Time Domain Simulations of Nonlinear Plasma Conductivity using Microwave Inductive Coupling

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Introduction

The use of inductive coupling in order to excite plasma has many advantages compared to capacitive coupling. The plasma reaches higher density and higher purity because ions do not touch the walls and the electrodes. However, the current paths are not predefined. Moreover, excitation at low power with the same arrangement is always in the capacitive mode.

Attaining a certain power, the device switches from the capacitive or so called E-mode to the inductive or H-mode. Simple observation of this transition is performed in steady state and the numeric analysis is mostly done in frequency domain, defining a clear level of power for the transition, the deposition power being directly connected with the electron density in the plasma.

The second aspect in the inductive coupling is the role of the strong, time dependent, magnetic field on the current paths. Usually one considers the plasma conductivity as a scalar; therefore the current density lines are parallel to the electric field lines. Since the magnetic field is the predominant force driving the currents, one expects a Lorentz force acting perpendicularly on the current paths, deviating their direction from the electric field lines. When the magnetic field is constant, one speaks of the Hall conductivity tensor. However, in the case of inductive coupling the RF or microwave magnetic field induces a time dependent conductivity tensor. Because of this time dependency, one expects harmonics in the induced currents. Indeed, such harmonics have been experimentally detected by 3 - 6 MHz excitation [1] and in our experiments at 2.45 GHz. Attempts to theoretically analyze this nonlinear behavior are few, either using a quasi-analytic way [2] or using a plasma-specific numerical analysis, the particle in cell [3].

The presented approach uses time domain analysis of the plasma, employing the Electromagnetic Waves Transient part of the RF Module of COMSOL. One result is a detailed representation of the current paths as a function of time, demonstrating that for intermediate electron densities the excitation oscillates within a period between E-mode and H-mode. The second result is that at high magnetic fields and high electron density, the current density evolution is no longer harmonic.

Experimental Set-up

The microwave device capable to inductively generate a plasma is sketched in Fig. 1. It consists of a copper block with a hole and a slit that represent electrically a one turn coil and a capacitor. In the hole a quartz tube is placed in which the plasma is excited. A detailed analysis of the plasma excitation regimes is presented in [4].

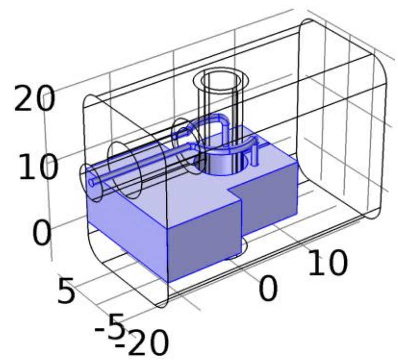

Figure 1. 3D geometry of the employed resonator; dimensions are in mm.

A time dependent analysis accounting for all technical details would be too complicated. The standard simplification is to consider a homogeneous plasma in the cross-section of the tube and also along the vertical z-axis. Therefore, the chosen 2D geometry for the actual simulations is that presented in Fig. 2.

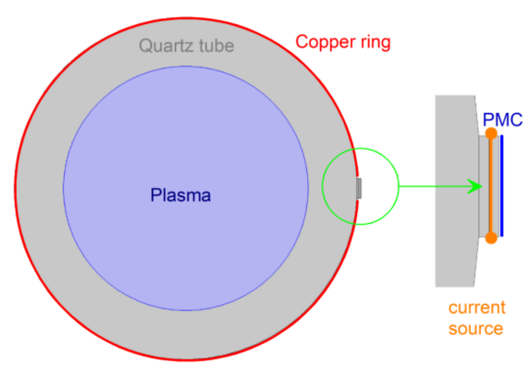

Figure 2. 2D geometry used in time domain simulations

The quartz tube has an outer diameter of 7 mm and an inner diameter 5 mm. The slit capacitor is not considered and the excitation is ideally represented by a current source (orange).

Behind it, the simulation area terminates with a perfect magnetic conductor (PMC, blue).

