

Studies of Sound Radiation From Beams with Acoustic Black Holes

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Abstract: Recently, Acoustic Black Holes (ABH), a new passive structural modification approach to control vibration and noise from mechanical structures has been developed and studied. An Acoustic Black Hole is usually a power-law taper profile due to which the wave velocity gradually reduces to zero. Also, the vibration energy gets concentrated at the locations of Acoustic Black Holes due to the progressive reduction of wavelength. This paper presents the work on the influence of acoustical black holes on the vibration and sound field of beams. The studies investigate the vibrations and acoustic near fields of cantilever beams. The aim of this study is to investigate the influence of the geometry of Acoustic Black Holes on the vibration noise radiated from the cantilever beams.

Keywords: Acoustic Black Hole, Structural Acoustics.

1. Introduction

Vibration and noise control of mechanical structures plays an important role in the design of many industrial systems. With the development of high-speed machinery, the vibration and noise control becomes important for mechanical system performance as well as environmental noise impact¹. The two well-known methods for passive control of structural vibrations, which also results in a reduction of noise are constrained layer damping and tuned dynamic absorber^{3,4}. The first method is based on using a viscoelastic layer attached to the structure and the second method needs an attachment of additional weight² to the target structure. Additionally, the active vibration control devices are also used for vibration damping⁵. However, these methods require consistent input energy and more particular design. So for the reason of limitation of size, budget or weight, sometimes it is not possible and also undesirable to use these above methods. There has been an effort in the literature to look for an additional effective passive approach for the control of vibrations⁶.

Recently a new approach for passive vibration control, Acoustic Black Holes (ABH), was developed. The ABH effect was first discovered by C.L. Pekeris in 1946⁶. He exploits the central physical principle of ABH: the phase velocity of sound waves that propagate in a stratified fluid is progressively decreasing to zero with depth increasing⁷. In 1988 M.A. Mironov, figured out that a flexural wave propagates in a thin plate slows down and needs an infinite time to reach a tapered edge. He also showed “the reflection coefficient can become quite large, quickly converging to one”⁸. Later, V.V. Krylov showed that the new approach of damping with ABH can significantly reduce these reflections with performing some experimental work^{9,10}, and also indicated that the ABH approach results in a significant amount of energy to be absorbed with a small amount of material attenuation near the edge. He also applied ABH to design some product such as turbofan blades¹¹. Recently, F. Semperlotti and S.C. Conlon used numerical modeling to analyze the ABH effect. They showed the application of ABH in vibration and noise control and indicated the design parameters for ABH^{12,13,14}. This paper presents the numerical analysis of cantilever beam with ABH at the free end. The study investigates the effect of ABH parameters.

2. Theoretical Analysis

M.A. Mironov⁹ showed that the bending wave speed goes to zero for beams and plates whose thickness decrease according to

$$h(x) = ax^m \quad (1)^{15}$$

However in reality, due to the limitation of manufacturing technics, it is impossible to build a zero thickness, so there will be a residual thickness b at the free end. Then the equation of the power-law curve becomes

$$h(x) = ax^m + b \quad (2),$$

where m is a positive rational number and a is a constant¹⁶, the scheme is shown in Figure 1.

The phase velocity C_p and group velocity C_g ¹⁷

$$C_p = \sqrt[4]{\frac{E}{12\rho(1-\nu^2)}\sqrt{\omega(ax^m + b)}} \quad (3)$$

$$C_g = \sqrt[4]{\frac{4E}{3\rho(1-\nu^2)}\sqrt{\omega(ax^m + b)}} \quad (4)$$

Where E is the Young's modulus, ν is the Poisson's ratio, ρ is the density, h is the thickness of the plate, and ω is the angular frequency of flexural wave.

When $b = 0$ and $x \rightarrow 0$, the phase velocity and group velocity tends to zero.^{18,19}

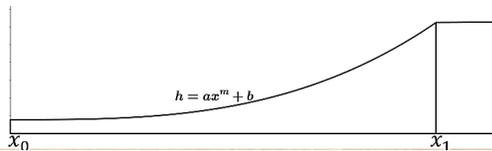


Figure 1. Power-law profile

Propagation time

$$T = \int_{x_1}^{x_0} 1/c_g dx$$

$$= \sqrt[4]{\frac{12\rho(1-\nu^2)}{E\omega^2}} \frac{1}{2-m} (x_0^{1-m/2} - x_1^{1-m/2}) \quad (5)$$

Where E is the Young's modulus, ν is the Poisson's ratio, ρ is the density, and ω is the angular frequency of flexural wave.

x_1 is the length of the ABH part, so when x_0 tends to 0, the T tends to infinity, only if $m \geq 2$.¹²

The theoretical analysis indicates the ABH can make wave speed decrease and also make the vibration energy concentrate at the ABH location.

3. Use of COMSOL Multiphysics® Software

Beam type structure is one of the basic element of vibration and acoustic analysis. The coefficient a and the power m in Equation (2) affect the geometry of the Acoustic Black Hole. The influence of m and a on the vibration and acoustic near field of several cantilever beams are studied with COMSOL. The beam dimensions should satisfy that the ratio of the length and the width is larger than 10:1 and the ratio of the thickness and

the width is larger than 1:20²⁰ to ignore the rotation and shear force.

