMM-Mitteilungen*, vol. 30, no. 1, pp. 185–194, 2007, doi: 10.1002/gamm.200790007.

[4] I. Nkurikiyimfura, Y. Wang, and Z. Pan, “Heat transfer enhancement by magnetic nanofluids — A review,” *Renew. Sustain. Energy Rev.*, vol. 21, pp. 548–561, 2013, doi: 10.1016/j.rser.2012.12.039.

[5] R. A. Ibrahim, *Liquid Sloshing Dynamics*. University Press, Cambridge, 2005.

[6] W. G. Price and Y. G. Chen, “A simulation of free surface waves for incompressible two-phase ows using a curvilinear level set formulation,” *Int. J. Numer. Methods Fluids*, vol. 51, no. May 2005, pp. 305–330, 2006.

[7] J. L. Bautista-Jacobo, E. Rodríguez-Morales, J. J. Montes-Rodríguez, and H. Gámez-Cuatzín, “Effect of Baffles on the Sloshing in Road Tankers Carrying LPG: A Comparative Numerical Study,” *Math. Probl. Eng.*, vol. 2015, 2015, doi: 10.1155/2015/359470.

[8] Q. Liu, M. F. Daqaq, and G. Li, “Performance Analysis of a Ferrofluid-Based Electromagnetic Energy Harvester,” *IEEE Trans. Magn.*, vol. 54, no. 5, 2018, doi: 10.1109/TMAG.2018.2801313.

[9] H. Ki, “Level set method for two-phase incompressible flows under magnetic fields,” *Comput. Phys. Commun.*, vol. 181, no. 6, pp. 999–1007, 2010, doi: 10.1016/j.cpc.2010.02.002.

[10] P. Capobianchi, M. Lappa, and M. S. N. Oliveira, “Deformation of a ferrofluid droplet in a simple shear flow under the effect of a constant magnetic field,” *Comput. Fluids*, vol. 173, pp. 313–323, 2018, doi: 10.1016/j.compfluid.2018.06.024.

[11] N. Anand, “Application of Ferrohydrodynamic Flows for Heat Transfer Enhancement and Energy Harvesting,” [Raleigh, North Carolina] : North Carolina State University, 2020., Raleigh, North Carolina, 2020.

[12] Q. Liu, S. F. Alazemi, M. F. Daqaq, and G. Li, “A ferrofluid based energy harvester: Computational modeling, analysis, and experimental validation,” *J. Magn. Magn. Mater.*, vol. 449, pp. 105–118, 2018, doi: 10.1016/j.jmmm.2017.09.064.

[13] S. Alazmi, Y. Xu, and M. F. Daqaq, “Harvesting energy from the sloshing motion of ferrofluids in an externally excited container: Analytical modeling and experimental validation,” *Phys. Fluids*, vol. 28, no. 7, 2016, doi: 10.1063/1.4954787.