Theoretical approach

In order to implement the time variation of the conductivity, the transient simulations have been implemented in short time slices Δt. In this short time the magnetic field is considered constant. In the 2D geometry the electric field has the E_x and E_y components and the magnetic field only H_z. The equation of motion, by taking into account the Lorentz force is:

$$
\begin{aligned}
m\left(\frac{dv_x}{dt} + \nu v_x\right) &= e\left(E_x + v_y B_z\right) \\
m\left(\frac{dv_y}{dt} + \nu v_y\right) &= e\left(E_y - v_x B_z\right),
\end{aligned} \tag{1}
$$

where m, e, and ν are the electron mass, charge and scattering frequency respectively and v_x, and v_y are their drift velocities in the x and y directions. If one makes the substitutions:

$$
\begin{aligned}
j &= nev \\
\sigma &= \frac{ne^2}{m\nu} \\
\omega_c &= \frac{eB}{m},
\end{aligned} \tag{2}
$$

where j is the current density, σ the DC conductivity, n the electron density, and ω_c the cyclotron frequency, the equations (1) can be written:

$$
\begin{aligned}
j_x' + \nu j_x &= E_x \nu\sigma + \omega_c j_y \\
j_y' + \nu j_y &= E_y \nu\sigma - \omega_c j_x
\end{aligned} \tag{3}
$$

The derivatives $j_{x,y}'$ are furthermore transformed into finite differences over Δt:

$$
j_x' = \frac{J_x - J_{x0}}{\Delta t} \ and\ j_y' = \frac{J_y - J_{y0}}{\Delta t} \tag{4}
$$

The current densities in the new time slice can be expressed as a function of the old values $J_{x,y0}$, the electric field $E_{x,y}$ and ω_c, or, alternatively, as a function of the below defined conductivity tensor.

$$
\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_d & \sigma_o \\ -\sigma_o & \sigma_d \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \tag{5}
$$

Using the above relations one can express the diagonal and off-diagonal conductivities at the end of a time slice Δt.

$$
\begin{aligned}
\sigma_d &= \frac{E_x J_{x0} + E_y J_{y0}}{E_x^2 + E_y^2} + \Delta t\nu\left(\sigma - \frac{E_x J_{x0} + E_y J_{y0}}{E_x^2 + E_y^2}\right) + \Delta t\omega_c \frac{E_x J_{y0} - E_y J_{x0}}{E_x^2 + E_y^2} \\
\sigma_o &= \Delta t\omega_c \frac{E_x J_{x0} + E_y J_{y0}}{E_x^2 + E_y^2} + (1 - \Delta t\nu)\frac{E_x J_{y0} - E_y J_{x0}}{E_x^2 + E_y^2}
\end{aligned} \tag{6}
$$

Obviously, if ω_c=0, the off-diagonal term σ_o=0 because of the symmetry: $E_x J_{y0} = E_y J_{x0}$ and when $\nu \to \infty$, $\sigma_d \to \sigma$ (the dc value).

The computational algorithm is as follows: COMSOL calculates for 20 periods the field evolution starting with zero initial values. The generic field from which all others are calculated is the vector potential **A**. In this initial evaluation the conductivity is set to the dc scalar and a homogeneous value for σ in the whole area of the plasma. At the end of this simulation, knowing the electric field and current density distribution as a function of the coordinates *(x,y)*, and using the equations (6), one calculates the distributions $\sigma_d(x,y)$ and $\sigma_o(x,y)$. These two distributions are saved and used in the calculation over the new time slice Δt. The initial values of the fields **A** and **A'** are also taken from the last values of the previous time slice Δt. The time discretization Δt is set to be 1/40 of a period of oscillation. After approx. 5 oscillations the simulation tends to a new stable regime.

In the software structure there are two COMSOL subroutines that are saved in Matlab language. The first subroutine calculates the initialization of the system over 20 periods. The second one calculates the evolution over the time slice Δt. The main Matlab program automatizes the saving and reading of data over the required number of time slices Δt.

Experimental Results

Fig. 3 shows our microwave miniature plasma source in two states, excited at low powers in capacitive mode and excited at high power in inductive mode.