This study focuses on the third mode of several Aluminum beams which are 10 in long, 1 inch wide and 1/8 inch thick, with various power-law profile tapered at the free end with the residual thickness b equal to 1/64 in as shown in Figure 2. The beam is excited with the load of $10 \sin \omega t$ N at 0.7 of the length of the targeted beam from the fixed end.

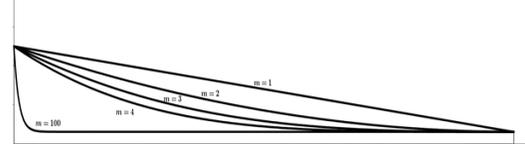


Figure 2. Various shape of power law profile of m

The mode shape analysis by COMSOL Structural Mechanics Module and the acoustic near field (0.1 inches below the beam) analysis by COMSOL Acoustics Modulus of several beams was compared to observe the change of nodes location with the change of m and a value.

Vibration energy is embodied by displacement of the beam. The displacement of beam without ABH and beam with ABH of $m=2$ are compared to show the vibration energy concentration effect of the ABH. To avoid the affect by natural frequency a length correction (decrease from 10 in to 9.4 in) is applied to the beam without ABH.

4. Numerical Analysis Result

4.1 Nodes Location of different m values

Table 1: The nodes locations and natural frequency for the third mode of various of a values

m value	node 1 location x_1/L	node 2 location x_2/L	Natural Frequency (Hz)
0	0.5	0.863	715.0
1	0.484	0.831	768.6
2	0.475	0.825	791.3
3	0.478	0.813	802.5
4	0.474	0.811	806.5
100	0.452	0.781	893.4

With fixed length of ABH part the m value actually affects the depth of the power-law profile which is shown in Figure 2. Table 1 shows the natural frequency and node locations of the third mode for different m values, where x_1 is the

distance between the first node and the fixed end, x_2 is the distance between the second node and the fixed end. The node values decrease and the frequencies increase with the increase of m value, which means when the wave propagates from fixed end to the free end, the wave speed decreases due to ABH, and decrease more quickly with the increase of m value due to the effect of ABH which is also shown in Figure 3.

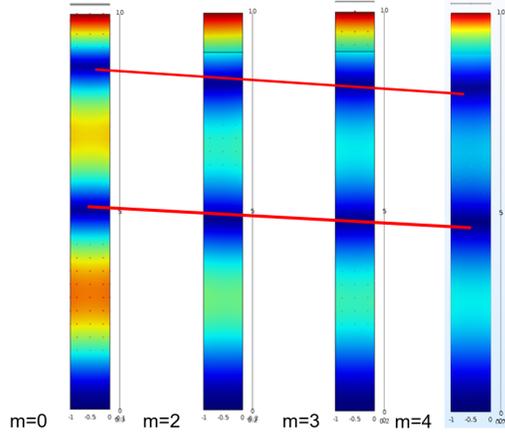


Figure 3. 3rd model shape simulation results. The bottom is the fixed end and the top is the free end. Blue color shows the node location and red color shows the peak value locations.

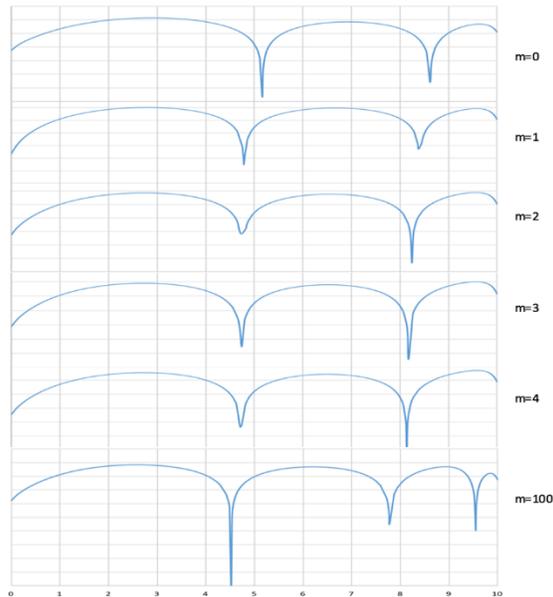


Figure 4. The acoustic near field for various m values. The figure is made by Excel with the data generated by COMSOL Acoustics Module.

Acoustic near field can express the vibration behavior of the target structural. Figure 4 shows the sound pressure level of near field of several beams. It is also seen that the node locations move towards the fixed end with the increase of m value.

4.2 Nodes Location of different a values

Equation 6 shows with the values of m and b fixed, the a value affects the length of the ABH cut end.

$$L_{ABH} = \sqrt[m]{\frac{h-b}{a}} \quad (6)$$

Where h is the thickness of the beam b is the residual thickness of the ABH part, and m is the power of the power-law profile. In this section, $m=3$, $h=1/8$ in and $b=1/64$ in.²¹

Table 2 shows the node location for the third mode for different a values with $m=3$, where x_1 is the distance between the first node and the fixed end, x_2 is the distance between the second node and the fixed end. Figure 5 shows the changes in node locations as a function of varying values of a which indicate the length of ABH for a given value of m . It is clear seen from the Table 2 and Figure 5 that for the value of ABH length 1.25 inch the nodes have moved closest to the fixed end.