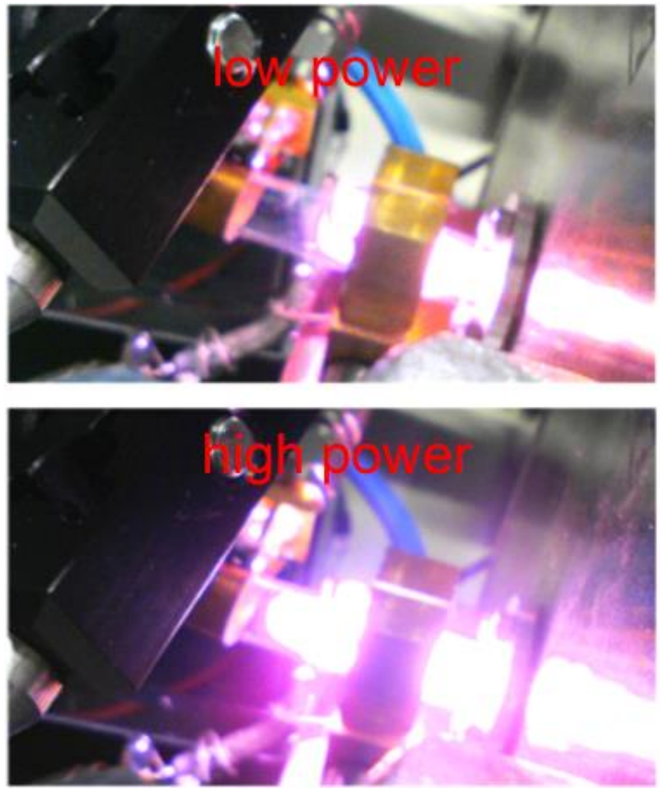

Figure 3. Plasma column in two excitation regimes E-mode (top) and H-mode (bottom).

A copper split ring around the quartz tube placed in the plasma column, but far from the excitation resonator, detects the magnetic field induced by the plasma currents. This little coil is connected to a spectrum analyzer Advantest U3751.

COMSOL simulations in frequency domain predict for low electron density current paths along the electric field lines, Fig. 4. The red thick line shows the essential expected behavior.

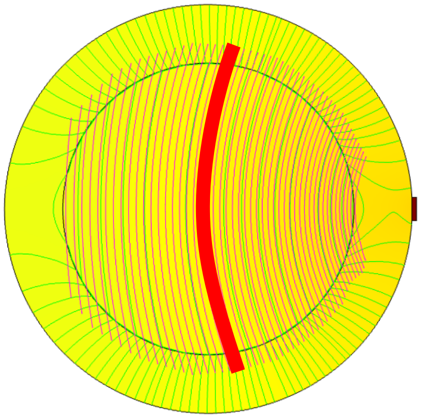

Figure 4. Electric field (green) and current density (red) typical field lines for low electron density, as simulated for a homogeneous plasma.

The experiment shows, however, a different current path, Fig. 5.

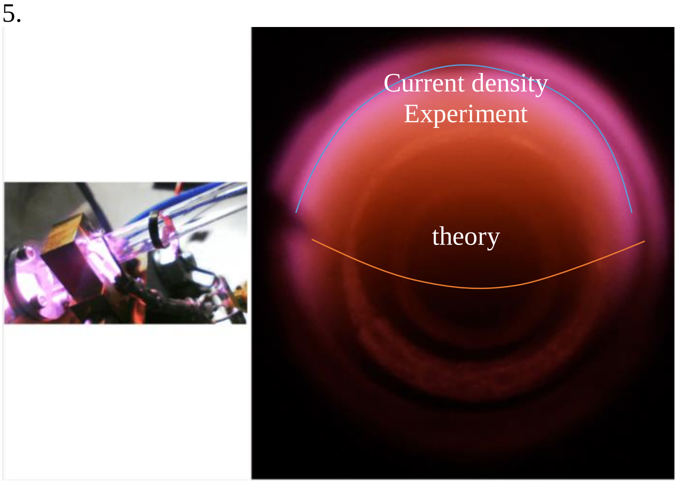

Figure 5. Left: Plasma source and the detecting coil. Right: Axial photograph showing the current distribution at low excitation power. The slit in this experimental picture is on the top.

Fig. 6 shows the presence of harmonics at high power excitation, compared to the low power, as measured with the spectrum analyzer.

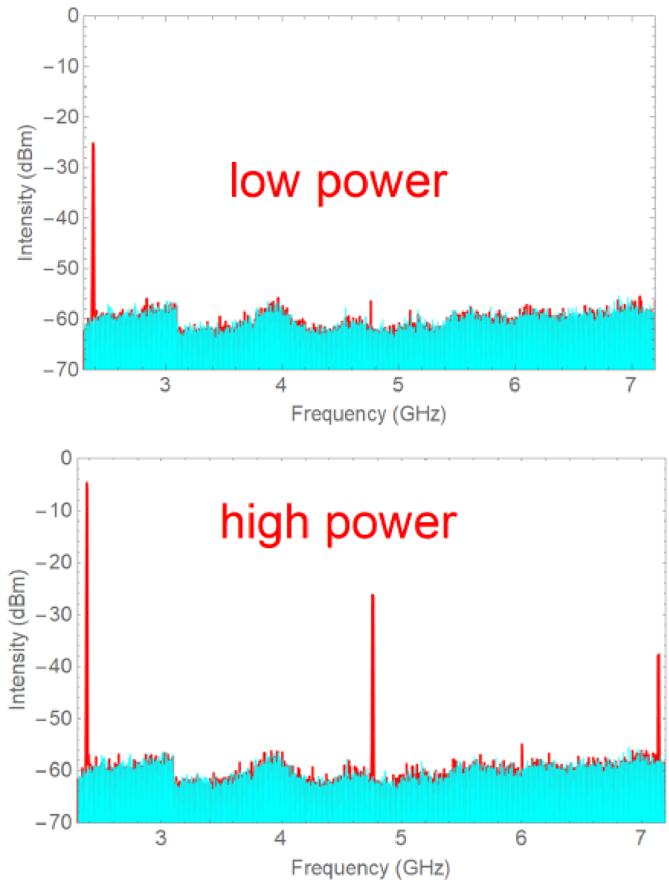

Figure 6. Second and third harmonic detection at 4.90 and 7.35 GHz at high power excitation.

Similar experiments, at the same power have been performed without plasma, in order to check whether the power amplifier generates at 40 W these harmonics. In this case, no harmonics within 50 dBm have been detected.

Simulation Results / Discussion

A simulation at low electron density (n_e=10^{17} m^{-3}), Fig. 7 shows a closer picture to the experiment presented in Fig. 5.

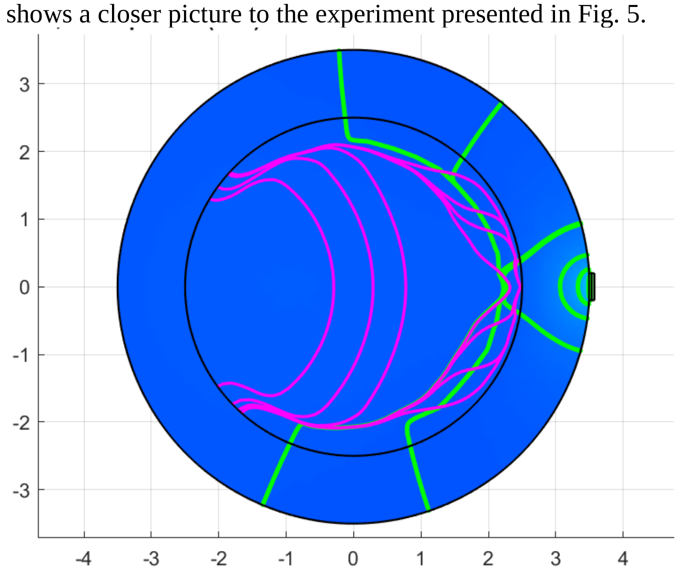

Figure 7. Current density (magenta) and electric field (green) distributions for low electron density. Dimensions are in mm. The slit in all simulations is on the right.

A typical distribution for high electron density (n_e=10^{19} m^{-3}) is plotted in Fig. 8, where the current paths are closed, (mostly) not touching the walls.

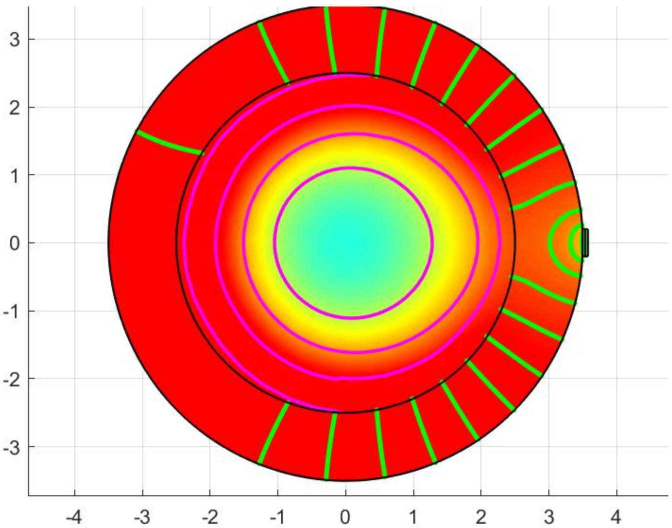

Figure 8. Current density (magenta) and electric field (green) distributions for high electron density. The surface shows the magnetic field intensity. Red means maximum in one direction, and cyan a minimum in center due to screening.

The nonlinearity can be seen in the time dependence of the current and of the magnetic field. A comparison of the variations for low and high density of electrons in plasma is shown in Fig. 9.
At least in the frame of the actual simulations the nonlinearity appears as a noisy curve.

Moreover, inspecting the currents at different times, one observes a quite complex evolution of the current paths, Fig. 10.