Table 2: The nodes locations and natural frequency for the third mode of various of a values

a value	Length of the ABH (in)	node 1 location x_1/L	node 2 location x_2/L
	0	0.5	0.863
7/64	1	0.478	0.813
7/125	1.25	0.473	0.809
7/216	1.5	0.475	0.813
7/343	1.75	0.48	0.829

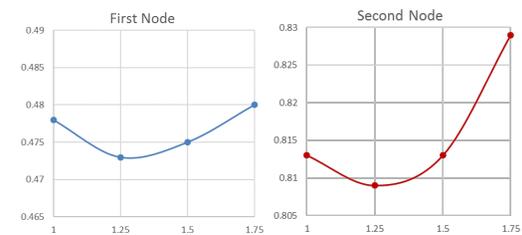


Figure 5. The relative first (left, blue line) and second (right, red line) node location for various length of the ABH.

4.3 Vibration Concentration Effect

The response of regular cantilever beam without cut end with length correction and beam with cut end in the third mode shape are shown in Figure 6 and Table 3.

With length correction the length of the beam, the natural frequency of the beams is the same (≈ 791 Hz) in 3rd mode.

Table 3: The peak values for 900 Hz force excitation

Peak Value	Beam w/o ABH with length correction 9.4 in	Beam with ABH 10 in
Anti-node 1	0.0019 in	0.0018 in
Anti-node 2	0.0021 in	0.0019 in
Free End	0.0024 in	0.0031 in

The displacement increases at the location of ABH and decreases at the other two anti-nodes location, because the input energy of the two beams are the same, due to energy conservation, the result indicate the vibration energy is concentrated at the location of ABH.

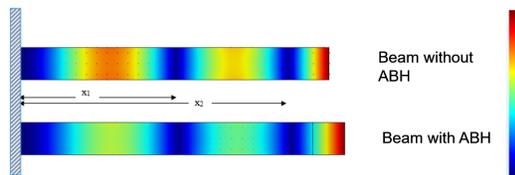


Figure 6. The third mode shape of normal cantilever beam (top) and with ABH (bottom, $m=2$), with left end fixed and right end free. The blue color is the displacement of zero (node), and the red color is the maximum displacement.

5. Experimental Result

Experimental work is developed as the flow chart shown in figure 7 and 8 respectively. All the data are the average value of five groups of test values.

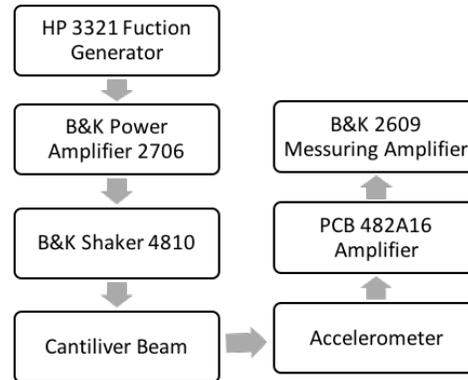


Figure 7. Flow chart of experiment



Figure 8. Experiment setup

5.1 Node locations

The experiment compares the third mode node locations of beam without ABH and beam with ABH of $m=2$. The experiment result show in Table 4.

Table 4: The nodes locations of third mode of beam without ABH and beam with ABH of $m=2$

Beam w/o ABH	Numerical	Experimental
x_1/L	0.87	0.84
x_2/L	0.51	0.43

Beam with ABH $m=2$	Numerical	Experimental
x_1/L	0.825	0.8
x_2/L	0.475	0.42

The experimental results also show the node location move towards the fixed end when the

beam is with ABH, which has the same effect as the simulation results.

5.2 Vibration Concentration Effect

Due to the limit of the equipment, it is difficult to directly measure the displacement of the vibration of the beam. The voltage generated by accelerometer can be shown the intensity of vibration energy. In the experiment, with the correction of length, the third natural frequency of the two beams are tested as 750 Hz (5% error compare with the simulation result 791 Hz). Table 5 shows the experiment results of the beams under 900 Hz excitation.

Peak Value (mV)	Beam w/o ABH	Beam with ABH
Anti-node 1	420	405
Anti-node 2	427.5	412.5
Free End	435	465

Table 5: Experimental results of Beam without ABH with correction and the Beam with ABH under 900 Hz excitation

Table 5 shows voltage output increases at the free end with ABH and decreases at the the other two anti-nodes location, comparing with the beam without ABH. The results show the same vibration concentration effect as the simulation results in Section 4.3.

6. Conclusions

This preliminary study on the ABH has shown that the parameters m and a in the power law profile have effect on the cantilever beam vibration. Also the study has shown the vibration concentration effect of ABH. The beam vibration analysis is 1-D field, and the future work will expend the topic in to higher dimensional area.

7. References

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