Discussion

As expressed in eq. (6), the competing terms in the expression of σ_d and σ_o are the excitation frequency ω, that appears indirectly in the time slice Δt, the scattering frequency ν and the cyclotron frequency ω_c. Our preliminary estimations show that ν is at least one order of magnitude higher than ω, while ω_c, for the estimated currents of 1 – 10 A is four orders of magnitude lower. This contradicts the statement that the Lorentz force is mainly responsible for the changed current path, as stated in [2, 3].

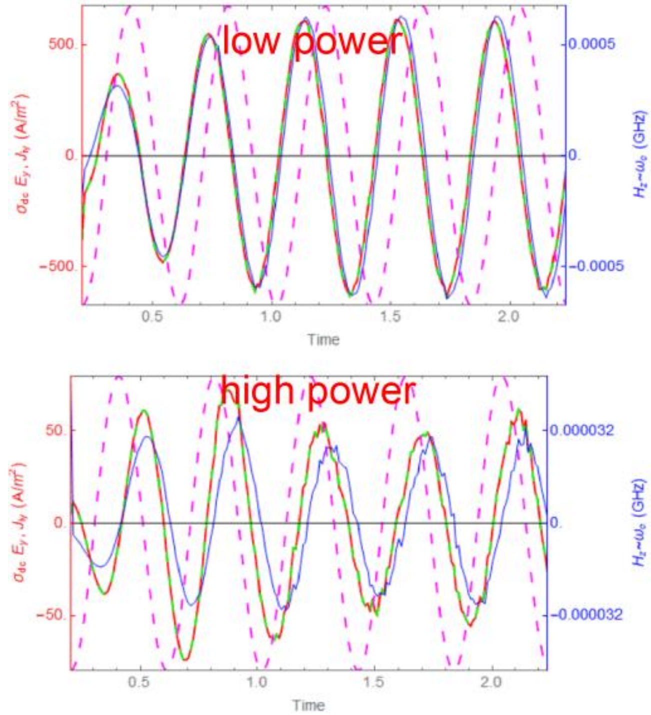

Figure 9. Current density (dashed red) and electric field (green) time variations for low and high electron density (low and high excitation power) measured in the center of the plasma column. The magnetic field is plotted with blue line and the excitation current with dashed magenta.

Conclusions

The presented tool offers the possibility to investigate in time domain the evolution of the current paths in an inductively coupled plasma. Further investigations should include different other contributions, like diffusion of ions and inhomogeneous plasma. Also one has to check the role of the Lorentz force, as stated in [1-3].

References

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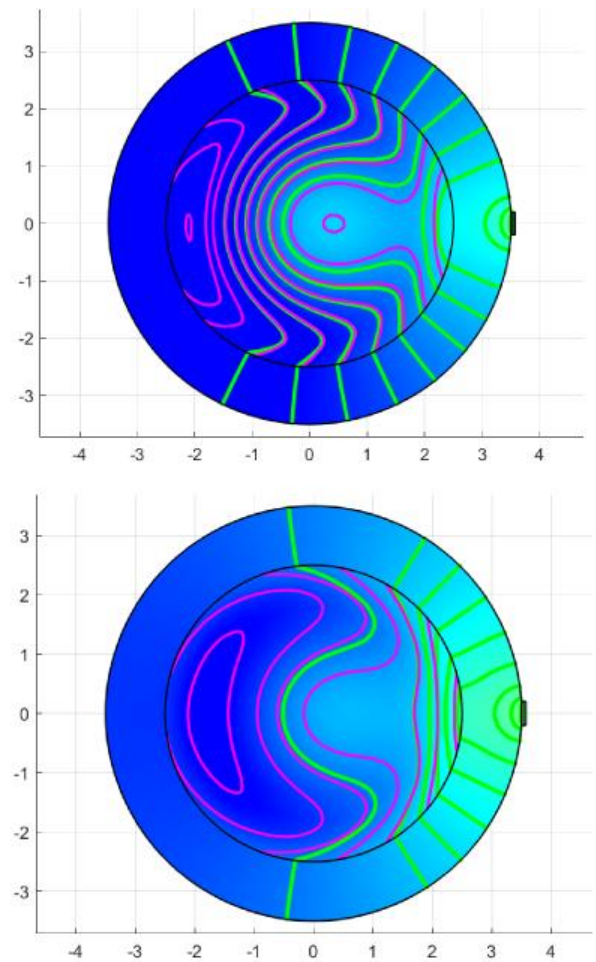

Figure 10. Current paths evolution during the time for an intermediate electron density (n_e=10^{18} m^{-3}).